CHAPTER 12

THREE-PHASE CIRCUITS

Society is never prepared to receive any invention. Every new thing is resisted, and it takes years for the inventor to get people to listen to him and years more before it can be introduced.

—Thomas Alva Edison

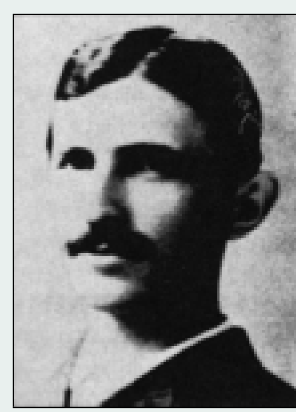
Historical Profiles

Thomas Alva Edison (1847–1931) was perhaps the greatest American inventor. He patented 1093 inventions, including such history-making inventions as the incandescent electric bulb, the phonograph, and the first commercial motion pictures.

Born in Milan, Ohio, the youngest of seven children, Edison received only three months of formal education because he hated school. He was home-schooled by his mother and quickly began to read on his own. In 1868, Edison read one of Faraday's books and found his calling. He moved to Menlo Park, New Jersey, in 1876, where he managed a well-staffed research laboratory. Most of his inventions came out of this laboratory. His laboratory served as a model for modern research organizations. Because of his diverse interests and the overwhelming number of his inventions and patents, Edison began to establish manufacturing companies for making the devices he invented. He designed the first electric power station to supply electric light. Formal electrical engineering education began in the mid-1880s with Edison as a role model and leader.

Nikola Tesla (1856–1943) was a Croatian-American engineer whose inventions—among them the induction motor and the first polyphase ac power system—greatly influenced the settlement of the ac versus dc debate in favor of ac. He was also responsible for the adoption of 60 Hz as the standard for ac power systems in the United States.

Born in Austria-Hungary (now Croatia), to a clergyman, Tesla had an incredible memory and a keen affinity for mathematics. He moved to the United States in 1884 and first worked for Thomas Edison. At that time, the country was in the "battle of the currents" with George Westinghouse (1846–1914) promoting ac and Thomas Edison rigidly leading the dc forces. Tesla left Edison and joined Westinghouse because of his interest in ac. Through Westinghouse, Tesla gained the reputation and acceptance of his polyphase ac generation, transmission, and distribution system. He held 700 patents in his lifetime. His other inventions include high-voltage apparatus (the tesla coil) and a wireless transmission system. The unit of magnetic flux density, the tesla, was named in honor of him.



12.1 INTRODUCTION

So far in this text, we have dealt with single-phase circuits. A single-phase ac power system consists of a generator connected through a pair of wires (a transmission line) to a load. Figure 12.1(a) depicts a single-phase two-wire system, where V_p is the magnitude of the source voltage and ϕ is the phase. What is more common in practice is a single-phase three-wire system, shown in Fig. 12.1(b). It contains two identical sources (equal magnitude and the same phase) which are connected to two loads by two outer wires and the neutral. For example, the normal household system is a single-phase three-wire system because the terminal voltages have the same magnitude and the same phase. Such a system allows the connection of both 120-V and 240-V appliances.

Historical note: Thomas Edison invented a *three-wire system*, using three wires instead of four.

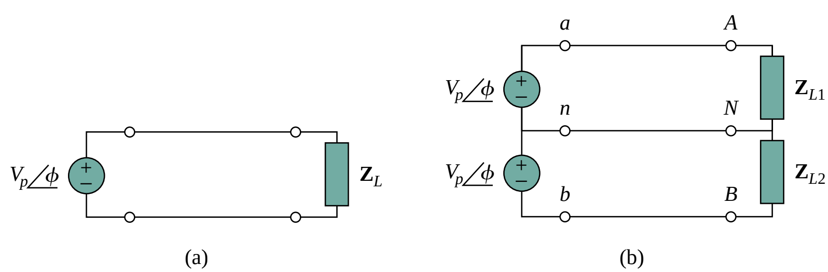


Figure 12.1 Single-phase systems: (a) two-wire type, (b) three-wire type.

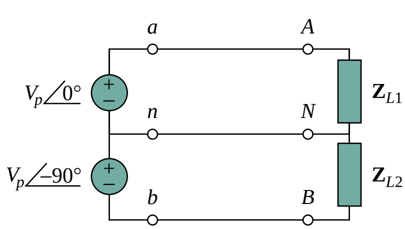


Figure 12.2 Two-phase three-wire system.

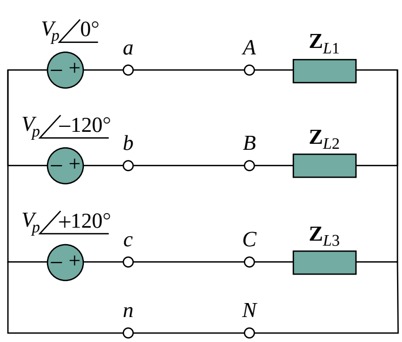


Figure 12.3 Three-phase four-wire system.

Circuits or systems in which the ac sources operate at the same frequency but different phases are known as *polyphase*. Figure 12.2 shows a two-phase three-wire system, and Fig. 12.3 shows a three-phase four-wire system. As distinct from a single-phase system, a two-phase system is produced by a generator consisting of two coils placed perpendicular to each other so that the voltage generated by one lags the other by 90°. By the same token, a three-phase system is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°. Since the three-phase system is by far the most prevalent and most economical polyphase system, discussion in this chapter is mainly on three-phase systems.

Three-phase systems are important for at least three reasons. First, nearly all electric power is generated and distributed in three-phase, at the operating frequency of 60 Hz (or $\omega = 377$ rad/s) in the United States or 50 Hz (or $\omega = 314$ rad/s) in some other parts of the world. When one-phase or two-phase inputs are required, they are taken from the three-phase system rather than generated independently. Even when more than three phases are needed—such as in the aluminum industry, where 48 phases are required for melting purposes—they can be provided by manipulating the three phases supplied. Second, the instantaneous power in a three-phase system can be constant (not pulsating), as we will see in Section 12.7. This results in uniform power transmission and less vibration of three-phase machines. Third, for the same amount of power, the three-phase system is more economical than the single-phase. The amount of wire required for a three-phase system is less than that required for an equivalent single-phase system.

We begin with a discussion of balanced three-phase voltages. Then we analyze each of the four possible configurations of balanced three-phase systems. We also discuss the analysis of unbalanced three-phase systems. We learn how to use *PSpice for Windows* to analyze a balanced or unbalanced three-phase system. Finally, we apply the concepts developed in this chapter to three-phase power measurement and residential electrical wiring.

12.2 BALANCED THREE-PHASE VOLTAGES

Three-phase voltages are often produced with a three-phase ac generator (or alternator) whose cross-sectional view is shown in Fig. 12.4. The generator basically consists of a rotating magnet (called the *rotor*) surrounded by a stationary winding (called the *stator*). Three separate windings or coils with terminals a-a', b-b', and c-c' are physically placed 120° apart around the stator. Terminals a and a', for example, stand for one of the ends of coils going into and the other end coming out of the page. As the rotor rotates, its magnetic field "cuts" the flux from the three coils and induces voltages in the coils. Because the coils are placed 120° apart, the induced voltages in the coils are equal in magnitude but out of phase by 120° (Fig. 12.5). Since each coil can be regarded as a single-phase generator by itself, the three-phase generator can supply power to both single-phase and three-phase loads.

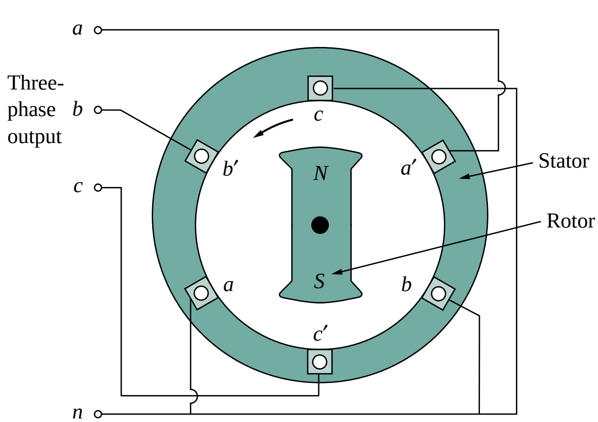


Figure 12.4 A three-phase generator.

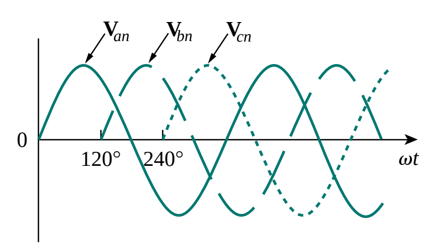


Figure 12.5 The generated voltages are 120° apart from each other.

A typical three-phase system consists of three voltage sources connected to loads by three or four wires (or transmission lines). (Three-phase current sources are very scarce.) A three-phase system is equivalent to three single-phase circuits. The voltage sources can be either wye-connected as shown in Fig. 12.6(a) or delta-connected as in Fig. 12.6(b).

Let us consider the wye-connected voltages in Fig. 12.6(a) for now. The voltages $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, and $\mathbf{V}_{cn}$ are respectively between lines a, b, and

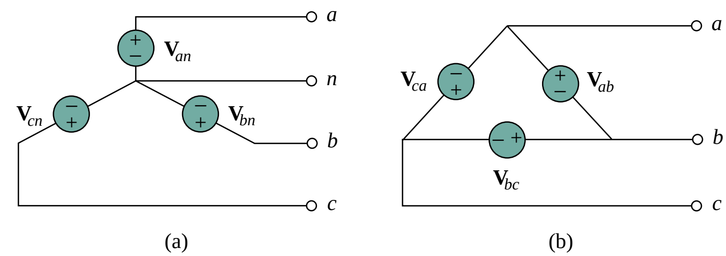


Figure 12.6 Three-phase voltage sources: (a) Y-connected source, (b) Δ-connected source.

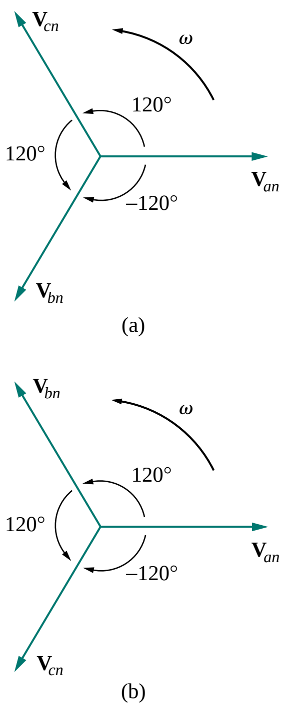


Figure 12.7 Phase sequences: (a) *abc* or positive sequence, (b) *acb* or negative sequence.

As a common tradition in power systems, voltage and current in this chapter are in rms values unless otherwise stated.

c, and the neutral line n. These voltages are called *phase voltages*. If the voltage sources have the same amplitude and frequency ω and are out of phase with each other by 120°, the voltages are said to be *balanced*. This implies that

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0 \tag{12.1}$$

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}| \tag{12.2}$$

Thus,

> Balanced phase voltages are equal in magnitude and are out of phase with each other by 120°.

Since the three-phase voltages are 120° out of phase with each other, there are two possible combinations. One possibility is shown in Fig. 12.7(a) and expressed mathematically as

$$\begin{aligned} \mathbf{V}_{an} &= V_p\underline{/0^\circ} \\ \mathbf{V}_{bn} &= V_p\underline{/-120^\circ} \\ \mathbf{V}_{cn} &= V_p\underline{/-240^\circ} = V_p\underline{/+120^\circ} \end{aligned} \tag{12.3}$$

where V_p is the effective or rms value. This is known as the *abc sequence* or *positive sequence*. In this phase sequence, $\mathbf{V}_{an}$ leads $\mathbf{V}_{bn}$, which in turn leads $\mathbf{V}_{cn}$. This sequence is produced when the rotor in Fig. 12.4 rotates counterclockwise. The other possibility is shown in Fig. 12.7(b) and is given by

$$\begin{aligned} \mathbf{V}_{an} &= V_p\underline{/0^\circ} \\ \mathbf{V}_{cn} &= V_p\underline{/-120^\circ} \\ \mathbf{V}_{bn} &= V_p\underline{/-240^\circ} = V_p\underline{/+120^\circ} \end{aligned} \tag{12.4}$$

This is called the *acb sequence* or *negative sequence*. For this phase sequence, $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$, which in turn leads $\mathbf{V}_{bn}$. The *acb* sequence is produced when the rotor in Fig. 12.4 rotates in the clockwise direction.

It is easy to show that the voltages in Eqs. (12.3) or (12.4) satisfy Eqs. (12.1) and (12.2). For example, from Eq. (12.3),

$$\begin{aligned}\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} &= V_p\underline{/0^\circ} + V_p\underline{/-120^\circ} + V_p\underline{/+120^\circ}\\ &= V_p(1.0 - 0.5 - j0.866 - 0.5 + j0.866) \qquad (12.5)\\ &= 0\end{aligned}$$

> The **phase sequence** is the time order in which the voltages pass through their respective maximum values.

The phase sequence may also be regarded as the order in which the phase voltages reach their peak (or maximum) values with respect to time.

The phase sequence is determined by the order in which the phasors pass through a fixed point in the phase diagram.

In Fig. 12.7(a), as the phasors rotate in the counterclockwise direction with frequency ω, they pass through the horizontal axis in a sequence *abcabca* . . . . Thus, the sequence is *abc* or *bca* or *cab*. Similarly, for the phasors in Fig. 12.7(b), as they rotate in the counterclockwise direction, they pass the horizontal axis in a sequence *acbacba* . . . . This describes the *acb* sequence. The phase sequence is important in three-phase power distribution. It determines the direction of the rotation of a motor connected to the power source, for example.

Reminder: As time increases, each phasor (or sinor) rotates at an angular velocity ω.

Like the generator connections, a three-phase load can be either wye-connected or delta-connected, depending on the end application. Figure 12.8(a) shows a wye-connected load, and Fig. 12.8(b) shows a delta-connected load. The neutral line in Fig. 12.8(a) may or may not be there, depending on whether the system is four- or three-wire. (And, of course, a neutral connection is topologically impossible for a delta connection.) A wye- or delta-connected load is said to be *unbalanced* if the phase impedances are not equal in magnitude or phase.

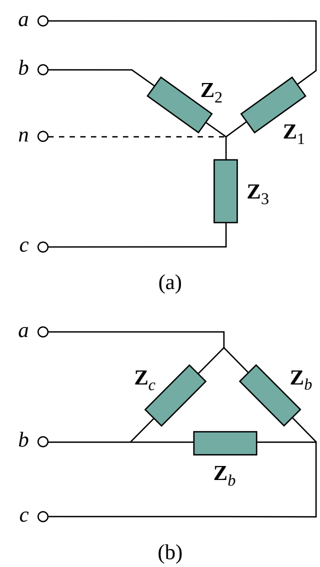


Figure 12.8 Two possible three-phase load configurations: (a) a Y-connected load, (b) a Δ-connected load

> A **balanced load** is one in which the phase impedances are equal in magnitude and in phase.

For a *balanced* wye-connected load,

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y \qquad (12.6)$$

where $\mathbf{Z}_Y$ is the load impedance per phase. For a *balanced* delta-connected load,

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta \qquad (12.7)$$

where $\mathbf{Z}_\Delta$ is the load impedance per phase in this case. We recall from Eq. (9.69) that

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad \text{or} \qquad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta \qquad (12.8)$$

Reminder: A Y-connected load consists of three impedances connected to a neutral node, while a Δ-connected load consists of three impedances connected around a loop. The load is balanced when the three impedances are equal in either case.

so we know that a wye-connected load can be transformed into a delta-connected load, or vice versa, using Eq. (12.8).

Since both the three-phase source and the three-phase load can be either wye- or delta-connected, we have four possible connections:

- Y-Y connection (i.e., Y-connected source with a Y-connected load).
- Y-Δ connection.
- Δ-Δ connection.
- Δ-Y connection.

In subsequent sections, we will consider each of these possible configurations.

It is appropriate to mention here that a balanced delta-connected load is more common than a balanced wye-connected load. This is due to the ease with which loads may be added or removed from each phase of a delta-connected load. This is very difficult with a wye-connected load because the neutral may not be accessible. On the other hand, delta-connected sources are not common in practice because of the circulating current that will result in the delta-mesh if the three-phase voltages are slightly unbalanced.

EXAMPLE 12.1

Determine the phase sequence of the set of voltages

$$v_{an} = 200\cos(\omega t + 10^\circ)$$

$$v_{bn} = 200\cos(\omega t - 230^\circ), \qquad v_{cn} = 200\cos(\omega t - 110^\circ)$$

Solution:

The voltages can be expressed in phasor form as

$$\mathbf{V}_{an} = 200\underline{/10^\circ}, \qquad \mathbf{V}_{bn} = 200\underline{/-230^\circ}, \qquad \mathbf{V}_{cn} = 200\underline{/-110^\circ}$$

We notice that $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ by 120° and $\mathbf{V}_{cn}$ in turn leads $\mathbf{V}_{bn}$ by 120°. Hence, we have an *acb* sequence.

PRACTICE PROBLEM 12.1

Given that $\mathbf{V}_{bn} = 110\underline{/30^\circ}$, find $\mathbf{V}_{an}$ and $\mathbf{V}_{cn}$, assuming a positive (*abc*) sequence.

Answer: $110\underline{/150^\circ}$, $110\underline{/-90^\circ}$.

12.3 BALANCED WYE-WYE CONNECTION

We begin with the Y-Y system, because any balanced three-phase system can be reduced to an equivalent Y-Y system. Therefore, analysis of this system should be regarded as the key to solving all balanced three-phase systems.

> A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.

Consider the balanced four-wire Y-Y system of Fig. 12.9, where a Y-connected load is connected to a Y-connected source. We assume a balanced load so that load impedances are equal. Although the impedance $\mathbf{Z}_Y$ is the total load impedance per phase, it may also be regarded as the sum of the source impedance $\mathbf{Z}_s$, line impedance $\mathbf{Z}_\ell$, and load impedance $\mathbf{Z}_L$ for each phase, since these impedances are in series. As illustrated in Fig. 12.9, $\mathbf{Z}_s$ denotes the internal impedance of the phase winding of the generator; $\mathbf{Z}_\ell$ is the impedance of the line joining a phase of the source with a phase of the load; $\mathbf{Z}_L$ is the impedance of each phase of the load; and $\mathbf{Z}_n$ is the impedance of the neutral line. Thus, in general

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L \tag{12.9}$$

$\mathbf{Z}_s$ and $\mathbf{Z}_\ell$ are often very small compared with $\mathbf{Z}_L$, so one can assume that $\mathbf{Z}_Y = \mathbf{Z}_L$ if no source or line impedance is given. In any event, by lumping the impedances together, the Y-Y system in Fig. 12.9 can be simplified to that shown in Fig. 12.10.

