Instructor's Solutions Manual
Part I

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to accompany

Thomas' Calculus, Early Transcendentals

Tenth Edition

Based on the original work by

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PRELIMINARY CHAPTER

P.1 LINES

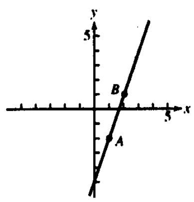
1. (a) $\Delta x = -1 - 1 = -2$
   $\Delta y = -1 - 2 = -3$

   (b) $\Delta x = -1 - (-3) = 2$
   $\Delta y = -2 - 2 = -4$

2. (a) $\Delta x = -8 - (-3) = -5$
   $\Delta y = 1 - 1 = 0$

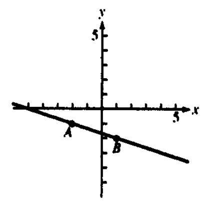
   (b) $\Delta x = 0 - 0 = 0$
   $\Delta y = -2 - 4 = -6$

3. (a)



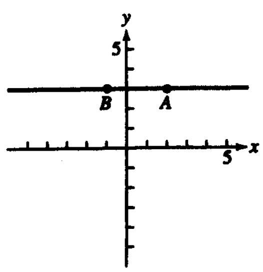
$$m = \frac{1-(-2)}{2-1} = \frac{3}{1} = 3$$

(b)



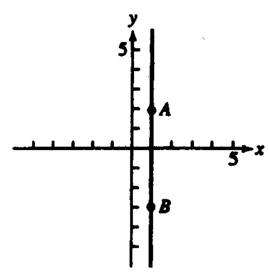
$$m = \frac{-2-(-1)}{1-(-2)} = \frac{-1}{3} = -\frac{1}{3}$$

4. (a)



$$m = \frac{3-3}{-1-2} = \frac{0}{-3} = 0$$

(b)



$$m = \frac{-3-2}{1-1} = \frac{-5}{0} \text{ (undefined)}$$

5. (a) $x = 2,\ y = 3$

   (b) $x = -1,\ y = \frac{4}{3}$

6. (a) $x = 0,\ y = -\sqrt{2}$

   (b) $x = -\pi,\ y = 0$

7. (a) $y = 1(x-1) + 1$

   (b) $y = -1[x-(-1)] + 1 = -1(x+1) + 1$

8. (a) $y = 2(x-0) + 3$

   (b) $y = -2[x-(-4)] + 0 = -2(x+4) + 0$

9. (a) $m = \frac{3-0}{2-0} = \frac{3}{2}$

   $y = \frac{3}{2}(x-0) + 0$

   $y = \frac{3}{2}x$

   (b) $m = \frac{1-1}{2-1} = \frac{0}{1} = 0$

   $y = 0(x-1) + 1$

   $y = 1$

$2y = 3x$

$3x - 2y = 0$

10. (a) $m = \frac{-2-0}{-2-(-2)} = \frac{-2}{0}$ (undefined)