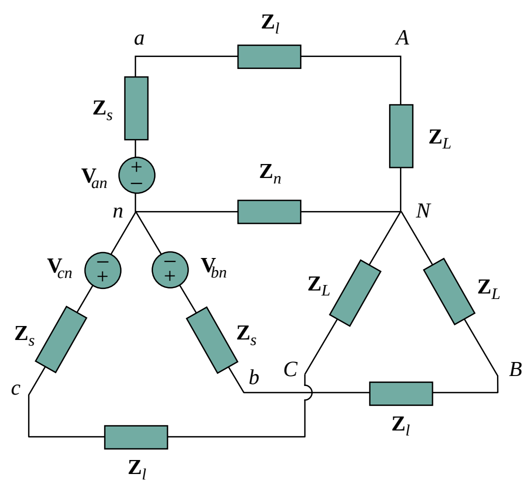


Figure 12.9 A balanced Y-Y system, showing the source, line, and load impedances.

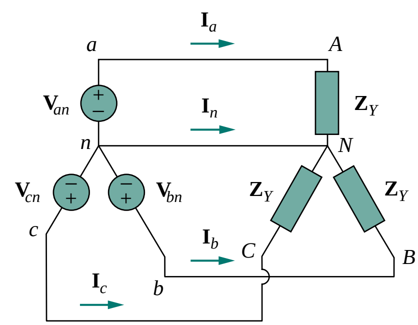


Figure 12.10 Balanced Y-Y connection.

Assuming the positive sequence, the *phase* voltages (or line-to-neutral voltages) are

$$\begin{gathered}\mathbf{V}_{an} = V_p\angle 0^\circ \\ \mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ\end{gathered} \tag{12.10}$$

The *line-to-line* voltages or simply *line* voltages $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$ are related to the phase voltages. For example,

$$\begin{aligned}\mathbf{V}_{ab} &= \mathbf{V}_{an} + \mathbf{V}_{nb} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\angle 0^\circ - V_p\angle -120^\circ \\ &= V_p\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V_p\angle 30^\circ\end{aligned} \tag{12.11a}$$

Similarly, we can obtain

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\angle -90^\circ \tag{12.11b}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\angle -210^\circ \tag{12.11c}$$

Thus, the magnitude of the line voltages V_L is $\sqrt{3}$ times the magnitude of the phase voltages V_p, or

$$V_L = \sqrt{3} V_p \tag{12.12}$$

where

$$V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}| \tag{12.13}$$

and

$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}| \tag{12.14}$$

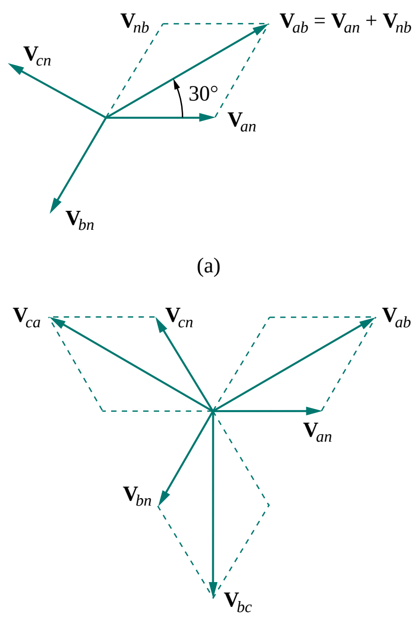


Figure 12.11 Phasor diagrams illustrating the relationship between line voltages and phase voltages.

Also the line voltages lead their corresponding phase voltages by 30°. Figure 12.11(a) illustrates this. Figure 12.11(a) also shows how to determine $\mathbf{V}_{ab}$ from the phase voltages, while Fig. 12.11(b) shows the same for the three line voltages. Notice that $\mathbf{V}_{ab}$ leads $\mathbf{V}_{bc}$ by 120°, and $\mathbf{V}_{bc}$ leads $\mathbf{V}_{ca}$ by 120°, so that the line voltages sum up to zero as do the phase voltages.

Applying KVL to each phase in Fig. 12.10, we obtain the line currents as

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -120^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -120^\circ$$
$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -240^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -240^\circ \tag{12.15}$$

We can readily infer that the line currents add up to zero,

$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0 \tag{12.16}$$

so that

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0 \tag{12.17a}$$

or

$$\mathbf{V}_{nN} = \mathbf{Z}_n \mathbf{I}_n = 0 \tag{12.17b}$$

that is, the voltage across the neutral wire is zero. The neutral line can thus be removed without affecting the system. In fact, in long distance power transmission, conductors in multiples of three are used with the earth itself acting as the neutral conductor. Power systems designed in this way are well grounded at all critical points to ensure safety.

While the *line* current is the current in each line, the *phase* current is the current in each phase of the source or load. In the Y-Y system, the line current is the same as the phase current. We will use single subscripts for line currents because it is natural and conventional to assume that line currents flow from the source to the load.

An alternative way of analyzing a balanced Y-Y system is to do so on a "per phase" basis. We look at one phase, say phase a, and analyze the single-phase equivalent circuit in Fig. 12.12. The single-phase analysis yields the line current $\mathbf{I}_a$ as

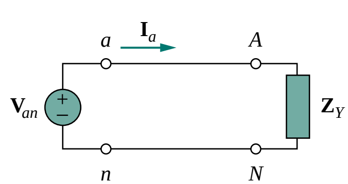


Figure 12.12 A single-phase equivalent circuit.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} \tag{12.18}$$

From $\mathbf{I}_a$, we use the phase sequence to obtain other line currents. Thus, as long as the system is balanced, we need only analyze one phase. We may do this even if the neutral line is absent, as in the three-wire system.

EXAMPLE 12.2

Calculate the line currents in the three-wire Y-Y system of Fig. 12.13.

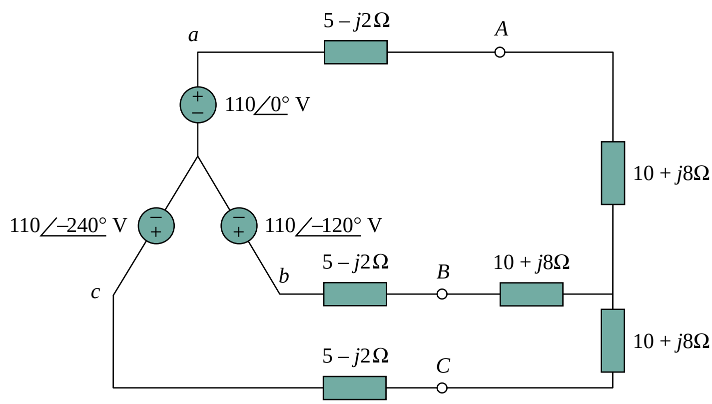


Figure 12.13 Three-wire Y-Y system; for Example 12.2.

Solution:

The three-phase circuit in Fig. 12.13 is balanced; we may replace it with its single-phase equivalent circuit such as in Fig. 12.12. We obtain $\mathbf{I}_a$ from the single-phase analysis as

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155\underline{/21.8^\circ}$. Hence,

$$\mathbf{I}_a = \frac{110\underline{/0^\circ}}{16.155\underline{/21.8^\circ}} = 6.81\underline{/-21.8^\circ} \text{ A}$$

Since the source voltages in Fig. 12.13 are in positive sequence and the line currents are also in positive sequence,

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 6.81\underline{/-141.8^\circ} \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/-240^\circ} = 6.81\underline{/-261.8^\circ} \text{ A} = 6.81\underline{/98.2^\circ} \text{ A}$$

PRACTICE PROBLEM 12.2

A Y-connected balanced three-phase generator with an impedance of $0.4 + j0.3$ Ω per phase is connected to a Y-connected balanced load with an impedance of $24 + j19$ Ω per phase. The line joining the generator and the load has an impedance of $0.6 + j0.7$ Ω per phase. Assuming a positive sequence for the source voltages and that $\mathbf{V}_{an} = 120\underline{/30^\circ}$ V, find: (a) the line voltages, (b) the line currents.

Answer: (a) $207.85\angle 60^\circ$ V, $207.85\angle -60^\circ$ V, $207.85\angle -180^\circ$ V,
(b) $3.75\angle -8.66^\circ$ A, $3.75\angle -128.66^\circ$ A, $3.75\angle -248.66^\circ$ A.

12.4 BALANCED WYE-DELTA CONNECTION

> A **balanced Y-Δ system** consists of a balanced Y-connected source feeding a balanced Δ-connected load.

This is perhaps the most practical three-phase system, as the three-phase sources are usually Y-connected while the three-phase loads are usually Δ-connected.

The balanced Y-delta system is shown in Fig. 12.14, where the source is wye-connected and the load is Δ-connected. There is, of course, no neutral connection from source to load for this case. Assuming the positive sequence, the phase voltages are again

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ \tag{12.19}$$

As shown in Section 12.3, the line voltages are

$$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90^\circ = \mathbf{V}_{BC}$$
$$\mathbf{V}_{ca} = \sqrt{3}V_p\angle -210^\circ = \mathbf{V}_{CA} \tag{12.20}$$

showing that the line voltages are equal to the voltages across the load impedances for this system configuration. From these voltages, we can obtain the phase currents as

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} \tag{12.21}$$

These currents have the same magnitude but are out of phase with each other by 120°.

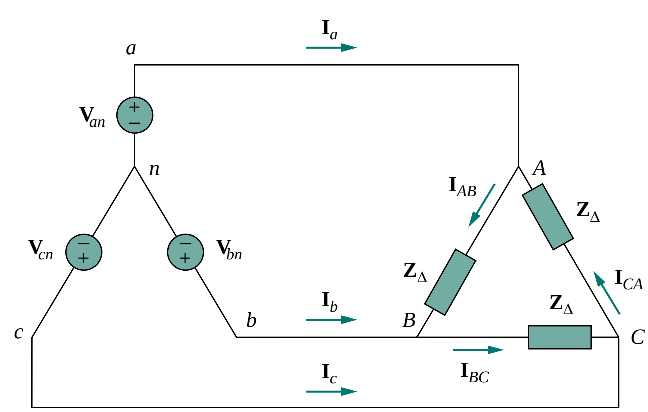


Figure 12.14 Balanced Y-Δ connection.

Another way to get these phase currents is to apply KVL. For example, applying KVL around loop $aABbna$ gives

$$-\mathbf{V}_{an} + \mathbf{Z}_{\Delta}\mathbf{I}_{AB} + \mathbf{V}_{bn} = 0$$

or

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{an} - \mathbf{V}_{bn}}{\mathbf{Z}_{\Delta}} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{\Delta}} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} \tag{12.22}$$

which is the same as Eq. (12.21). This is the more general way of finding the phase currents.

The line currents are obtained from the phase currents by applying KCL at nodes A, B, and C. Thus,

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} \tag{12.23}$$

Since $\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/-240^\circ}$,

$$\begin{aligned}\mathbf{I}_a &= \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\underline{/-240^\circ}) \\ &= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ}\end{aligned} \tag{12.24}$$

showing that the magnitude I_L of the line current is $\sqrt{3}$ times the magnitude I_p of the phase current, or

$$\boxed{I_L = \sqrt{3}I_p} \tag{12.25}$$

where

$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c| \tag{12.26}$$

and

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}| \tag{12.27}$$

Also, the line currents lag the corresponding phase currents by 30°, assuming the positive sequence. Figure 12.15 is a phasor diagram illustrating the relationship between the phase and line currents.

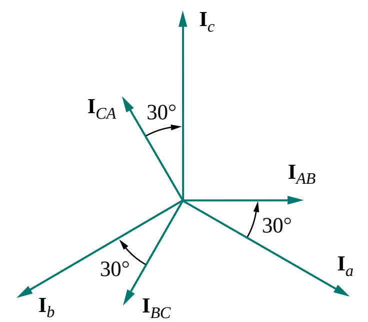


Figure 12.15 Phasor diagram illustrating the relationship between phase and line currents.

An alternative way of analyzing the Y-Δ circuit is to transform the Δ-connected load to an equivalent Y-connected load. Using the Δ-Y transformation formula in Eq. (9.69),

$$\boxed{\mathbf{Z}_Y = \frac{\mathbf{Z}_{\Delta}}{3}} \tag{12.28}$$

After this transformation, we now have a Y-Y system as in Fig. 12.10. The three-phase Y-Δ system in Fig. 12.14 can be replaced by the single-phase equivalent circuit in Fig. 12.16. This allows us to calculate only the line currents. The phase currents are obtained using Eq. (12.25) and utilizing the fact that each of the phase currents leads the corresponding line current by 30°.

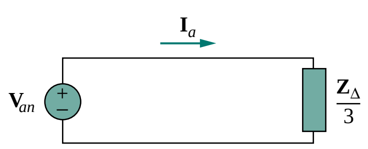


Figure 12.16 A single-phase equivalent circuit of a balanced Y-Δ circuit.

EXAMPLE 12.3

A balanced abc-sequence Y-connected source with $\mathbf{V}_{an} = 100\underline{/10^\circ}$ V is connected to a Δ-connected balanced load $(8 + j4)\ \Omega$ per phase. Calculate the phase and line currents.

Solution:

This can be solved in two ways.

METHOD 1 The load impedance is

$$\mathbf{Z}_\Delta = 8 + j4 = 8.944\underline{/26.57^\circ}\ \Omega$$

If the phase voltage $\mathbf{V}_{an} = 100\underline{/10^\circ}$, then the line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\underline{/30^\circ} = 100\sqrt{3}\underline{/10^\circ + 30^\circ} = \mathbf{V}_{AB}$$

or

$$\mathbf{V}_{AB} = 173.2\underline{/40^\circ}\ \text{V}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{173.2\underline{/40^\circ}}{8.944\underline{/26.57^\circ}} = 19.36\underline{/13.43^\circ}\ \text{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\underline{/-120^\circ} = 19.36\underline{/-106.57^\circ}\ \text{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/+120^\circ} = 19.36\underline{/133.43^\circ}\ \text{A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ} = \sqrt{3}(19.36)\underline{/13.43^\circ - 30^\circ}$$
$$= 33.53\underline{/-16.57^\circ}\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 33.53\underline{/-136.57^\circ}\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ} = 33.53\underline{/103.43^\circ}\ \text{A}$$

METHOD 2 Alternatively, using single-phase analysis,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\Delta/3} = \frac{100\underline{/10^\circ}}{2.981\underline{/26.57^\circ}} = 33.54\underline{/-16.57^\circ}\ \text{A}$$

as above. Other line currents are obtained using the *abc* phase sequence.

PRACTICE PROBLEM 12.3

One line voltage of a balanced Y-connected source is $\mathbf{V}_{AB} = 180\underline{/-20^\circ}$ V. If the source is connected to a Δ-connected load of $20\underline{/40^\circ}\ \Omega$, find the phase and line currents. Assume the *abc* sequence.

Answer: $9\underline{/-60^\circ}$, $9\underline{/-180^\circ}$, $9\underline{/60^\circ}$, $15.59\underline{/-90^\circ}$, $15.59\underline{/-210^\circ}$, $15.59\underline{/30^\circ}$ A.

12.5 BALANCED DELTA-DELTA CONNECTION

A balanced Δ-Δ system is one in which both the balanced source and balanced load are Δ-connected.

The source as well as the load may be delta-connected as shown in Fig. 12.17. Our goal is to obtain the phase and line currents as usual. Assuming a positive sequence, the phase voltages for a delta-connected source are

$$\mathbf{V}_{ab} = V_p\underline{/0^\circ}$$
$$\mathbf{V}_{bc} = V_p\underline{/-120^\circ}, \qquad \mathbf{V}_{ca} = V_p\underline{/+120^\circ} \tag{12.29}$$

The line voltages are the same as the phase voltages. From Fig. 12.17, assuming there is no line impedances, the phase voltages of the delta-connected source are equal to the voltages across the impedances; that is,

$$\mathbf{V}_{ab} = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \qquad \mathbf{V}_{ca} = \mathbf{V}_{CA} \tag{12.30}$$

Hence, the phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_\Delta}$$
$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_\Delta} \tag{12.31}$$

Since the load is delta-connected just as in the previous section, some of the formulas derived there apply here. The line currents are obtained from the phase currents by applying KCL at nodes A, B, and C, as we did in the previous section:

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} \tag{12.32}$$

Also, as shown in the last section, each line current lags the corresponding phase current by 30°; the magnitude I_L of the line current is $\sqrt{3}$ times the magnitude I_p of the phase current,

$$I_L = \sqrt{3}I_p \tag{12.33}$$

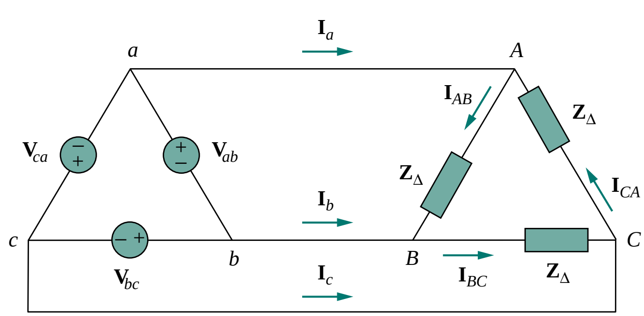


Figure 12.17 A balanced Δ-Δ connection.

An alternative way of analyzing the Δ-Δ circuit is to convert both the source and the load to their Y equivalents. We already know that $\mathbf{Z}_Y = \mathbf{Z}_\Delta/3$. To convert a Δ-connected source to a Y-connected source, see the next section.

EXAMPLE 12.4

A balanced Δ-connected load having an impedance $20 - j15\ \Omega$ is connected to a Δ-connected, positive-sequence generator having $\mathbf{V}_{ab} = 330\underline{/0^\circ}$ V. Calculate the phase currents of the load and the line currents.

Solution:

The load impedance per phase is

$$\mathbf{Z}_\Delta = 20 - j15 = 25\underline{/-36.87^\circ}\ \Omega$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{330\underline{/0^\circ}}{25\underline{/-36.87}} = 13.2\underline{/36.87^\circ}\ \text{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\underline{/-120^\circ} = 13.2\underline{/-83.13^\circ}\ \text{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/+120^\circ} = 13.2\underline{/156.87^\circ}\ \text{A}$$

For a delta load, the line current always lags the corresponding phase current by 30° and has a magnitude $\sqrt{3}$ times that of the phase current. Hence, the line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ} = (13.2\underline{/36.87^\circ})(\sqrt{3}\underline{/-30^\circ})$$
$$= 22.86\underline{/6.87^\circ}\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 22.86\underline{/-113.13^\circ}\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ} = 22.86\underline{/126.87^\circ}\ \text{A}$$

PRACTICE PROBLEM 12.4

A positive-sequence, balanced Δ-connected source supplies a balanced Δ-connected load. If the impedance per phase of the load is $18 + j12\ \Omega$ and $\mathbf{I}_a = 22.5\underline{/35^\circ}$ A, find $\mathbf{I}_{AB}$ and $\mathbf{V}_{AB}$.

Answer: $13\underline{/65^\circ}$ A, $281.2\underline{/98.69^\circ}$ V.

12.6 BALANCED DELTA-WYE CONNECTION

A balanced Δ-Y system consists of a balanced Δ-connected source feeding a balanced Y-connected load.

Consider the Δ-Y circuit in Fig. 12.18. Again, assuming the *abc* sequence, the phase voltages of a delta-connected source are

$$\mathbf{V}_{ab} = V_p\underline{/0^\circ}, \qquad \mathbf{V}_{bc} = V_p\underline{/-120^\circ}$$
$$\mathbf{V}_{ca} = V_p\underline{/+120^\circ} \tag{12.34}$$

These are also the line voltages as well as the phase voltages.

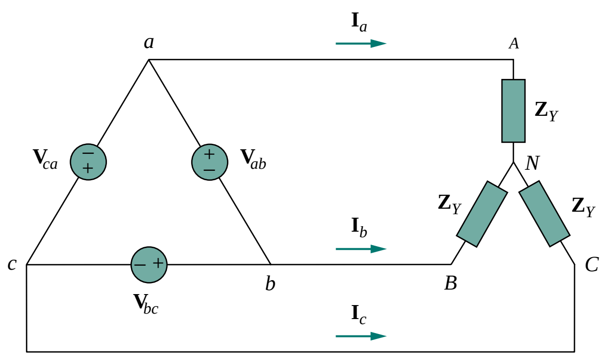


Figure 12.18 A balanced Δ-Y connection.

We can obtain the line currents in many ways. One way is to apply KVL to loop $aANBba$ in Fig. 12.18, writing

$$-\mathbf{V}_{ab} + \mathbf{Z}_Y\mathbf{I}_a - \mathbf{Z}_Y\mathbf{I}_b = 0$$

or

$$\mathbf{Z}_Y(\mathbf{I}_a - \mathbf{I}_b) = \mathbf{V}_{ab} = V_p\underline{/0^\circ}$$

Thus,

$$\mathbf{I}_a - \mathbf{I}_b = \frac{V_p\underline{/0^\circ}}{\mathbf{Z}_Y} \tag{12.35}$$

But $\mathbf{I}_b$ lags $\mathbf{I}_a$ by 120°, since we assumed the abc sequence; that is, $\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ}$. Hence,

$$\begin{aligned}\mathbf{I}_a - \mathbf{I}_b &= \mathbf{I}_a(1 - 1\underline{/-120^\circ}) \\ &= \mathbf{I}_a\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \mathbf{I}_a\sqrt{3}\underline{/30^\circ}\end{aligned} \tag{12.36}$$

Substituting Eq. (12.36) into Eq. (12.35) gives

$$\mathbf{I}_a = \frac{V_p/\sqrt{3}\underline{/-30^\circ}}{\mathbf{Z}_Y} \tag{12.37}$$

From this, we obtain the other line currents $\mathbf{I}_b$ and $\mathbf{I}_c$ using the positive phase sequence, i.e., $\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ}$, $\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ}$. The phase currents are equal to the line currents.

Another way to obtain the line currents is to replace the delta-connected source with its equivalent wye-connected source, as shown in Fig. 12.19. In Section 12.3, we found that the line-to-line voltages of a wye-connected source lead their corresponding phase voltages by 30°. Therefore, we obtain each phase voltage of the equivalent wye-connected source by dividing the corresponding line voltage of the delta-connected source by $\sqrt{3}$ and shifting its phase by $-30°$. Thus, the equivalent wye-connected source has the phase voltages

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\underline{/-30^\circ}$$
$$\mathbf{V}_{bn} = \frac{V_p}{\sqrt{3}}\underline{/-150^\circ}, \qquad \mathbf{V}_{cn} = \frac{V_p}{\sqrt{3}}\underline{/+90^\circ} \tag{12.38}$$

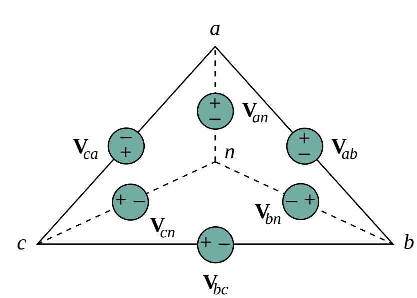


Figure 12.19 Transforming a Δ-connected source to an equivalent Y-connected source.

If the delta-connected source has source impedance $\mathbf{Z}_s$ per phase, the equivalent wye-connected source will have a source impedance of $\mathbf{Z}_s/3$ per phase, according to Eq. (9.69).

Once the source is transformed to wye, the circuit becomes a wye-wye system. Therefore, we can use the equivalent single-phase circuit shown in Fig. 12.20, from which the line current for phase a is

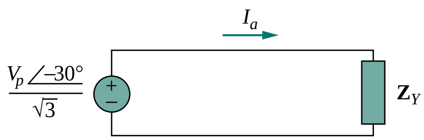


Figure 12.20 The single-phase equivalent circuit.

$$\mathbf{I}_a = \frac{V_p/\sqrt{3}\angle -30^\circ}{\mathbf{Z}_Y} \qquad (12.39)$$

which is the same as Eq. (12.37).

Alternatively, we may transform the wye-connected load to an equivalent delta-connected load. This results in a delta-delta system, which can be analyzed as in Section 12.5. Note that

$$\mathbf{V}_{AN} = \mathbf{I}_a\mathbf{Z}_Y = \frac{V_p}{\sqrt{3}}\angle -30^\circ \qquad (12.40)$$

$$\mathbf{V}_{BN} = \mathbf{V}_{AN}\angle -120^\circ, \qquad \mathbf{V}_{CN} = \mathbf{V}_{AN}\angle +120^\circ$$

As stated earlier, the delta-connected load is more desirable than the wye-connected load. It is easier to alter the loads in any one phase of the delta-connected loads, as the individual loads are connected directly across the lines. However, the delta-connected source is hardly used in practice, because any slight imbalance in the phase voltages will result in unwanted circulating currents.

Table 12.1 presents a summary of the formulas for phase currents and voltages and line currents and voltages for the four connections. Students are advised not to memorize the formulas but to understand how they are derived. The formulas can always be obtained by directly applying KCL and KVL to the appropriate three-phase circuits.

TABLE 12.1 Summary of phase and line voltages/currents for balanced three-phase systems[1].

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

[1] *Positive or abc sequence is assumed.*

TABLE 12.1 (*continued*)

Connection	Phase voltages/currents	Line voltages/currents
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p\angle +120^\circ$	
	$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p\angle +120^\circ$	
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

EXAMPLE 12.5

A balanced Y-connected load with a phase resistance of 40 Ω and a reactance of 25 Ω is supplied by a balanced, positive sequence Δ-connected source with a line voltage of 210 V. Calculate the phase currents. Use $\mathbf{V}_{ab}$ as reference.

Solution:

The load impedance is

$$\mathbf{Z}_Y = 40 + j25 = 47.17\angle 32^\circ\ \Omega$$

and the source voltage is

$$\mathbf{V}_{ab} = 210\angle 0^\circ\ \text{V}$$

When the Δ-connected source is transformed to a Y-connected source,

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle -30^\circ = 121.2\angle -30^\circ\ \text{V}$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2\angle -30^\circ}{47.12\angle 32^\circ} = 2.57\angle -62^\circ\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 2.57\angle -182^\circ\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120^\circ = 2.57\angle 58^\circ\ \text{A}$$

which are the same as the phase currents.

PRACTICE PROBLEM 12.5

In a balanced Δ-Y circuit, $\mathbf{V}_{ab} = 240\underline{/15^\circ}$ and $\mathbf{Z}_Y = (12 + j15)\ \Omega$. Calculate the line currents.

Answer: $7.21\underline{/-66.34^\circ}$, $7.21\underline{/-186.34^\circ}$, $7.21\underline{/53.66^\circ}$ A.

12.7 POWER IN A BALANCED SYSTEM

Let us now consider the power in a balanced three-phase system. We begin by examining the instantaneous power absorbed by the load. This requires that the analysis be done in the time domain. For a Y-connected load, the phase voltages are

$$\begin{gathered} v_{AN} = \sqrt{2}V_p \cos\omega t, \qquad v_{BN} = \sqrt{2}V_p \cos(\omega t - 120^\circ) \\ v_{CN} = \sqrt{2}V_p \cos(\omega t + 120^\circ) \end{gathered} \tag{12.41}$$

where the factor $\sqrt{2}$ is necessary because V_p has been defined as the rms value of the phase voltage. If $\mathbf{Z}_Y = Z\underline{/\theta}$, the phase currents lag behind their corresponding phase voltages by θ. Thus,

$$\begin{gathered} i_a = \sqrt{2}I_p \cos(\omega t - \theta), \qquad i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120^\circ) \\ i_c = \sqrt{2}I_p \cos(\omega t - \theta + 120^\circ) \end{gathered} \tag{12.42}$$

where I_p is the rms value of the phase current. The total instantaneous power in the load is the sum of the instantaneous powers in the three phases; that is,

$$\begin{aligned} p = p_a + p_b + p_c &= v_{AN}i_a + v_{BN}i_b + v_{CN}i_c \\ &= 2V_pI_p[\cos\omega t \cos(\omega t - \theta) \\ &\quad + \cos(\omega t - 120^\circ)\cos(\omega t - \theta - 120^\circ) \\ &\quad + \cos(\omega t + 120^\circ)\cos(\omega t - \theta + 120^\circ)] \end{aligned} \tag{12.43}$$

Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)] \tag{12.44}$$

gives

$$\begin{aligned} p &= V_pI_p[3\cos\theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240^\circ) \\ &\qquad + \cos(2\omega t - \theta + 240^\circ)] \\ &= V_pI_p[3\cos\theta + \cos\alpha + \cos\alpha\cos 240^\circ + \sin\alpha\sin 240^\circ \\ &\qquad + \cos\alpha\cos 240^\circ - \sin\alpha\sin 240^\circ] \\ &\qquad \text{where } \alpha = 2\omega t - \theta \\ &= V_pI_p\left[3\cos\theta + \cos\alpha + 2\left(-\frac{1}{2}\right)\cos\alpha\right] = 3V_pI_p\cos\theta \end{aligned} \tag{12.45}$$

Thus the total instantaneous power in a balanced three-phase system is constant—it does not change with time as the instantaneous power of each phase does. This result is true whether the load is Y- or Δ-connected.

This is one important reason for using a three-phase system to generate and distribute power. We will look into another reason a little later.

Since the total instantaneous power is independent of time, the average power per phase P_p for either the Δ-connected load or the Y-connected load is $p/3$, or

$$P_p = V_p I_p \cos\theta \tag{12.46}$$

and the reactive power per phase is

$$Q_p = V_p I_p \sin\theta \tag{12.47}$$

The apparent power per phase is

$$S_p = V_p I_p \tag{12.48}$$

The complex power per phase is

$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^* \tag{12.49}$$

where $\mathbf{V}_p$ and $\mathbf{I}_p$ are the phase voltage and phase current with magnitudes V_p and I_p, respectively. The total average power is the sum of the average powers in the phases:

$$P = P_a + P_b + P_c = 3P_p = 3V_p I_p \cos\theta = \sqrt{3} V_L I_L \cos\theta \tag{12.50}$$

For a Y-connected load, $I_L = I_p$ but $V_L = \sqrt{3}V_p$, whereas for a Δ-connected load, $I_L = \sqrt{3}I_p$ but $V_L = V_p$. Thus, Eq. (12.50) applies for both Y-connected and Δ-connected loads. Similarly, the total reactive power is

$$Q = 3V_p I_p \sin\theta = 3Q_p = \sqrt{3} V_L I_L \sin\theta \tag{12.51}$$

and the total complex power is

$$\boxed{\mathbf{S} = 3\mathbf{S}_p = 3\mathbf{V}_p \mathbf{I}_p^* = 3I_p^2 \mathbf{Z}_p = \frac{3V_p^2}{\mathbf{Z}_p^*}} \tag{12.52}$$

where $\mathbf{Z}_p = Z_p \angle\theta$ is the load impedance per phase. ($\mathbf{Z}_p$ could be $\mathbf{Z}_Y$ or $\mathbf{Z}_\Delta$.) Alternatively, we may write Eq. (12.52) as

$$\boxed{\mathbf{S} = P + jQ = \sqrt{3} V_L I_L \angle\theta} \tag{12.53}$$

Remember that V_p, I_p, V_L, and I_L are all rms values and that θ is the angle of the load impedance or the angle between the phase voltage and the phase current.

A second major advantage of three-phase systems for power distribution is that the three-phase system uses a lesser amount of wire than the single-phase system for the same line voltage V_L and the same absorbed power P_L. We will compare these cases and assume in both that the wires are of the same material (e.g., copper with resistivity ρ), of the same length ℓ, and that the loads are resistive (i.e., unity power factor). For the two-wire single-phase system in Fig. 12.21(a), $I_L = P_L/V_L$, so the power loss in the two wires is

$$P_{\text{loss}} = 2I_L^2 R = 2R\frac{P_L^2}{V_L^2} \tag{12.54}$$

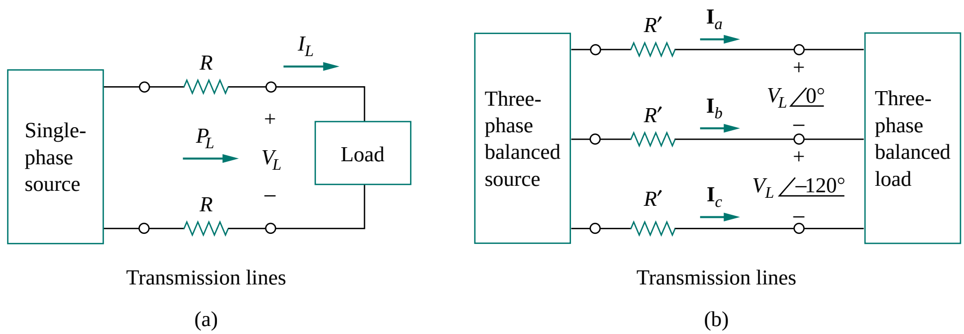


Figure 12.21 Comparing the power loss in (a) a single-phase system, and (b) a three-phase system.

For the three-wire three-phase system in Fig. 12.21(b), $I'_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c| = P_L/\sqrt{3}V_L$ from Eq. (12.50). The power loss in the three wires is

$$P'_{\text{loss}} = 3(I'_L)^2 R' = 3R'\frac{P_L^2}{3V_L^2} = R'\frac{P_L^2}{V_L^2} \tag{12.55}$$

Equations (12.54) and (12.55) show that for the same total power delivered P_L and same line voltage V_L,

$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2R}{R'} \tag{12.56}$$

But from Chapter 2, $R = \rho\ell/\pi r^2$ and $R' = \rho\ell/\pi r'^2$, where r and r' are the radii of the wires. Thus,

$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2r'^2}{r^2} \tag{12.57}$$

If the same power loss is tolerated in both systems, then $r^2 = 2r'^2$. The ratio of material required is determined by the number of wires and their volumes, so

$$\begin{aligned}\frac{\text{Material for single-phase}}{\text{Material for three-phase}} &= \frac{2(\pi r^2\ell)}{3(\pi r'^2\ell)} = \frac{2r^2}{3r'^2}\\ &= \frac{2}{3}(2) = 1.333\end{aligned} \tag{12.58}$$

since $r^2 = 2r'^2$. Equation (12.58) shows that the single-phase system uses 33 percent more material than the three-phase system or that the three-phase system uses only 75 percent of the material used in the equivalent single-phase system. In other words, considerably less material is needed to deliver the same power with a three-phase system than is required for a single-phase system.

EXAMPLE 12.6

Refer to the circuit in Fig. 12.13 (in Example 12.2). Determine the total average power, reactive power, and complex power at the source and at the load.

Solution:

It is sufficient to consider one phase, as the system is balanced. For phase a,

$$\mathbf{V}_p = 110\underline{/0^\circ}\text{ V} \qquad \text{and} \qquad \mathbf{I}_p = 6.81\underline{/-21.8^\circ}\text{ A}$$

Thus, at the source, the complex power supplied is

$$\mathbf{S}_s = -3\mathbf{V}_p\mathbf{I}_p^* = 3(110\underline{/0^\circ})(6.81\underline{/21.8^\circ})$$
$$= -2247\underline{/21.8^\circ} = -(2087 + j834.6)\text{ VA}$$

The real or average power supplied is −2087 W and the reactive power is −834.6 VAR.

At the load, the complex power absorbed is

$$\mathbf{S}_L = 3|\mathbf{I}_p|^2\mathbf{Z}_p$$

where $\mathbf{Z}_p = 10 + j8 = 12.81\underline{/38.66^\circ}$ and $\mathbf{I}_p = \mathbf{I}_a = 6.81\underline{/-21.8^\circ}$. Hence

$$\mathbf{S}_L = 3(6.81)^2 12.81\underline{/38.66^\circ} = 1782\underline{/38.66}$$
$$= (1392 + j1113)\text{ VA}$$

The real power absorbed is 1391.7 W and the reactive power absorbed is 1113.3 VAR. The difference between the two complex powers is absorbed by the line impedance $(5 - j2)\ \Omega$. To show that this is the case, we find the complex power absorbed by the line as

$$\mathbf{S}_\ell = 3|\mathbf{I}_p|^2\mathbf{Z}_\ell = 3(6.81)^2(5 - j2) = 695.6 - j278.3\text{ VA}$$

which is the difference between $\mathbf{S}_s$ and $\mathbf{S}_L$, that is, $\mathbf{S}_s + \mathbf{S}_\ell + \mathbf{S}_L = 0$, as expected.

PRACTICE PROBLEM 12.6

For the Y-Y circuit in Practice Prob. 12.2, calculate the complex power at the source and at the load.

Answer: $(1054 + j843.3)$ VA, $(1012 + j801.6)$ VA.

EXAMPLE 12.7

A three-phase motor can be regarded as a balanced Y-load. A three-phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor.

Solution:

The apparent power is

$$S = \sqrt{3}V_L I_L = \sqrt{3}(220)(18.2) = 6935.13\text{ VA}$$

Since the real power is

$$P = S\cos\theta = 5600\text{ W}$$

the power factor is

$$\text{pf} = \cos\theta = \frac{P}{S} = \frac{5600}{6935.13} = 0.8075$$

PRACTICE PROBLEM 12.7

Calculate the line current required for a 30-kW three-phase motor having a power factor of 0.85 lagging if it is connected to a balanced source with a line voltage of 440 V.

Answer: 50.94 A.

EXAMPLE 12.8

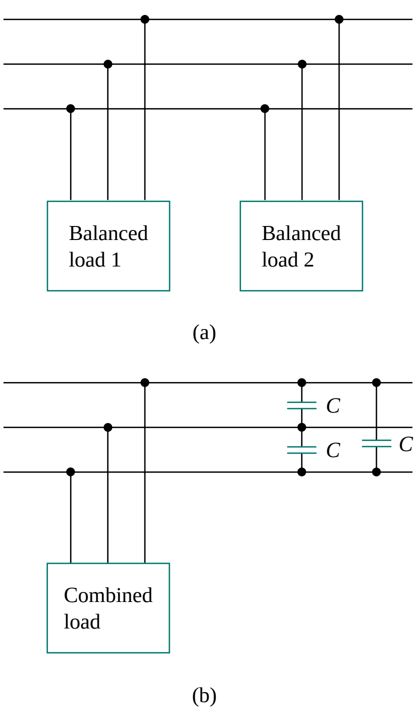


Figure 12.22 For Example 12.8: (a) The original balanced loads, (b) the combined load with improved power factor.

Two balanced loads are connected to a 240-kV rms 60-Hz line, as shown in Fig. 12.22(a). Load 1 draws 30 kW at a power factor of 0.6 lagging, while load 2 draws 45 kVAR at a power factor of 0.8 lagging. Assuming the *abc* sequence, determine: (a) the complex, real, and reactive powers absorbed by the combined load, (b) the line currents, and (c) the kVAR rating of the three capacitors Δ-connected in parallel with the load that will raise the power factor to 0.9 lagging and the capacitance of each capacitor.

Solution:

(a) For load 1, given that $P_1 = 30$ kW and $\cos\theta_1 = 0.6$, then $\sin\theta_1 = 0.8$. Hence,

$$S_1 = \frac{P_1}{\cos\theta_1} = \frac{30 \text{ kW}}{0.6} = 50 \text{ kVA}$$

and $Q_1 = S_1 \sin\theta_1 = 50(0.8) = 40$ kVAR. Thus, the complex power due to load 1 is

$$\mathbf{S}_1 = P_1 + jQ_1 = 30 + j40 \text{ kVA} \tag{12.8.1}$$

For load 2, if $Q_2 = 45$ kVAR and $\cos\theta_2 = 0.8$, then $\sin\theta_2 = 0.6$. We find

$$S_2 = \frac{Q_2}{\sin\theta_2} = \frac{45 \text{ kVA}}{0.6} = 75 \text{ kVA}$$

and $P_2 = S_2 \cos\theta_2 = 75(0.8) = 60$ kW. Therefore the complex power due to load 2 is

$$\mathbf{S}_2 = P_2 + jQ_2 = 60 + j45 \text{ kVA} \tag{12.8.2}$$

From Eqs. (12.8.1) and (12.8.2), the total complex power absorbed by the load is

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = 90 + j85 \text{ kVA} = 123.8\underline{/43.36^\circ} \text{ kVA} \tag{12.8.3}$$

which has a power factor of $\cos 43.36^\circ = 0.727$ lagging. The real power is then 90 kW, while the reactive power is 85 kVAR.

(b) Since $S = \sqrt{3}V_L I_L$, the line current is

$$I_L = \frac{S}{\sqrt{3}V_L} \tag{12.8.4}$$

We apply this to each load, keeping in mind that for both loads, $V_L = 240$ kV. For load 1,

$$I_{L1} = \frac{50{,}000}{\sqrt{3}\,240{,}000} = 120.28 \text{ mA}$$

Since the power factor is lagging, the line current lags the line voltage by $\theta_1 = \cos^{-1} 0.6 = 53.13^\circ$. Thus,

$$\mathbf{I}_{a1} = 120.28\angle{-53.13^\circ}$$

For load 2,

$$I_{L2} = \frac{75{,}000}{\sqrt{3}\,240{,}000} = 180.42 \text{ mA}$$

and the line current lags the line voltage by $\theta_2 = \cos^{-1} 0.8 = 36.87^\circ$. Hence,

$$\mathbf{I}_{a2} = 180.42\angle{-36.87^\circ}$$

The total line current is

$$\begin{aligned}\mathbf{I}_a = \mathbf{I}_{a1} + \mathbf{I}_{a2} &= 120.28\angle{-53.13^\circ} + 180.42\angle{-36.87^\circ}\\ &= (72.168 - j96.224) + (144.336 - j108.252)\\ &= 216.5 - j204.472 = 297.8\angle{-43.36^\circ} \text{ mA}\end{aligned}$$

Alternatively, we could obtain the current from the total complex power using Eq. (12.8.4) as

$$I_L = \frac{123{,}800}{\sqrt{3}\,240{,}000} = 297.82 \text{ mA}$$

and

$$\mathbf{I}_a = 297.82\angle{-43.36^\circ} \text{ mA}$$

which is the same as before. The other line currents, $\mathbf{I}_{b2}$ and $\mathbf{I}_{ca}$, can be obtained according to the abc sequence (i.e., $\mathbf{I}_b = 297.82\angle{-163.36^\circ}$ mA and $\mathbf{I}_c = 297.82\angle{76.64^\circ}$ mA).

(c) We can find the reactive power needed to bring the power factor to 0.9 lagging using Eq. (11.59),

$$Q_C = P(\tan\theta_{\text{old}} - \tan\theta_{\text{new}})$$

where $P = 90$ kW, $\theta_{\text{old}} = 43.36^\circ$, and $\theta_{\text{new}} = \cos^{-1} 0.9 = 25.84^\circ$. Hence,

$$Q_C = 90{,}000(\tan 43.36^\circ - \tan 25.04^\circ) = 41.4 \text{ kVAR}$$

This reactive power is for the three capacitors. For each capacitor, the rating $Q'_C = 13.8$ kVAR. From Eq. (11.60), the required capacitance is

$$C = \frac{Q'_C}{\omega V_{\text{rms}}^2}$$

Since the capacitors are Δ-connected as shown in Fig. 12.22(b), V_{rms} in the above formula is the line-to-line or line voltage, which is 240 kV. Thus,

$$C = \frac{13{,}800}{(2\pi 60)(240{,}000)^2} = 635.5 \text{ pF}$$

PRACTICE PROBLEM 12.8

Assume that the two balanced loads in Fig. 12.22(a) are supplied by an 840-V rms 60-Hz line. Load 1 is Y-connected with $30 + j40\ \Omega$ per phase, while load 2 is a balanced three-phase motor drawing 48 kW at a power factor of 0.8 lagging. Assuming the *abc* sequence, calculate: (a) the complex power absorbed by the combined load, (b) the kVAR rating of each of the three capacitors Δ-connected in parallel with the load to raise the power factor to unity, and (c) the current drawn from the supply at unity power factor condition.

Answer: (a) $56.47 + j47.29$ kVA, (b) 15.7 kVAR, (c) 38.813 A.

@ Network Analysis

†12.8 UNBALANCED THREE-PHASE SYSTEMS

This chapter would be incomplete without mentioning unbalanced three-phase systems. An unbalanced system is caused by two possible situations: (1) the source voltages are not equal in magnitude and/or differ in phase by angles that are unequal, or (2) load impedances are unequal. Thus,

An unbalanced system is due to unbalanced voltage sources or an unbalanced load.

To simplify analysis, we will assume balanced source voltages, but an unbalanced load.

A special technique for handling unbalanced three-phase systems is the method of *symmetrical components*, which is beyond the scope of this text.

Unbalanced three-phase systems are solved by direct application of mesh and nodal analysis. Figure 12.23 shows an example of an unbalanced three-phase system that consists of balanced source voltages (not shown in the figure) and an unbalanced Y-connected load (shown in the figure). Since the load is unbalanced, $\mathbf{Z}_A$, $\mathbf{Z}_B$, and $\mathbf{Z}_C$ are not equal. The line currents are determined by Ohm's law as

$$\mathbf{I}_a = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_A}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_B}, \qquad \mathbf{I}_c = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_C} \tag{12.59}$$

This set of unbalanced line currents produces current in the neutral line, which is not zero as in a balanced system. Applying KCL at node N gives the neutral line current as

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) \tag{12.60}$$

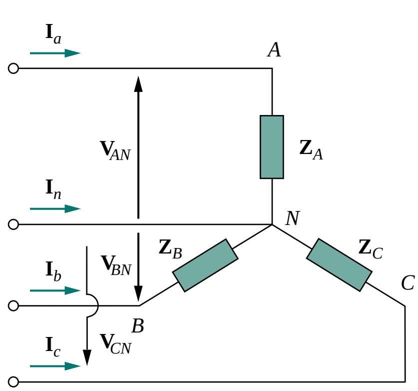


Figure 12.23 Unbalanced three-phase Y-connected load.

In a three-wire system where the neutral line is absent, we can still find the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ using mesh analysis. At node N, KCL must be satisfied so that $\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$ in this case. The same could be done for a Δ-Y, Y-Δ, or Δ-Δ three-wire system. As mentioned earlier, in long distance power transmission, conductors in multiples of three (multiple three-wire systems) are used, with the earth itself acting as the neutral conductor.

To calculate power in an unbalanced three-phase system requires that we find the power in each phase using Eqs. (12.46) to (12.49). The total power is not simply three times the power in one phase but the sum of the powers in the three phases.

EXAMPLE 12.9

The unbalanced Y-load of Fig. 12.23 has balanced voltages of 100 V and the *acb* sequence. Calculate the line currents and the neutral current. Take $\mathbf{Z}_A = 15\ \Omega,\ \mathbf{Z}_B = 10 + j5\ \Omega,\ \mathbf{Z}_C = 6 - j8\ \Omega$.

Solution:

Using Eq. (12.59), the line currents are

$$\mathbf{I}_a = \frac{100\underline{/0^\circ}}{15} = 6.67\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_b = \frac{100\underline{/120^\circ}}{10 + j5} = \frac{100\underline{/120^\circ}}{11.18\underline{/26.56^\circ}} = 8.94\underline{/93.44^\circ}\text{ A}$$

$$\mathbf{I}_c = \frac{100\underline{/-120^\circ}}{6 - j8} = \frac{100\underline{/-120^\circ}}{10\underline{/-53.13^\circ}} = 10\underline{/-66.87^\circ}\text{ A}$$

Using Eq. (12.60), the current in the neutral line is

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = -(6.67 - 0.54 + j8.92 + 3.93 - j9.2)$$
$$= -10.06 + j0.28 = 10.06\underline{/178.4^\circ}\text{ A}$$

PRACTICE PROBLEM 12.9

The unbalanced Δ-load of Fig. 12.24 is supplied by balanced voltages of 200 V in the positive sequence. Find the line currents. Take $\mathbf{V}_{ab}$ as reference.

Answer: $18.05\underline{/-41.06^\circ},\ 29.15\underline{/220.2^\circ},\ 31.87\underline{/74.27^\circ}$ A.

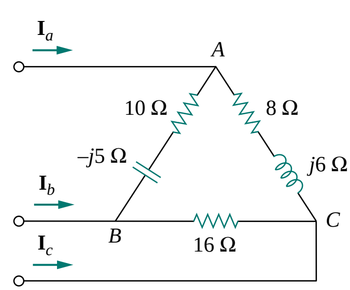


Figure 12.24 Unbalanced Δ-load; for Practice Prob. 12.9.

EXAMPLE 12.10

For the unbalanced circuit in Fig. 12.25, find: (a) the line currents, (b) the total complex power absorbed by the load, and (c) the total complex power supplied by the source.

Solution:

(a) We use mesh analysis to find the required currents. For mesh 1,

$$120\underline{/-120^\circ} - 120\underline{/0^\circ} + (10 + j5)\mathbf{I}_1 - 10\mathbf{I}_2 = 0$$

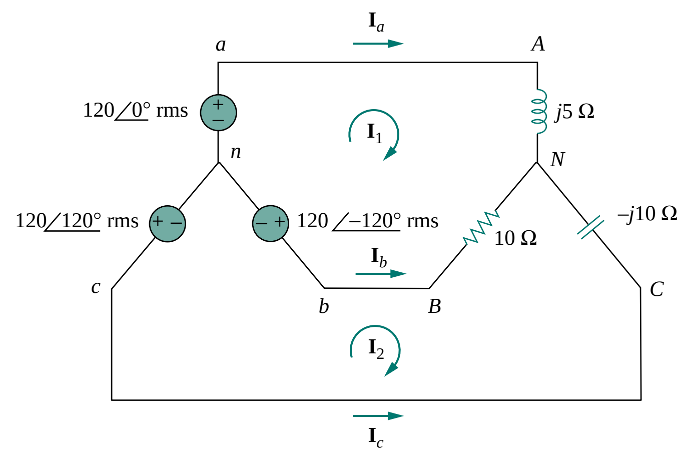


Figure 12.25 For Example 12.10.

or

$$(10 + j5)\mathbf{I}_1 - 10\mathbf{I}_2 = 120\sqrt{3}\underline{/30^\circ} \qquad (12.10.1)$$

For mesh 2,

$$120\underline{/120^\circ} - 120\underline{/-120^\circ} + (10 - j10)\mathbf{I}_2 - 10\mathbf{I}_1 = 0$$

or

$$-10\mathbf{I}_1 + (10 - j10)\mathbf{I}_2 = 120\sqrt{3}\underline{/-90^\circ} \qquad (12.10.2)$$

Equations (12.10.1) and (12.10.2) form a matrix equation:

$$\begin{bmatrix} 10 + j5 & -10 \\ -10 & 10 - j10 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 120\sqrt{3}\underline{/30^\circ} \\ 120\sqrt{3}\underline{/-90^\circ} \end{bmatrix}$$

The determinants are

$$\Delta = \begin{vmatrix} 10 + j5 & -10 \\ -10 & 10 - j10 \end{vmatrix} = 50 - j50 = 70.71\underline{/-45^\circ}$$

$$\Delta_1 = \begin{vmatrix} 120\sqrt{3}\underline{/30^\circ} & -10 \\ 120\sqrt{3}\underline{/-90^\circ} & 10 - j10 \end{vmatrix} = 207.85(13.66 - j13.66)$$

$$= 4015\underline{/-45^\circ}$$

$$\Delta_2 = \begin{vmatrix} 10 + j5 & 120\sqrt{3}\underline{/30^\circ} \\ -10 & 120\sqrt{3}\underline{/-90^\circ} \end{vmatrix} = 207.85(13.66 - j5)$$

$$= 3023\underline{/-20.1^\circ}$$

The mesh currents are

$$\mathbf{I}_1 = \frac{\triangle_1}{\triangle} = \frac{4015.23\underline{/-45^\circ}}{70.71\underline{/-45^\circ}} = 56.78 \text{ A}$$

$$\mathbf{I}_2 = \frac{\triangle_2}{\triangle} = \frac{3023.4\underline{/-20.1^\circ}}{70.71\underline{/-45^\circ}} = 42.75\underline{/24.9^\circ} \text{ A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_1 = 56.78 \text{ A}, \qquad \mathbf{I}_c = -\mathbf{I}_2 = 42.75\underline{/-155.1^\circ} \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_1 = 38.78 + j18 - 56.78 = 25.46\underline{/135^\circ} \text{ A}$$

(b) We can now calculate the complex power absorbed by the load. For phase A,

$$\mathbf{S}_A = |\mathbf{I}_a|^2\mathbf{Z}_A = (56.78)^2(j5) = j16{,}120 \text{ VA}$$

For phase B,

$$\mathbf{S}_B = |\mathbf{I}_b|^2\mathbf{Z}_B = (25.46)^2(10) = 6480 \text{ VA}$$

For phase C,

$$\mathbf{S}_C = |\mathbf{I}_c|^2\mathbf{Z}_C = (42.75)^2(-j10) = -j18{,}276 \text{ VA}$$

The total complex power absorbed by the load is

$$\mathbf{S}_L = \mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C = 6480 - j2156 \text{ VA}$$

(c) We check the result above by finding the power supplied by the source. For the voltage source in phase a,

$$\mathbf{S}_a = -\mathbf{V}_{an}\mathbf{I}_a^* = -(120\underline{/0^\circ})(56.78) = -6813.6 \text{ VA}$$

For the source in phase b,

$$\begin{aligned}\mathbf{S}_b = -\mathbf{V}_{bn}\mathbf{I}_b^* &= -(120\underline{/-120^\circ})(25.46\underline{/-135^\circ})\\ &= -3055.2\underline{/105^\circ} = 790 - j2951.1 \text{ VA}\end{aligned}$$

For the source in phase c,

$$\begin{aligned}\mathbf{S}_c = -\mathbf{V}_{bn}\mathbf{I}_c^* &= -(120\underline{/120^\circ})(42.75\underline{/155.1^\circ})\\ &= -5130\underline{/275.1^\circ} = -456.03 + j5109.7 \text{ VA}\end{aligned}$$

The total complex power supplied by the three-phase source is

$$\mathbf{S}_s = \mathbf{S}_a + \mathbf{S}_b + \mathbf{S}_c = -6480 + j2156 \text{ VA}$$

showing that $\mathbf{S}_s + \mathbf{S}_L = 0$ and confirming the conservation principle of ac power.

PRACTICE PROBLEM 12.10

Find the line currents in the unbalanced three-phase circuit of Fig. 12.26 and the real power absorbed by the load.

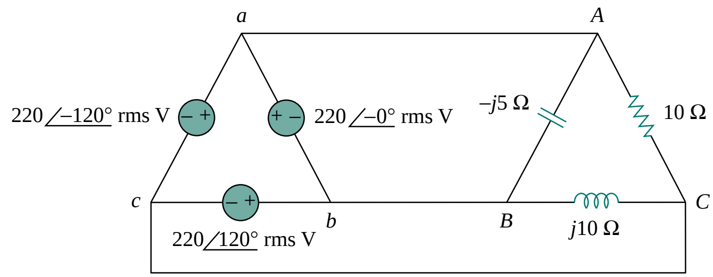


Figure 12.26 For Practice Prob. 12.10.

Answer: $64\angle 80.1^\circ$, $38.1\angle -60^\circ$, $42.5\angle 225^\circ$ A, 4.84 kW.

12.9 *PSPICE* FOR THREE-PHASE CIRCUITS

PSpice can be used to analyze three-phase balanced or unbalanced circuits in the same way it is used to analyze single-phase ac circuits. However, a delta-connected source presents two major problems to *PSpice*. First, a delta-connected source is a loop of voltage sources—which *PSpice* does not like. To avoid this problem, we insert a resistor of negligible resistance (say, 1 $\mu\Omega$ per phase) into each phase of the delta-connected source. Second, the delta-connected source does not provide a convenient node for the ground node, which is necessary to run *PSpice*. This problem can be eliminated by inserting balanced wye-connected large resistors (say, 1 MΩ per phase) in the delta-connected source so that the neutral node of the wye-connected resistors serves as the ground node 0. Example 12.12 will illustrate this.

EXAMPLE 12.11

For the balanced Y-Δ circuit in Fig. 12.27, use *PSpice* to find the line current $\mathbf{I}_{aA}$, the phase voltage $\mathbf{V}_{AB}$, and the phase current $\mathbf{I}_{AC}$. Assume that the source frequency is 60 Hz.

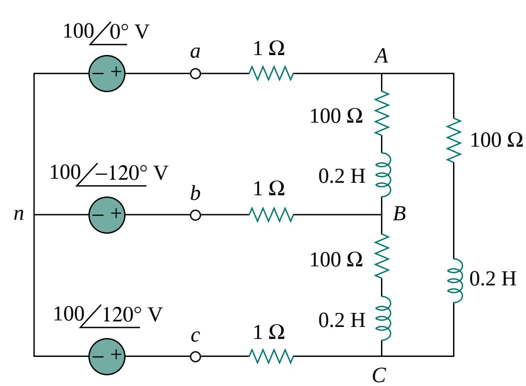


Figure 12.27 For Example 12.10.

Solution:

The schematic is shown in Fig. 12.28. The pseudocomponents IPRINT are inserted in the appropriate lines to obtain $\mathbf{I}_{aA}$ and $\mathbf{I}_{AC}$, while VPRINT2 is inserted between nodes A and B to print differential voltage $\mathbf{V}_{AB}$. We set the attributes of IPRINT and VPRINT2 each to $AC = yes$, $MAG = yes$, $PHASE = yes$, to print only the magnitude and phase of the currents and voltages. As a single-frequency analysis, we select **Analysis/Setup/AC Sweep** and enter *Total Pts* = 1, *Start Freq* = 60, and *Final Freq* = 60. Once the circuit is saved, it is simulated by selecting **Analysis/Simulate**. The output file includes the following:

```
FREQ         V(A,B)      VP(A,B)
6.000E+01    1.699E+02   3.081E+01

FREQ         IM(V_PRINT2)  IP(V_PRINT2)
6.000E+01    2.350E+00     -3.620E+01

FREQ         IM(V_PRINT3)  IP(V_PRINT3)
6.000E+01    1.357E+00      -6.620E+01
```

From this, we obtain

$$\mathbf{I}_{aA} = 2.35\underline{/-36.2^\circ} \text{ A}$$

$$\mathbf{V}_{AB} = 169.9\underline{/30.81^\circ} \text{ V}, \qquad \mathbf{I}_{AC} = 1.357\underline{/-66.2^\circ} \text{ A}$$

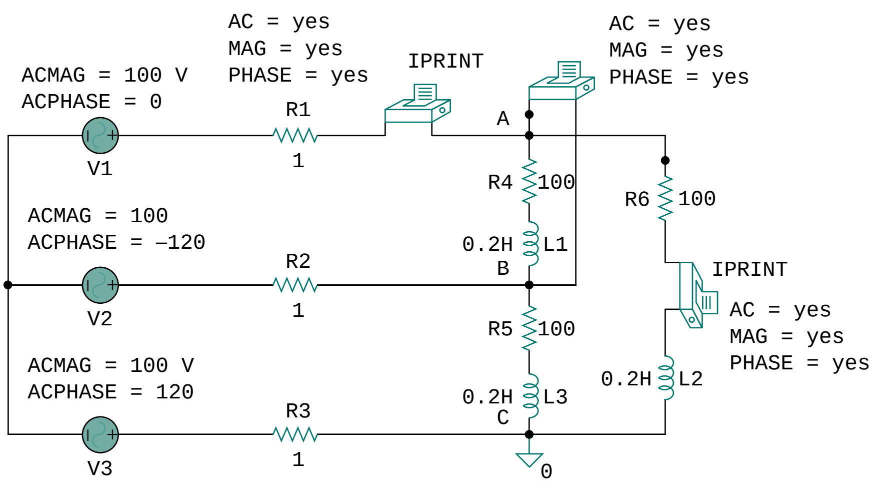


Figure 12.28 Schematic for the circuit in Fig. 12.27.

PRACTICE PROBLEM 12.11

Refer to the balanced Y-Y circuit of Fig. 12.29. Use *PSpice* to find the line current $\mathbf{I}_{bB}$ and the phase voltage $\mathbf{V}_{AN}$. Take $f = 100$ Hz.

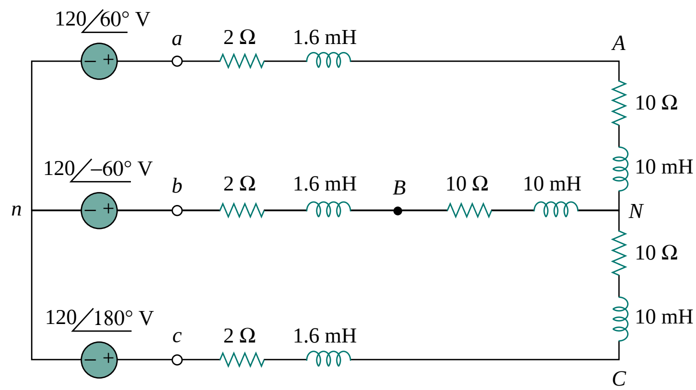


Figure 12.29 For Practice Prob. 12.11.

Answer: $100.9\underline{/60.87^\circ}$ V, $8.547\underline{/-91.27^\circ}$ A.

EXAMPLE 12.12

Consider the unbalanced Δ-Δ circuit in Fig. 12.30. Use *PSpice* to find the generator current $\mathbf{I}_{ab}$, the line current $\mathbf{I}_{bB}$, and the phase current $\mathbf{I}_{BC}$.

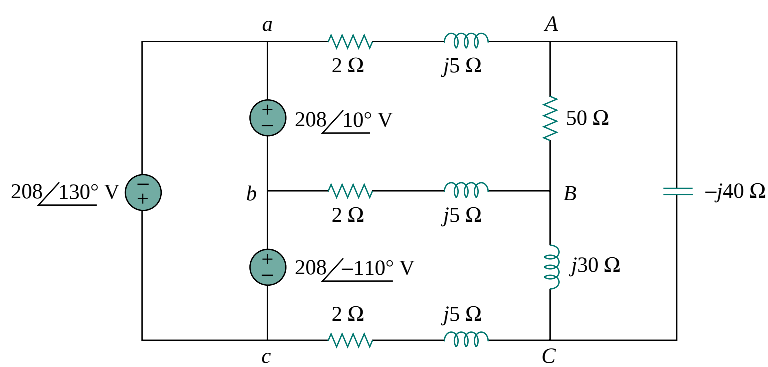


Figure 12.30 For Example 12.12.

Solution:

As mentioned above, we avoid the loop of voltage sources by inserting a 1-$\mu\Omega$ series resistor in the delta-connected source. To provide a ground node 0, we insert balanced wye-connected resistors (1 MΩ per phase) in the delta-connected source, as shown in the schematic in Fig. 12.31. Three IPRINT pseudocomponents with their attributes are inserted to be able to get the required currents $\mathbf{I}_{ab}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{BC}$. Since the operating frequency is not given and the inductances and capacitances should be specified instead of impedances, we assume $\omega = 1$ rad/s so that $f = 1/2\pi = 0.159155$ Hz. Thus,

$$L = \frac{X_L}{\omega} \qquad \text{and} \qquad C = \frac{1}{\omega X_C}$$

We select **Analysis/Setup/AC Sweep** and enter *Total Pts* = 1, *Start Freq* = 0.159155, and *Final Freq* = 0.159155. Once the schematic is saved, we select **Analysis/Simulate** to simulate the circuit. The output file includes:

```
FREQ        IM(V_PRINT1)   IP(V_PRINT1)
1.592E-01   9.106E+00      1.685E+02

FREQ        IM(V_PRINT2)   IP(V_PRINT2)
1.592E-01   5.959E+00      2.821E+00

FREQ        IM(V_PRINT3)   IP(V_PRINT3)
1.592E-01   5.500E+00      -7.532E+00
```

from which we get

$$\mathbf{I}_{ab} = 5.96\angle 2.82^\circ \text{ A}$$

$$\mathbf{I}_{bB} = 9.106\angle 168.5^\circ \text{ A}, \qquad \mathbf{I}_{BC} = 5.5\angle -7.53^\circ \text{ A}$$

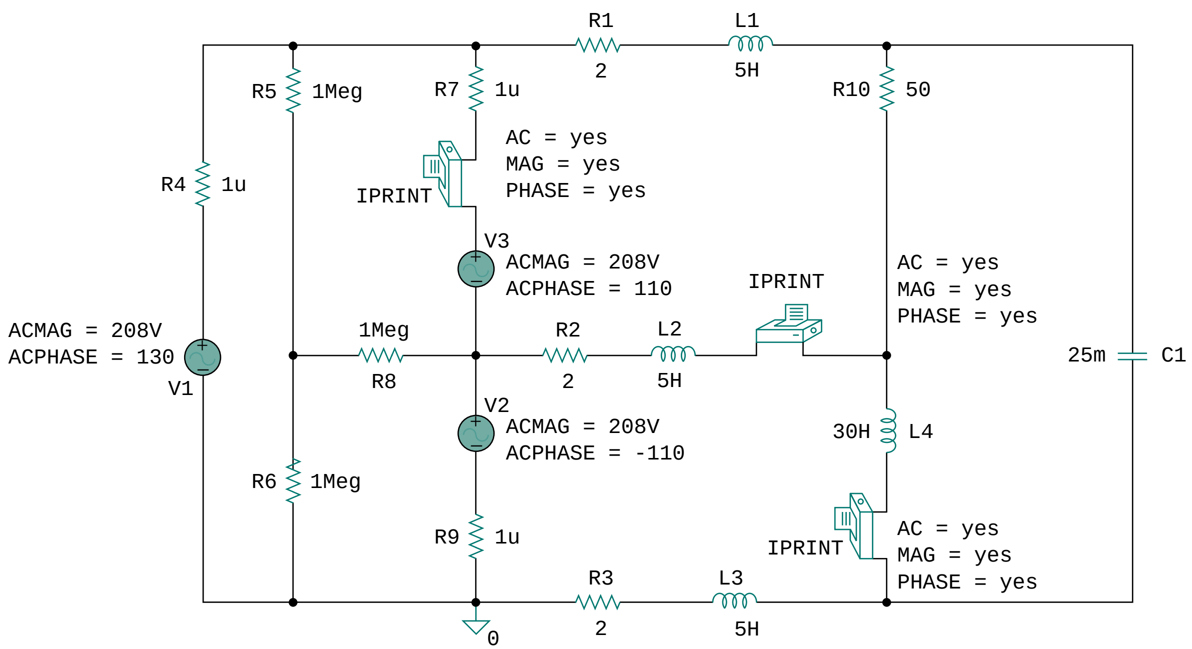


Figure 12.31 Schematic for the circuit in Fig. 12.30.

PRACTICE PROBLEM 12.12

For the unbalanced circuit in Fig. 12.32, use *PSpice* to find the generator current $\mathbf{I}_{ca}$, the line current $\mathbf{I}_{cC}$, and the phase current $\mathbf{I}_{AB}$.

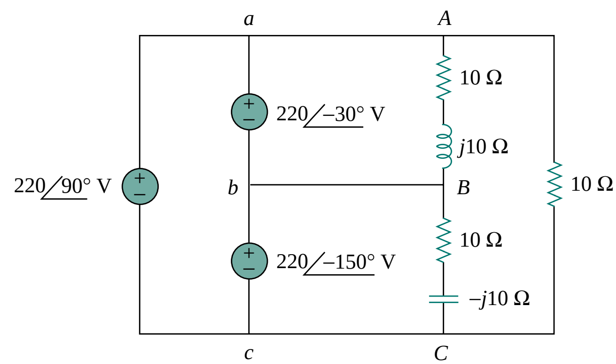


Figure 12.32 For Practice Prob. 12.12.

Answer: $24.68\underline{/-90^\circ}$ A, $15.56\underline{/105^\circ}$ A, $37.24\underline{/83.79^\circ}$ A.

†12.10 APPLICATIONS

Both wye and delta source connections have important practical applications. The wye source connection is used for long distance transmission of electric power, where resistive losses (I^2R) should be minimal. This is due to the fact that the wye connection gives a line voltage that is $\sqrt{3}$ greater than the delta connection; hence, for the same power, the line current is $\sqrt{3}$ smaller. The delta source connection is used when three single-phase circuits are desired from a three-phase source. This conversion from three-phase to single-phase is required in residential wiring, because household lighting and appliances use single-phase power. Three-phase power is used in industrial wiring where a large power is required. In some applications, it is immaterial whether the load is wye- or delta-connected. For example, both connections are satisfactory with induction motors. In fact, some manufacturers connect a motor in delta for 220 V and in wye for 440 V so that one line of motors can be readily adapted to two different voltages.

Here we consider two practical applications of those concepts covered in this chapter: power measurement in three-phase circuits and residential wiring.

12.10.1 Three-Phase Power Measurement

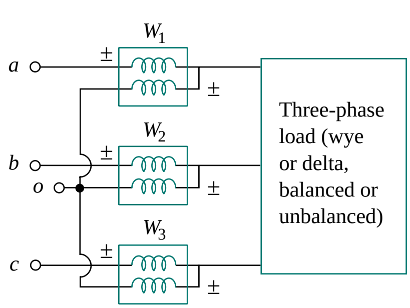


Figure 12.33 Three-wattmeter method for measuring three-phase power.

Section 11.9 presented the wattmeter as the instrument for measuring the average (or real) power in single-phase circuits. A single wattmeter can also measure the average power in a three-phase system that is balanced, so that $P_1 = P_2 = P_3$; the total power is three times the reading of that one wattmeter. However, two or three single-phase wattmeters are necessary to measure power if the system is unbalanced. The *three-wattmeter method* of power measurement, shown in Fig. 12.33, will work regardless of whether the load is balanced or unbalanced, wye- or delta-connected.

The three-wattmeter method is well suited for power measurement in a three-phase system where the power factor is constantly changing. The total average power is the algebraic sum of the three wattmeter readings,

$$P_T = P_1 + P_2 + P_3 \tag{12.61}$$

where P_1, P_2, and P_3 correspond to the readings of wattmeters W_1, W_2, and W_3, respectively. Notice that the common or reference point o in Fig. 12.33 is selected arbitrarily. If the load is wye-connected, point o can be connected to the neutral point n. For a delta-connected load, point o can be connected to any point. If point o is connected to point b, for example, the voltage coil in wattmeter W_2 reads zero and $P_2 = 0$, indicating that wattmeter W_2 is not necessary. Thus, two wattmeters are sufficient to measure the total power.

The *two-wattmeter method* is the most commonly used method for three-phase power measurement. The two wattmeters must be properly connected to any two phases, as shown typically in Fig. 12.34. Notice that the current coil of each wattmeter measures the line current, while the respective voltage coil is connected between the line and the third line and measures the line voltage. Also notice that the ± terminal of the voltage coil is connected to the line to which the corresponding current coil is connected. Although the individual wattmeters no longer read the power taken by any particular phase, the algebraic sum of the two wattmeter readings equals the total average power absorbed by the load, regardless of whether it is wye- or delta-connected, balanced or unbalanced. The total real power is equal to the algebraic sum of the two wattmeter readings,

$$P_T = P_1 + P_2 \tag{12.62}$$

We will show here that the method works for a balanced three-phase system.

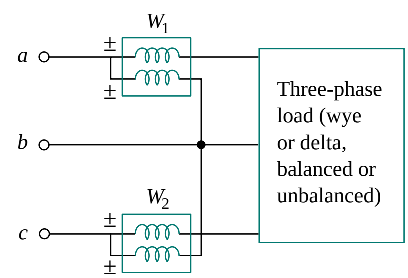


Figure 12.34 Two-wattmeter method for measuring three-phase power.

Consider the balanced, wye-connected load in Fig. 12.35. Our objective is to apply the two-wattmeter method to find the average power absorbed by the load. Assume the source is in the abc sequence and the load impedance $\mathbf{Z}_Y = Z_Y\underline{/\theta}$. Due to the load impedance, each voltage coil leads its current coil by θ, so that the power factor is $\cos\theta$. We recall that each line voltage leads the corresponding phase voltage by 30°. Thus, the total phase difference between the phase current $\mathbf{I}_a$ and line voltage

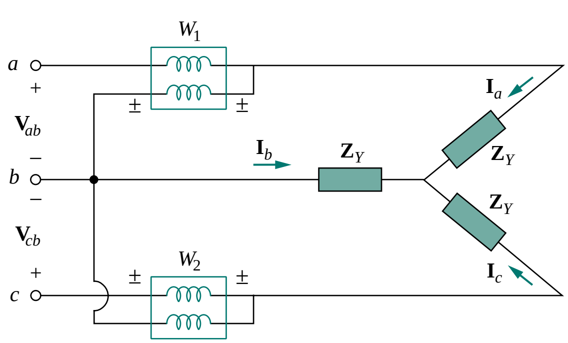


Figure 12.35 Two-wattmeter method applied to a balanced wye load.

$\mathbf{V}_{ab}$ is $\theta + 30^\circ$, and the average power read by wattmeter W_1 is

$$P_1 = \text{Re}[\mathbf{V}_{ab}\mathbf{I}_a^*] = V_{ab}I_a \cos(\theta + 30^\circ) = V_L I_L \cos(\theta + 30^\circ) \quad (12.63)$$

Similarly, we can show that the average power read by wattmeter 2 is

$$P_2 = \text{Re}[\mathbf{V}_{cb}\mathbf{I}_c^*] = V_{cb}I_c \cos(\theta - 30^\circ) = V_L I_L \cos(\theta - 30^\circ) \quad (12.64)$$

We now use the trigonometric identities

$$\begin{aligned} \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \end{aligned} \quad (12.65)$$

to find the sum and the difference of the two wattmeter readings in Eqs. (12.63) and (12.64):

$$\begin{aligned} P_1 + P_2 &= V_L I_L[\cos(\theta + 30^\circ) + \cos(\theta - 30^\circ)] \\ &= V_l I_L(\cos\theta \cos 30^\circ - \sin\theta \sin 30^\circ \\ &\qquad + \cos\theta \cos 30^\circ + \sin\theta \sin 30^\circ) \\ &= V_L I_L 2\cos 30^\circ \cos\theta = \sqrt{3} V_L I_L \cos\theta \end{aligned} \quad (12.66)$$

since $2\cos 30^\circ = \sqrt{3}$. Comparing Eq. (12.66) with Eq. (12.50) shows that the sum of the wattmeter readings gives the total average power,

$$\boxed{P_T = P_1 + P_2} \quad (12.67)$$

Similarly,

$$\begin{aligned} P_1 - P_2 &= V_L I_L[\cos(\theta + 30^\circ) - \cos(\theta - 30^\circ)] \\ &= V_l I_L(\cos\theta \cos 30^\circ - \sin\theta \sin 30^\circ \\ &\qquad - \cos\theta \cos 30^\circ - \sin\theta \sin 30^\circ) \\ &= -V_L I_L 2\sin 30^\circ \sin\theta \\ P_2 - P_1 &= V_L I_L \sin\theta \end{aligned} \quad (12.68)$$

since $2\sin 30^\circ = 1$. Comparing Eq. (12.68) with Eq. (12.51) shows that the difference of the wattmeter readings is proportional to the total reactive power, or

$$\boxed{Q_T = \sqrt{3}(P_2 - P_1)} \quad (12.69)$$

From Eqs. (12.67) and (12.69), the total apparent power can be obtained as

$$S_T = \sqrt{P_T^2 + Q_T^2} \quad (12.70)$$

Dividing Eq. (12.69) by Eq. (12.67) gives the tangent of the power factor angle as

$$\tan\theta = \frac{Q_T}{P_T} = \sqrt{3}\frac{P_2 - P_1}{P_2 + P_1} \quad (12.71)$$

from which we can obtain the power factor as $\text{pf} = \cos\theta$. Thus, the two-wattmeter method not only provides the total real and reactive powers, it can also be used to compute the power factor. From Eqs. (12.67), (12.69), and (12.71), we conclude that:

1. If $P_2 = P_1$, the load is resistive.
2. If $P_2 > P_1$, the load is inductive.
3. If $P_2 < P_1$, the load is capacitive.

Although these results are derived from a balanced wye-connected load, they are equally valid for a balanced delta-connected load. However, the two-wattmeter method cannot be used for power measurement in a three-phase four-wire system unless the current through the neutral line is zero. We use the three-wattmeter method to measure the real power in a three-phase four-wire system.

EXAMPLE 12.13

Three wattmeters W_1, W_2, and W_3 are connected, respectively, to phases a, b, and c to measure the total power absorbed by the unbalanced wye-connected load in Example 12.9 (see Fig. 12.23). (a) Predict the wattmeter readings. (b) Find the total power absorbed.

Solution:

Part of the problem is already solved in Example 12.9. Assume that the wattmeters are properly connected as in Fig. 12.36.

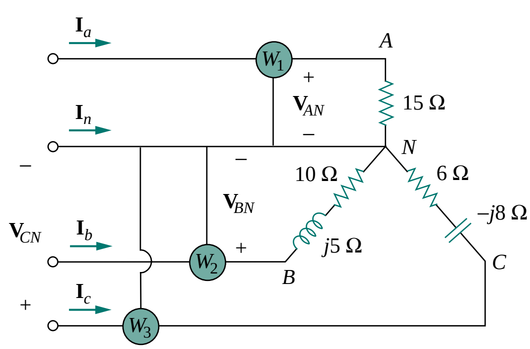


Figure 12.36 For Example 12.13.

(a) From Example 12.9,

$$\mathbf{V}_{AN} = 100\angle 0^\circ, \qquad \mathbf{V}_{BN} = 100\angle 120^\circ, \qquad \mathbf{V}_{CN} = 100\angle -120^\circ \text{ V}$$

while

$$\mathbf{I}_a = 6.67\angle 0^\circ, \qquad \mathbf{I}_b = 8.94\angle 93.44^\circ, \qquad \mathbf{I}_c = 10\angle -66.87^\circ \text{ A}$$

We calculate the wattmeter readings as follows:

$$\begin{aligned}
P_1 = \text{Re}(\mathbf{V}_{AN}\mathbf{I}_a^*) &= V_{AN} I_a \cos(\theta_{\mathbf{V}_{AN}} - \theta_{\mathbf{I}_a}) \\
&= 100 \times 6.67 \times \cos(0^\circ - 0^\circ) = 667 \text{ W} \\
P_2 = \text{Re}(\mathbf{V}_{BN}\mathbf{I}_b^*) &= V_{BN} I_b \cos(\theta_{\mathbf{V}_{BN}} - \theta_{\mathbf{I}_b}) \\
&= 100 \times 8.94 \times \cos(120^\circ - 93.44^\circ) = 800 \text{ W} \\
P_3 = \text{Re}(\mathbf{V}_{CN}\mathbf{I}_c^*) &= V_{CN} I_c \cos(\theta_{\mathbf{V}_{CN}} - \theta_{\mathbf{I}_c}) \\
&= 100 \times 10 \times \cos(-120^\circ + 66.87^\circ) = 600 \text{ W}
\end{aligned}$$

(b) The total power absorbed is

$$P_T = P_1 + P_2 + P_3 = 667 + 800 + 600 = 2067 \text{ W}$$

We can find the power absorbed by the resistors in Fig. 12.36 and use that to check or confirm this result.

$$\begin{aligned} P_T &= |I_a|^2(15) + |I_b|^2(10) + |I_c|^2(6) \\ &= 6.67^2(15) + 8.94^2(10) + 10^2(6) \\ &= 667 + 800 + 600 = 2067 \text{ W} \end{aligned}$$

which is exactly the same thing.

PRACTICE PROBLEM 12.13

Repeat Example 12.13 for the network in Fig. 12.24 (see Practice Prob. 12.9). *Hint:* Connect the reference point o in Fig. 12.33 to point B.

Answer: (a) 2961 W, 0 W, 4339 W, (b) 7300 W.

EXAMPLE 12.14

The two-wattmeter method produces wattmeter readings $P_1 = 1560$ W and $P_2 = 2100$ W when connected to a delta-connected load. If the line voltage is 220 V, calculate: (a) the per-phase average power, (b) the per-phase reactive power, (c) the power factor, and (d) the phase impedance.

Solution:

We can apply the given results to the delta-connected load.
(a) The total real or average power is

$$P_T = P_1 + P_2 = 1560 + 2100 = 3660 \text{ W}$$

The per-phase average power is then

$$P_p = \frac{1}{3}P_T = 1220 \text{ W}$$

(b) The total reactive power is

$$Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(2100 - 1560) = 935.3 \text{ VAR}$$

so that the per-phase reactive power is

$$Q_p = \frac{1}{3}Q_T = 311.77 \text{ VAR}$$

(c) The power angle is

$$\theta = \tan^{-1}\frac{Q_T}{P_T} = \tan^{-1}\frac{935.3}{3660} = 14.33^\circ$$

Hence, the power factor is

$$\cos\theta = 0.9689 \text{ (leading)}$$

It is a leading pf because Q_T is positive or $P_2 > P_1$.

(c) The phase impedance is $\mathbf{Z}_p = Z_p\angle\theta$. We know that θ is the same as the pf angle; that is, $\theta = 14.57^\circ$.

$$Z_p = \frac{V_p}{I_p}$$

We recall that for a delta-connected load, $V_p = V_L = 220$ V. From Eq. (12.46),

$$P_p = V_p I_p \cos\theta \qquad \Longrightarrow \qquad I_p = \frac{1220}{220 \times 0.9689} = 5.723 \text{ A}$$

Hence,

$$Z_p = \frac{V_p}{I_p} = \frac{220}{5.723} = 38.44\ \Omega$$

and

$$\mathbf{Z}_p = 38.44\angle 14.33^\circ\ \Omega$$

PRACTICE PROBLEM 12.14

Let the line voltage $V_L = 208$ V and the wattmeter readings of the balanced system in Fig. 12.35 be $P_1 = -560$ W and $P_2 = 800$ W. Determine:

(a) the total average power

(b) the total reactive power

(c) the power factor

(d) the phase impedance

Is the impedance inductive or capacitive?

Answer: (a) 240 W, (b) 2355.6 VAR, (c) 0.1014, (d) $18.25\angle 84.18^\circ\ \Omega$, inductive.

EXAMPLE 12.15

The three-phase balanced load in Fig. 12.35 has impedance per phase of $\mathbf{Z}_Y = 8 + j6\ \Omega$. If the load is connected to 208-V lines, predict the readings of the wattmeters W_1 and W_2. Find P_T and Q_T.

Solution:

The impedance per phase is

$$\mathbf{Z}_Y = 8 + j6 = 10\angle 36.87^\circ\ \Omega$$

so that the pf angle is 36.87°. Since the line voltage $V_L = 208$ V, the line current is

$$I_L = \frac{V_p}{|\mathbf{Z}_Y|} = \frac{208/\sqrt{3}}{10} = 12 \text{ A}$$

Then

$$P_1 = V_L I_L \cos(\theta + 30^\circ) = 208 \times 12 \times \cos(36.87^\circ + 30^\circ)$$
$$= 980.48 \text{ W}$$
$$P_2 = V_L I_L \cos(\theta - 30^\circ) = 208 \times 12 \times \cos(36.87^\circ - 30^\circ)$$
$$= 2478.1 \text{ W}$$

Thus, wattmeter 1 reads 980.48 W, while wattmeter 2 reads 2478.1 W. Since $P_2 > P_1$, the load is inductive. This is evident from the load $\mathbf{Z}_Y$ itself. Next,

$$P_T = P_1 + P_2 = 3.4586 \text{ kW}$$

and

$$Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(1497.6) \text{ VAR} = 2.594 \text{ kVAR}$$

PRACTICE PROBLEM 12.15

If the load in Fig. 12.35 is delta-connected with impedance per phase of $\mathbf{Z}_p = 30 - j40\ \Omega$ and $V_L = 440$ V, predict the readings of the wattmeters W_1 and W_2. Calculate P_T and Q_T.

Answer: 6.166 kW, 0.8021 kW, 6.968 kW, −9.291 kVAR.

12.10.2 Residential Wiring

In the United States, most household lighting and appliances operate on 120-V, 60-Hz, single-phase alternating current. (The electricity may also be supplied at 110, 115, or 117 V, depending on the location.) The local power company supplies the house with a three-wire ac system. Typically, as in Fig. 12.37, the line voltage of, say, 12,000 V is stepped down to 120/240 V with a transformer (more details on transformers

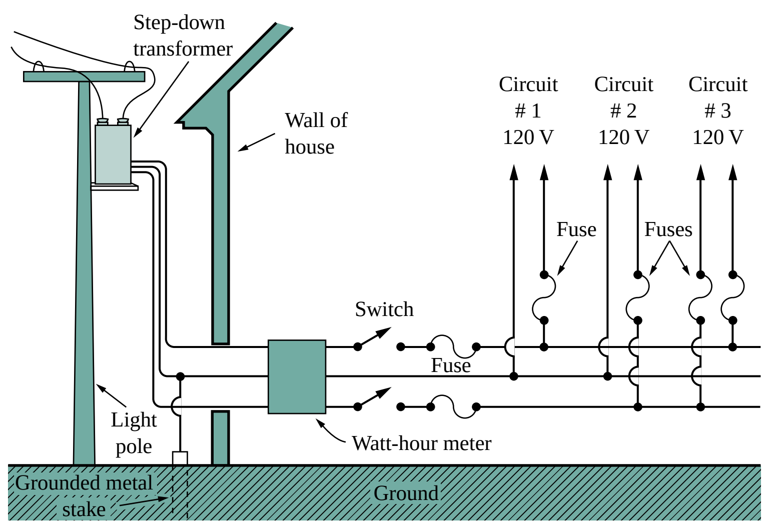


Figure 12.37 A 120/240 household power system.
(Source: A. Marcus and C. M. Thomson, Electricity for Technicians, 2nd ed. [Englewood Cliffs, NJ: Prentice Hall, 1975], p. 324.)

in the next chapter). The three wires coming from the transformer are typically colored red (hot), black (hot), and white (neutral). As shown in Fig. 12.38, the two 120-V voltages are opposite in phase and hence add up to zero. That is, $\mathbf{V}_W = 0\underline{/0^\circ}$, $\mathbf{V}_B = 120\underline{/0^\circ}$, $\mathbf{V}_R = 120\underline{/180^\circ} = -\mathbf{V}_B$.

$$\mathbf{V}_{BR} = \mathbf{V}_B - \mathbf{V}_R = \mathbf{V}_B - (-\mathbf{V}_B) = 2\mathbf{V}_B = 240\underline{/0^\circ} \qquad (12.72)$$

Since most appliances are designed to operate with 120 V, the lighting and appliances are connected to the 120-V lines, as illustrated in Fig. 12.39 for a room. Notice in Fig. 12.37 that all appliances are connected in parallel. Heavy appliances that consume large currents, such as air conditioners, dishwashers, ovens, and laundry machines, are connected to the 240-V power line.

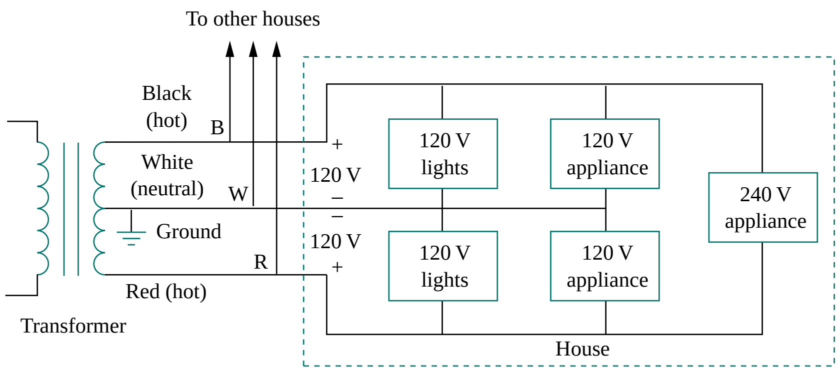


Figure 12.38 Single-phase three-wire residential wiring.

Because of the dangers of electricity, house wiring is carefully regulated by a code drawn by local ordinances and by the National Electrical Code (NEC). To avoid trouble, insulation, grounding, fuses, and circuit breakers are used. Modern wiring codes require a third wire for a separate ground. The ground wire does not carry power like the neutral wire but enables appliances to have a separate ground connection. Figure 12.40 shows the connection of the receptacle to a 120-V rms line and to the ground. As shown in the figure, the neutral line is connected to the ground (the earth) at many critical locations. Although the ground line

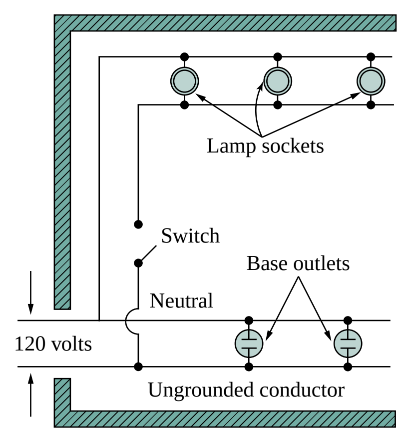


Figure 12.39 A typical wiring diagram of a room.
(Source: A. Marcus and C. M. Thomson, Electricity for Technicians, 2nd ed. [Englewood Cliffs, NJ: Prentice Hall, 1975], p. 325.)

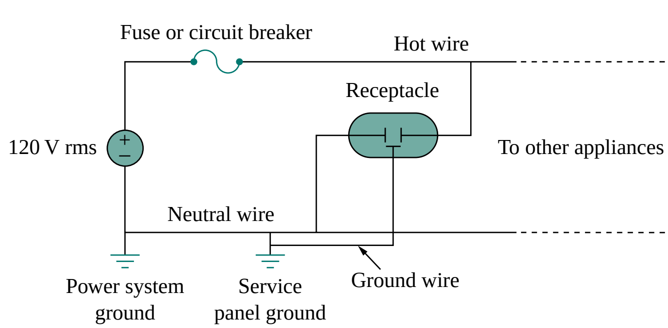


Figure 12.40 Connection of a receptacle to the hot line and to the ground.

seems redundant, grounding is important for many reasons. First, it is required by NEC. Second, grounding provides a convenient path to ground for lightning that strikes the power line. Third, grounds minimize the risk of electric shock. What causes shock is the passage of current from one part of the body to another. The human body is like a big resistor R. If V is the potential difference between the body and the ground, the current through the body is determined by Ohm's law as

$$I = \frac{V}{R} \tag{12.73}$$

The value of R varies from person to person and depends on whether the body is wet or dry. How great or how deadly the shock is depends on the amount of current, the pathway of the current through the body, and the length of time the body is exposed to the current. Currents less than 1 mA may not be harmful to the body, but currents greater than 10 mA can cause severe shock. A modern safety device is the *ground-fault circuit interrupter* (GFCI), used in outdoor circuits and in bathrooms, where the risk of electric shock is greatest. It is essentially a circuit breaker that opens when the sum of the currents i_R, i_W, and i_B through the red, white, and the black lines is not equal to zero, or $i_R + i_W + i_B \neq 0$.

The best way to avoid electric shock is to follow safety guidelines concerning electrical systems and appliances. Here are some of them:

- Never assume that an electrical circuit is dead. Always check to be sure.
- Use safety devices when necessary, and wear suitable clothing (insulated shoes, gloves, etc.).
- Never use two hands when testing high-voltage circuits, since the current through one hand to the other hand has a direct path through your chest and heart.
- Do not touch an electrical appliance when you are wet. Remember that water conducts electricity.
- Be extremely careful when working with electronic appliances such as radio and TV because these appliances have large capacitors in them. The capacitors take time to discharge after the power is disconnected.
- Always have another person present when working on a wiring system, just in case of an accident.

12.11 SUMMARY

1. The phase sequence is the order in which the phase voltages of a three-phase generator occur with respect to time. In an abc sequence of balanced source voltages, $\mathbf{V}_{an}$ leads $\mathbf{V}_{bn}$ by 120°, which in turn leads $\mathbf{V}_{cn}$ by 120°. In an acb sequence of balanced voltages, $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ by 120°, which in turn leads $\mathbf{V}_{bn}$ by 120°.
2. A balanced wye- or delta-connected load is one in which the three-phase impedances are equal.
3. The easiest way to analyze a balanced three-phase circuit is to transform both the source and the load to a Y-Y system and then

analyze the single-phase equivalent circuit. Table 12.1 presents a summary of the formulas for phase currents and voltages and line currents and voltages for the four possible configurations.

4. The line current I_L is the current flowing from the generator to the load in each transmission line in a three-phase system. The line voltage V_L is the voltage between each pair of lines, excluding the neutral line if it exists. The phase current I_p is the current flowing through each phase in a three-phase load. The phase voltage V_p is the voltage of each phase. For a wye-connected load,
$$V_L = \sqrt{3}V_p \qquad \text{and} \qquad I_L = I_p$$
For a delta-connected load,
$$V_L = V_p \qquad \text{and} \qquad I_L = \sqrt{3}I_p$$
5. The total instantaneous power in a balanced three-phase system is constant and equal to the average power.
6. The total complex power absorbed by a balanced three-phase Y-connected or Δ-connected load is
$$\mathbf{S} = P + jQ = \sqrt{3}V_L I_L \underline{/\theta}$$
where θ is the angle of the load impedances.
7. An unbalanced three-phase system can be analyzed using nodal or mesh analysis.
8. *PSpice* is used to analyze three-phase circuits in the same way as it is used for analyzing single-phase circuits.
9. The total real power is measured in three-phase systems using either the three-wattmeter method or the two-wattmeter method.
10. Residential wiring uses a 120/240-V, single-phase, three-wire system.

REVIEW QUESTIONS

12.1 What is the phase sequence of a three-phase motor for which $\mathbf{V}_{AN} = 220\underline{/-100^\circ}$ V and $\mathbf{V}_{BN} = 220\underline{/140^\circ}$ V?
(a) *abc* (b) *acb*

12.2 If in an *acb* phase sequence, $\mathbf{V}_{an} = 100\underline{/-20^\circ}$, then $\mathbf{V}_{cn}$ is:
(a) $100\underline{/-140^\circ}$ (b) $100\underline{/100^\circ}$
(c) $100\underline{/-50^\circ}$ (d) $100\underline{/10^\circ}$

12.3 Which of these is not a required condition for a balanced system:
(a) $|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$
(b) $\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$
(c) $V_{an} + V_{bn} + V_{cn} = 0$
(d) Source voltages are 120° out of phase with each other.
(e) Load impedances for the three phases are equal.

12.4 In a Y-connected load, the line current and phase current are equal.
(a) True (b) False

12.5 In a Δ-connected load, the line current and phase current are equal.
(a) True (b) False

12.6 In a Y-Y system, a line voltage of 220 V produces a phase voltage of:
(a) 381 V (b) 311 V (c) 220 V
(d) 156 V (e) 127 V

12.7 In a Δ-Δ system, a phase voltage of 100 V produces a line voltage of:
(a) 58 V (b) 71 V (c) 100 V
(d) 173 V (e) 141 V

12.8 When a Y-connected load is supplied by voltages in *abc* phase sequence, the line voltages lag the corresponding phase voltages by 30°.
(a) True (b) False

12.9 In a balanced three-phase circuit, the total instantaneous power is equal to the average power.
(a) True (b) False

12.10 The total power supplied to a balanced Δ-load is found in the same way as for a balanced Y-load.
(a) True (b) False

Answers: 12.1a, 12.2a, 12.3c, 12.4a, 12.5b, 12.6e, 12.7c, 12.8b, 12.9a, 12.10a.

PROBLEMS[1]

Section 12.2 Balanced Three-Phase Voltages

12.1 If $\mathbf{V}_{ab} = 400$ V in a balanced Y-connected three-phase generator, find the phase voltages, assuming the phase sequence is:
(a) *abc* (b) *acb*

12.2 What is the phase sequence of a balanced three-phase circuit for which $\mathbf{V}_{an} = 160\angle 30^\circ$ V and $\mathbf{V}_{cn} = 160\angle -90^\circ$ V? Find $\mathbf{V}_{bn}$.

12.3 Determine the phase sequence of a balanced three-phase circuit in which $\mathbf{V}_{bn} = 208\angle 130^\circ$ V and $\mathbf{V}_{cn} = 208\angle 10^\circ$ V. Obtain $\mathbf{V}_{an}$.

12.4 Assuming the *abc* sequence, if $\mathbf{V}_{ca} = 208\angle 20^\circ$ V in a balanced three-phase circuit, find $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, $\mathbf{V}_{an}$, and $\mathbf{V}_{bn}$.

12.5 Given that the line voltages of a three-phase circuit are

$$\mathbf{V}_{ab} = 420\angle 0^\circ, \quad \mathbf{V}_{bc} = 420\angle -120^\circ$$
$$\mathbf{V}_{ac} = 420\angle 120^\circ \text{ V}$$

find the phase voltages $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, and $\mathbf{V}_{cn}$.

Section 12.3 Balanced Wye-Wye Connection

12.6 For the Y-Y circuit of Fig. 12.41, find the line currents, the line voltages, and the load voltages.

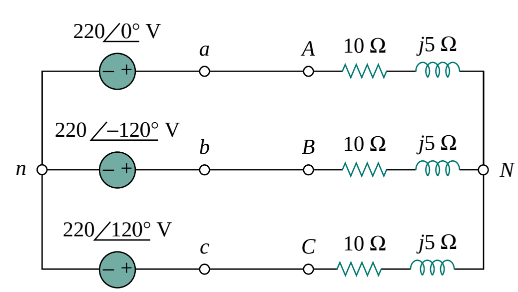


Figure 12.41 For Prob. 12.6.

12.7 Obtain the line currents in the three-phase circuit of Fig. 12.42 below.

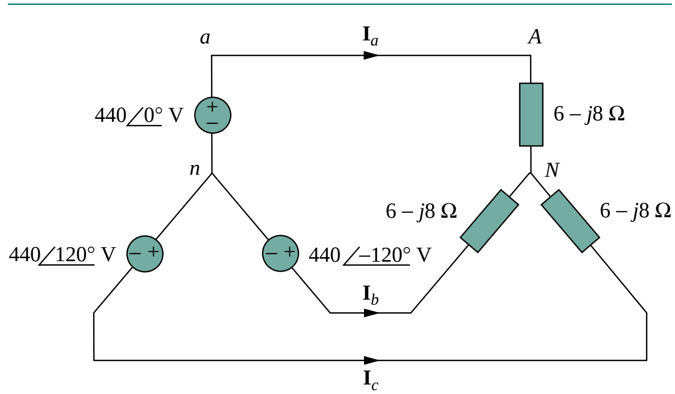


Figure 12.42 For Prob. 12.7.

[1]Remember that unless stated otherwise, all given voltages and currents are rms values.

12.8 A balanced Y-connected load with a phase impedance of $16 + j9\ \Omega$ is connected to a balanced three-phase source with a line voltage of 220 V. Calculate the line current I_L.

12.9 A balanced Y-Y four-wire system has phase voltages

$$\mathbf{V}_{an} = 120\underline{/0^\circ}, \quad \mathbf{V}_{bn} = 120\underline{/-120^\circ}$$
$$\mathbf{V}_{cn} = 120\underline{/120^\circ}\ \text{V}$$

The load impedance per phase is $19 + j13\ \Omega$, and the line impedance per phase is $1 + j2\ \Omega$. Solve for the line currents and neutral current.

12.10 For the circuit in Fig. 12.43, determine the current in the neutral line.

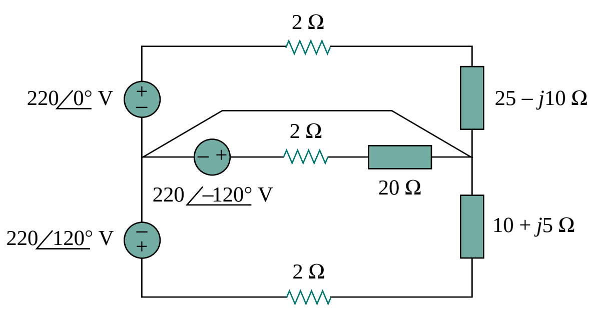


Figure 12.43 For Prob. 12.10.

Section 12.4 Balanced Wye-Delta Connection

12.11 For the three-phase circuit of Fig. 12.44, $\mathbf{I}_{bB} = 30\underline{/60^\circ}$ A and $\mathbf{V}_{BC} = 220\underline{/10^\circ}$ V. Find $\mathbf{V}_{an}$, $\mathbf{V}_{AB}$, $\mathbf{I}_{AC}$, and $\mathbf{Z}$.

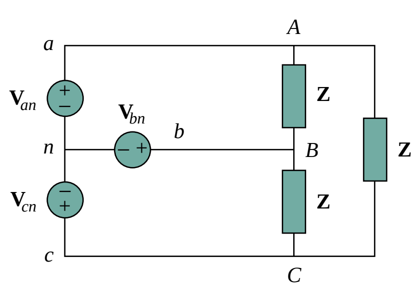


Figure 12.44 For Prob. 12.11.

12.12 Solve for the line currents in the Y-Δ circuit of Fig. 12.45. Take $\mathbf{Z}_\Delta = 60\underline{/45^\circ}\ \Omega$.

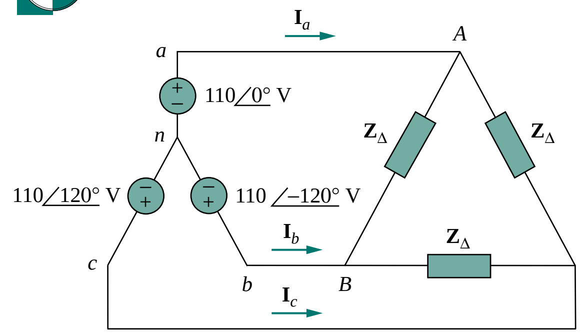


Figure 12.45 For Prob. 12.12.

12.13 The circuit in Fig. 12.46 is excited by a balanced three-phase source with a line voltage of 210 V. If $\mathbf{Z}_\ell = 1 + j1\ \Omega$, $\mathbf{Z}_\Delta = 24 - j30\ \Omega$, and $\mathbf{Z}_Y = 12 + j5\ \Omega$, determine the magnitude of the line current of the combined loads.

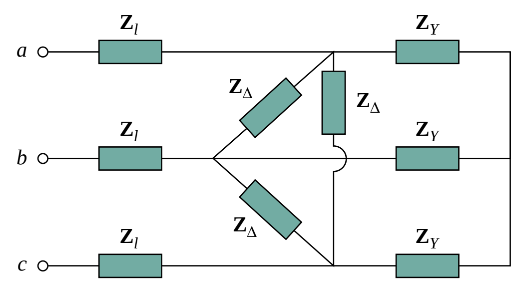


Figure 12.46 For Prob. 12.13.

12.14 A balanced delta-connected load has a phase current $\mathbf{I}_{AC} = 10\underline{/-30^\circ}$ A.

(a) Determine the three line currents assuming that the circuit operates in the positive phase sequence.

(b) Calculate the load impedance if the line voltage is $\mathbf{V}_{AB} = 110\underline{/0^\circ}$ V.

12.15 In a wye-delta three-phase circuit, the source is a balanced, positive phase sequence with $\mathbf{V}_{an} = 120\underline{/0^\circ}$ V. It feeds a balanced load with $\mathbf{Z}_\Delta = 9 + j12\ \Omega$ per phase through a balanced line with $\mathbf{Z}_\ell = 1 + j0.5\ \Omega$ per phase. Calculate the phase voltages and currents in the load.

12.16 If $\mathbf{V}_{an} = 440\angle 60^\circ$ V in the network of Fig. 12.47, find the load phase currents $\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$, and $\mathbf{I}_{CA}$.

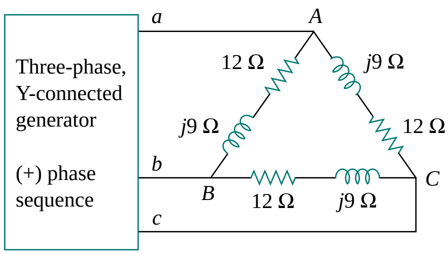


Figure 12.47 For Prob. 12.16.

Section 12.5 Balanced Delta-Delta Connection

12.17 For the Δ-Δ circuit of Fig. 12.48, calculate the phase and line currents.

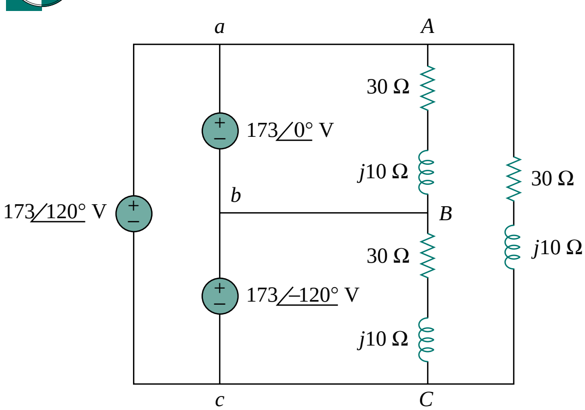


Figure 12.48 For Prob. 12.17.

12.18 Refer to the Δ-Δ circuit in Fig. 12.49. Find the line and phase currents. Assume that the load impedance is $12 + j9\ \Omega$ per phase.

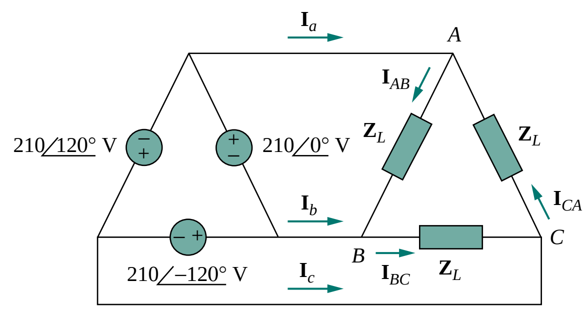


Figure 12.49 For Prob. 12.18.

12.19 Find the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ in the three-phase network of Fig. 12.50 below. Take $\mathbf{Z}_\Delta = 12 - j15\ \Omega$, $\mathbf{Z}_Y = 4 + j6\ \Omega$, and $\mathbf{Z}_\ell = 2\ \Omega$.

12.20 A balanced delta-connected source has phase voltage $\mathbf{V}_{ab} = 416\angle 30^\circ$ V and a positive phase sequence. If this is connected to a balanced delta-connected load, find the line and phase currents. Take the load impedance per phase as $60\angle 30^\circ\ \Omega$ and line impedance per phase as $1 + j1\ \Omega$.

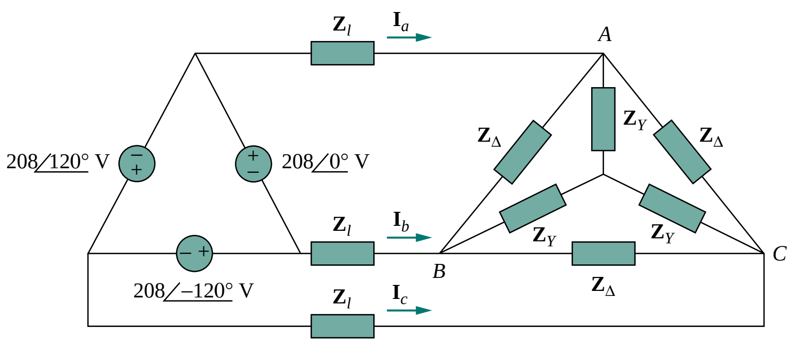


Figure 12.50 For Prob. 12.19.

Section 12.6 Balanced Delta-Wye Connection

12.21 In the circuit of Fig. 12.51, if $\mathbf{V}_{ab} = 440\underline{/10^\circ}$, $\mathbf{V}_{bc} = 440\underline{/250^\circ}$, $\mathbf{V}_{ca} = 440\underline{/130^\circ}$ V, find the line currents.

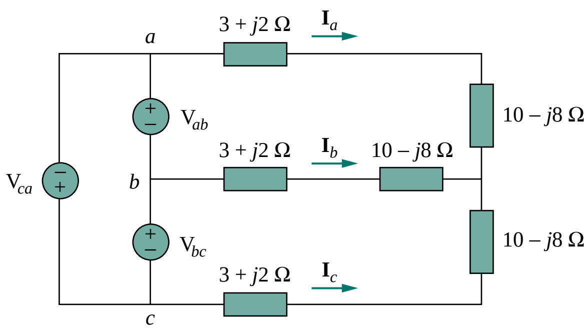


Figure 12.51 For Prob. 12.21.

12.22 For the balanced circuit in Fig. 12.52, $\mathbf{V}_{ab} = 125\underline{/0^\circ}$ V. Find the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

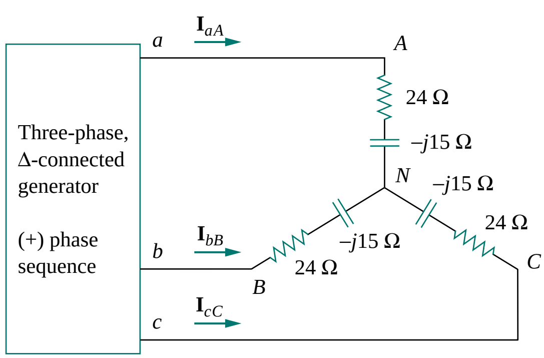


Figure 12.52 For Prob. 12.22.

12.23 In a balanced three-phase Δ-Y circuit, the source is connected in the positive sequence, with $\mathbf{V}_{ab} = 220\underline{/20^\circ}$ V and $\mathbf{Z}_Y = 20 + j15\ \Omega$. Find the line currents.

12.24 A delta-connected generator supplies a balanced wye-connected load with an impedance of $30\underline{/-60^\circ}\ \Omega$. If the line voltages of the generator have a magnitude of 400 V and are in the positive phase sequence, find the line current I_L and phase voltage V_p at the load.

Section 12.7 Power in a Balanced System

12.25 A balanced wye-connected load absorbs a total power of 5 kW at a leading power factor of 0.6 when connected to a line voltage of 240 V. Find the impedance of each phase and the total complex power of the load.

12.26 A balanced wye-connected load absorbs 50 kVA at a 0.6 lagging power factor when the line voltage is 440 V. Find the line current and the phase impedance.

12.27 A three-phase source delivers 4800 VA to a wye-connected load with a phase voltage of 208 V and a power factor of 0.9 lagging. Calculate the source line current and the source line voltage.

12.28 A balanced wye-connected load with a phase impedance of $10 - j16\ \Omega$ is connected to a balanced three-phase generator with a line voltage of 220 V. Determine the line current and the complex power absorbed by the load.

12.29 The total power measured in a three-phase system feeding a balanced wye-connected load is 12 kW at a power factor of 0.6 leading. If the line voltage is 208 V, calculate the line current I_L and the load impedance $\mathbf{Z}_Y$.

12.30 Given the circuit in Fig. 12.53 below, find the total complex power absorbed by the load.

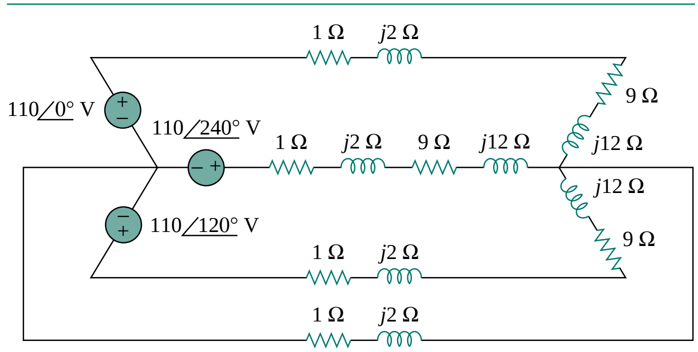


Figure 12.53 For Prob. 12.30.

12.31 Find the real power absorbed by the load in Fig. 12.54.

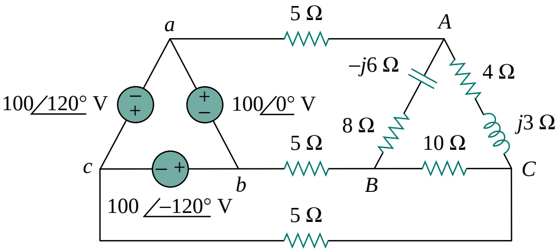


Figure 12.54 For Prob. 12.31.

12.32 For the three-phase circuit in Fig. 12.55, find the average power absorbed by the delta-connected load with $\mathbf{Z}_\Delta = 21 + j24\ \Omega$.

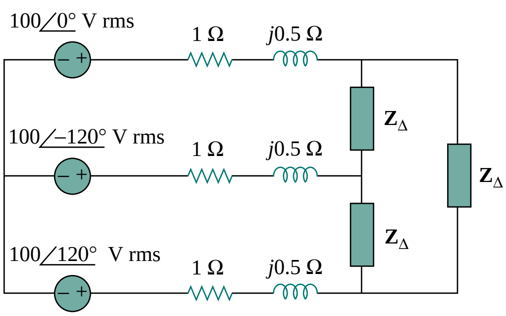


Figure 12.55 For Prob. 12.32.

12.33 A balanced delta-connected load draws 5 kW at a power factor of 0.8 lagging. If the three-phase system has an effective line voltage of 400 V, find the line current.

12.34 A balanced three-phase generator delivers 7.2 kW to a wye-connected load with impedance $30 - j40\ \Omega$ per phase. Find the line current I_L and the line voltage V_L.

12.35 Refer to Fig. 12.46. Obtain the complex power absorbed by the combined loads.

12.36 A three-phase line has an impedance of $1 + j3\ \Omega$ per phase. The line feeds a balanced delta-connected load, which absorbs a total complex power of $12 + j5$ kVA. If the line voltage at the load end has a magnitude of 240 V, calculate the magnitude of the line voltage at the source end and the source power factor.

12.37 A balanced wye-connected load is connected to the generator by a balanced transmission line with an impedance of $0.5 + j2\ \Omega$ per phase. If the load is rated at 450 kW, 0.708 power factor lagging, 440-V line voltage, find the line voltage at the generator.

12.38 A three-phase load consists of three 100-Ω resistors that can be wye- or delta-connected. Determine which connection will absorb the most average power from a three-phase source with a line voltage of 110 V. Assume zero line impedance.

12.39 The following three parallel-connected three-phase loads are fed by a balanced three-phase source.

Load 1: 250 kVA, 0.8 pf lagging
Load 2: 300 kVA, 0.95 pf leading
Load 3: 450 kVA, unity pf

If the line voltage is 13.8 kV, calculate the line current and the power factor of the source. Assume that the line impedance is zero.

Section 12.8 Unbalanced Three-Phase Systems

12.40 For the circuit in Fig. 12.56, $\mathbf{Z}_a = 6 - j8\ \Omega$, $\mathbf{Z}_b = 12 + j9\ \Omega$, and $\mathbf{Z}_c = 15\ \Omega$. Find the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$.

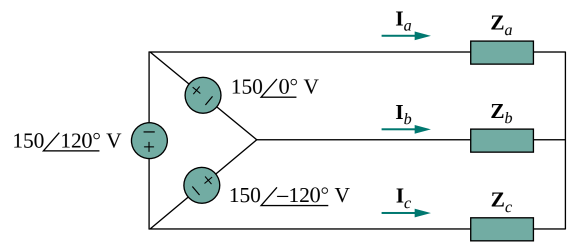


Figure 12.56 For Prob. 12.40.

12.41 A four-wire wye-wye circuit has

$$\mathbf{V}_{an} = 120\angle 120^\circ, \qquad \mathbf{V}_{bn} = 120\angle 0^\circ$$
$$\mathbf{V}_{cn} = 120\angle -120^\circ \text{ V}$$

If the impedances are

$$\mathbf{Z}_{AN} = 20\angle 60^\circ, \qquad \mathbf{Z}_{BN} = 30\angle 0^\circ$$
$$\mathbf{Z}_{cn} = 40\angle 30^\circ\ \Omega$$

find the current in the neutral line.

12.42 For the wye-connected load of Fig. 12.57, the line voltages all have a magnitude of 250 V and are in a positive phase sequence. Calculate the line currents and the neutral current.

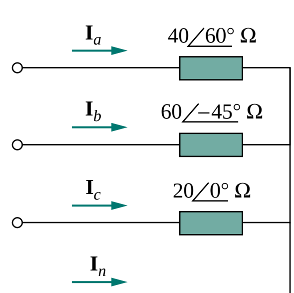


Figure 12.57 For Prob. 12.42.

12.43 A delta-connected load whose phase impedances are $\mathbf{Z}_{AB} = 50\ \Omega$, $\mathbf{Z}_{BC} = -j50\ \Omega$, and $\mathbf{Z}_{CA} = j50\ \Omega$ is fed by a balanced wye-connected three-phase source with $V_p = 100$ V. Find the phase currents.

12.44 A balanced three-phase wye-connected generator with $V_p = 220$ V supplies an unbalanced wye-connected load with $\mathbf{Z}_{AN} = 60 + j80\ \Omega$, $\mathbf{Z}_{BN} = 100 - j120\ \Omega$, and $\mathbf{Z}_{CN} = 30 + j40\ \Omega$. Find the total complex power absorbed by the load.

12.45 Refer to the unbalanced circuit of Fig. 12.58. Calculate:

(a) the line currents

(b) the real power absorbed by the load

(c) the total complex power supplied by the source

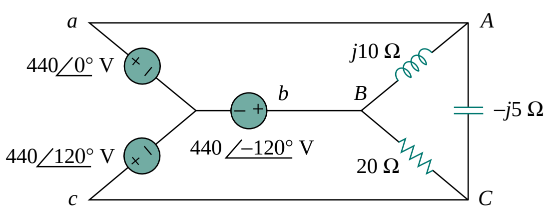


Figure 12.58 For Prob. 12.45.

Section 12.9 *PSpice* for Three-Phase Circuits

12.46 Solve Prob. 12.10 using *PSpice*.

12.47 The source in Fig. 12.59 is balanced and exhibits a positive phase sequence. If $f = 60$ Hz, use *PSpice* to find $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$.

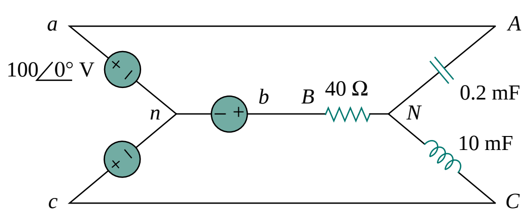


Figure 12.59 For Prob. 12.47.

12.48 Use *PSpice* to determine $\mathbf{I}_o$ in the single-phase, three-wire circuit of Fig. 12.60. Let $\mathbf{Z}_1 = 15 - j10\ \Omega$, $\mathbf{Z}_2 = 30 + j20\ \Omega$, and $\mathbf{Z}_3 = 12 + j5\ \Omega$.

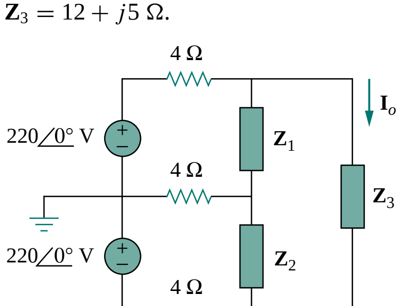


Figure 12.60 For Prob. 12.48.

12.49 Given the circuit in Fig. 12.61, use *PSpice* to determine currents $\mathbf{I}_{aA}$ and voltage $\mathbf{V}_{BN}$.

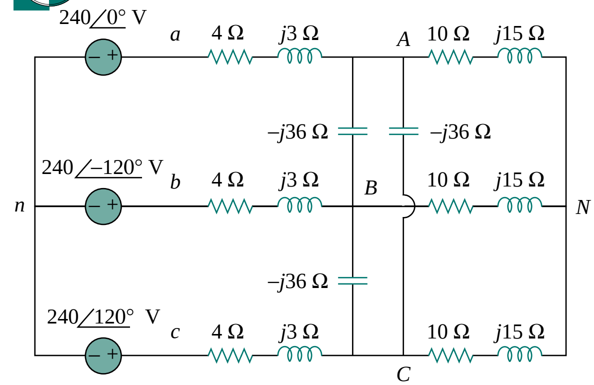


Figure 12.61 For Prob. 12.49.

12.50 The circuit in Fig. 12.62 operates at 60 Hz. Use *PSpice* to find the source current $\mathbf{I}_{ab}$ and the line current $\mathbf{I}_{bB}$.

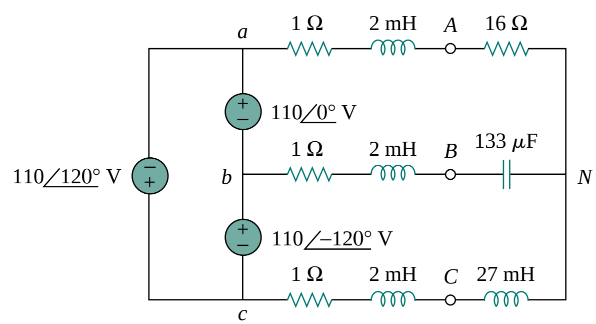


Figure 12.62 For Prob. 12.50.

12.51 For the circuit in Fig. 12.54, use *PSpice* to find the line currents and the phase currents.

12.52 A balanced three-phase circuit is shown in Fig. 12.63 on the next page. Use *PSpice* to find the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

Section 12.10 Applications

12.53 A three-phase, four-wire system operating with a 208-V line voltage is shown in Fig. 12.64. The source voltages are balanced. The power absorbed by the resistive wye-connected load is measured by the three-wattmeter method. Calculate:

(a) the voltage to neutral

(b) the currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_n$

(c) the readings of the wattmeters

(d) the total power absorbed by the load

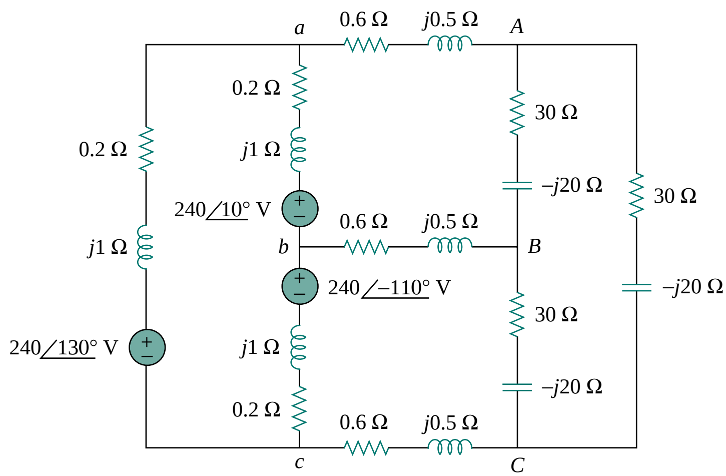


Figure 12.63 For Prob. 12.52.

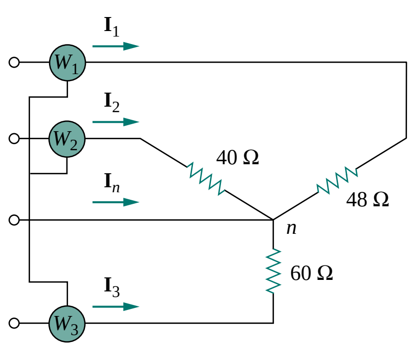


Figure 12.64 For Prob. 12.53.

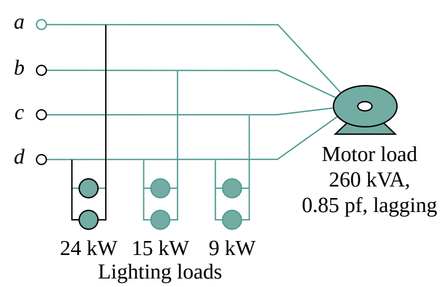


Figure 12.65 For Prob. 12.54.

*12.54 As shown in Fig. 12.65, a three-phase four-wire line with a phase voltage of 120 V supplies a balanced motor load at 260 kVA at 0.85 pf lagging. The motor load is connected to the three main lines marked *a*, *b*, and *c*. In addition, incandescent lamps (unity pf) are connected as follows: 24 kW from line *a* to the neutral, 15 kW from line *b* to the neutral, and 9 kW from line *a* to the neutral.

(a) If three wattmeters are arranged to measure the power in each line, calculate the reading of each meter.

(b) Find the current in the neutral line.

12.55 Meter readings for a three-phase wye-connected alternator supplying power to a motor indicate that the line voltages are 330 V, the line currents are 8.4 A, and the total line power is 4.5 kW. Find:

(a) the load in VA

(b) the load pf

(c) the phase current

(d) the phase voltage

12.56 The two-wattmeter method gives $P_1 = 1200$ W and $P_2 = -400$ W for a three-phase motor running on a 240-V line. Assume that the motor load is wye-connected and that it draws a line current of 6 A. Calculate the pf of the motor and its phase impedance.

*An asterisk indicates a challenging problem.

12.57 In Fig. 12.66, two wattmeters are properly connected to the unbalanced load supplied by a balanced source such that $\mathbf{V}_{ab} = 208\underline{/0^\circ}$ V with positive phase sequence.

(a) Determine the reading of each wattmeter.

(b) Calculate the total apparent power absorbed by the load.

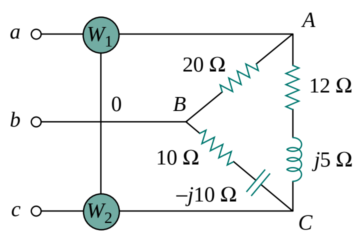


Figure 12.66 For Prob. 12.57.

12.58 If wattmeters W_1 and W_2 are properly connected respectively between lines a and b and lines b and c to measure the power absorbed by the delta-connected load in Fig. 12.44, predict their readings.

12.59 For the circuit displayed in Fig. 12.67, find the wattmeter readings.

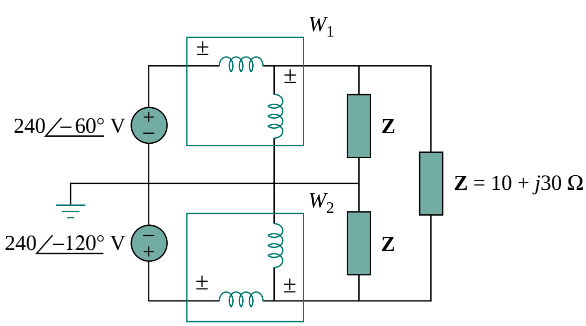


Figure 12.67 For Prob. 12.59.

12.60 Predict the wattmeter readings for the circuit in Fig. 12.68.

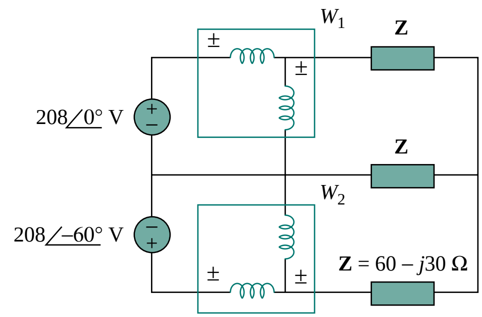


Figure 12.68 For Prob. 12.60.

12.61 A man has a body resistance of 600 Ω. How much current flows through his ungrounded body:

(a) when he touches the terminals of a 12-V autobattery?

(b) when he sticks his finger into a 120-V light socket?

12.62 Show that the I^2R losses will be higher for a 120-V appliance than for a 240-V appliance if both have the same power rating.

COMPREHENSIVE PROBLEMS

12.63 A three-phase generator supplied 3.6 kVA at a power factor of 0.85 lagging. If 2500 W are delivered to the load and line losses are 80 W per phase, what are the losses in the generator?

12.64 A three-phase 440-V, 51-kW, 60-kVA inductive load operates at 60 Hz and is wye-connected. It is desired to correct the power factor to 0.95 lagging. What value of capacitor should be placed in parallel with each load impedance?

12.65 A balanced three-phase generator has an abc phase sequence with phase voltage $\mathbf{V}_{an} = 255\underline{/0^\circ}$ V. The generator feeds an induction motor which may be represented by a balanced Y-connected load with an impedance of $12 + j5$ Ω per phase. Find the line currents and the load voltages. Assume a line impedance of 2 Ω per phase.

12.66 Three balanced loads are connected to a distribution line as depicted in Fig. 12.69. The loads are

Transformer: 12 kVA at 0.6 pf lagging
Motor: 16 kVA at 0.8 pf lagging
Unknown load: — — ——

If the line voltage is 220 V, the line current is 120 A, and the power factor of the combined load is 0.95 lagging, determine the unknown load.

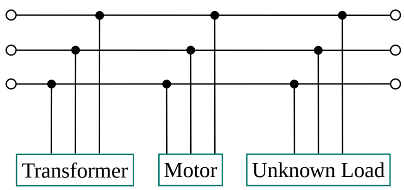


Figure 12.69 For Prob. 12.66.

12.67 A professional center is supplied by a balanced three-phase source. The center has four plants, each a balanced three-phase load as follows:

Load 1: 150 kVA at 0.8 pf leading
Load 2: 100 kW at unity pf
Load 3: 200 kVA at 0.6 pf lagging
Load 4: 80 kW and 95 kVAR (inductive)

If the line impedance is $0.02 + j0.05\ \Omega$ per phase and the line voltage at the loads is 480 V, find the magnitude of the line voltage at the source.

***12.68** Figure 12.70 displays a three-phase delta-connected motor load which is connected to a line voltage of 440 V and draws 4 kVA at a power factor of 72 percent lagging. In addition, a single 1.8 kVAR capacitor is connected between lines a and b, while a 800-W lighting load is connected between line c and neutral. Assuming the abc sequence and taking $\mathbf{V}_{an} = V_p\underline{/0^\circ}$, find the magnitude and phase angle of currents $\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$, and $\mathbf{I}_n$.

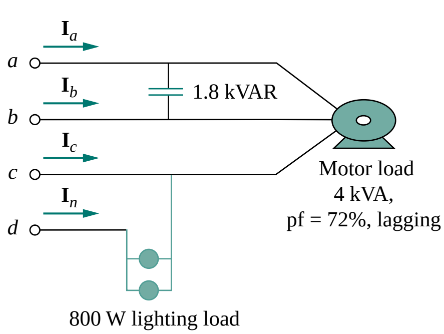


Figure 12.70 For Prob. 12.68.

12.69 Design a three-phase heater with suitable symmetric loads using wye-connected pure resistance. Assume that the heater is supplied by a 240-V line voltage and is to give 27 kW of heat.

12.70 For the single-phase three-wire system in Fig. 12.71, find currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{nN}$.

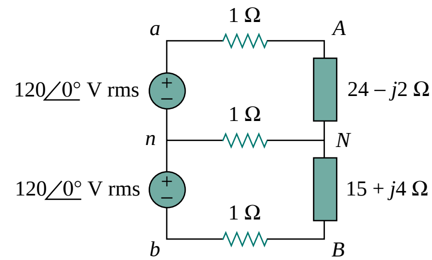


Figure 12.71 For Prob. 12.70.

12.71 Consider the single-phase three-wire system shown in Fig. 12.72. Find the current in the neutral wire and the complex power supplied by each source. Take $\mathbf{V}_s$ as a $115\underline{/0^\circ}$-V, 60-Hz source.

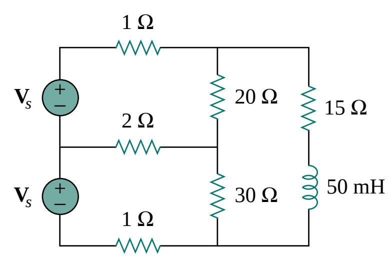


Figure 12.72 For Prob. 12.71.