Vertical line: $x = -2$

(b) $m = \frac{-2-1}{2-(-2)} = \frac{-3}{4} = -\frac{3}{4}$

$y = -\frac{3}{4}[x-(-2)] + 1$

$4y = -3(x+2) + 4$

$4y = -3x - 2$

$3x + 4y = -2$

11. (a) $y = 3x - 2$

(b) $y = -1x + 2$ or $y = -x + 2$

12. (a) $y = -\frac{1}{2}x - 3$

(b) $y = \frac{1}{3}x - 1$

13. The line contains $(0,0)$ and $(10,25)$.

$m = \frac{25-0}{10-0} = \frac{25}{10} = \frac{5}{2}$

$y = \frac{5}{2}x$

14. The line contains $(0,0)$ and $(5,2)$.

$m = \frac{2-0}{5-0} = \frac{2}{5}$

$y = \frac{2}{5}x$

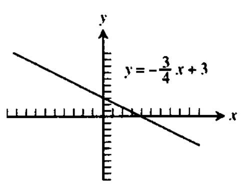
15. (a) $3x + 4y = 12$

$4y = -3x + 12$

$y = -\frac{3}{4}x + 3$

i) Slope: $-\frac{3}{4}$

ii) y-intercept: 3

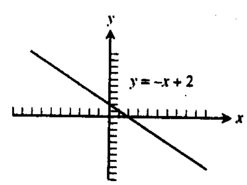


(b) $x + y = 2$

$y = -x + 2$

i) Slope: -1

ii) y-intercept: 2



16. (a) $\frac{x}{3} + \frac{y}{4} = 1$

$\frac{y}{4} = -\frac{x}{3} + 1$

$y = -\frac{4}{3}x + 4$

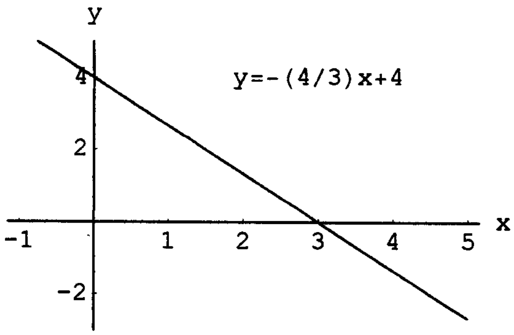
i) Slope: $-\frac{4}{3}$

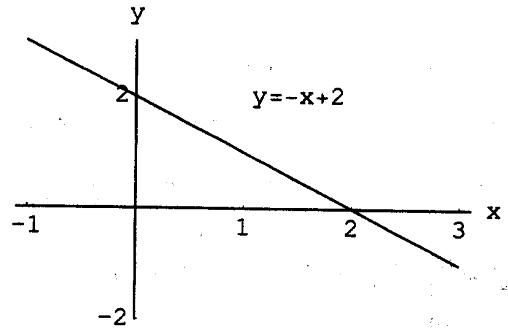
ii) y-intercept: 4

(b) $y = 2x + 4$

i) Slope: 2

ii) y-intercept: 4





17. (a) i) The desired line has slope -1 and passes through $(0,0)$: $y = -1(x-0)+0$ or $y = -x$.

ii) The desired line has slope $\frac{-1}{-1} = 1$ and passes through $(0,0)$: $y = 1(x-0)+0$ or $y = x$.

(b) i) The given equation is equivalent to $y = -2x+4$. The desired line has slope -2 and passes through $(-2,2)$: $y = -2(x+2)+2$ or $y = -2x-2$.

ii) The desired line has slope $\frac{-1}{-2} = \frac{1}{2}$ and passes through $(-2,2)$: $y = \frac{1}{2}(x+2)+2$ or $y = \frac{1}{2}x+3$.

18. (a) i) The given line is vertical, so we seek a vertical line through $(-2,4)$: $x = -2$.

ii) We seek a horizontal line through $(-2,4)$: $y = 4$.

(b) i) The given line is horizontal, so we seek a horizontal line through $\left(-1,\frac{1}{2}\right)$: $y = \frac{1}{2}$.

ii) We seek a vertical line through $\left(-1,\frac{1}{2}\right)$: $x = -1$.

19. $m = \frac{9-2}{3-1} = \frac{7}{2}$

$f(x) = \frac{7}{2}(x-1)+2 = \frac{7}{2}x - \frac{3}{2}$

Check: $f(5) = \frac{7}{2}(5) - \frac{3}{2} = 16$, as expected.

Since $f(x) = \frac{7}{2}x - \frac{3}{2}$, we have $m = \frac{7}{2}$ and $b = -\frac{3}{2}$.

20. $m = \frac{-4-(-1)}{4-2} = \frac{-3}{2} = -\frac{3}{2}$

$f(x) = -\frac{3}{2}(x-2)+(-1) = -\frac{3}{2}x+2$

Check: $f(6) = -\frac{3}{2}(6)+2 = -7$, as expected.

Since $f(x) = -\frac{3}{2}x+2$, we have $m = -\frac{3}{2}$ and $b = 2$.

21. $-\frac{2}{3} = \frac{y-3}{4-(-2)}$

$-\frac{2}{3}(6) = y-3$

$-4 = y-3$

$-1 = y$

22. $2 = \frac{2-(-2)}{x-(-8)}$

$2(x+8) = 4$

$x+8 = 2$

$x = -6$

23. $y = 1\cdot(x-3)+4$
$y = x-3+4$
$y = x+1$
This is the same as the equation obtained in Example 5.

24. (a) When $y = 0$, we have $\frac{x}{c} = 1$, so $x = c$.

When $x = 0$, we have $\frac{y}{d} = 1$, so $y = d$.

(b) When $y = 0$, we have $\frac{x}{c} = 2$, so $x = 2c$.

When $x = 0$, we have $\frac{y}{d} = 2$, so $y = 2d$.

The x-intercept is $2c$ and the y-intercept is $2d$.

25. (a) The given equations are equivalent to $y = -\frac{2}{k}x + \frac{3}{k}$ and $y = -x + 1$, respectively, so the slopes are $-\frac{2}{k}$ and -1. The lines are parallel when $-\frac{2}{k} = -1$, so $k = 2$.

(b) The lines are perpendicular when $-\frac{2}{k} = \frac{-1}{-1}$, so $k = -2$.

26. (a) $m \approx \frac{68 - 69.5}{0.4 - 0} = \frac{-1.5}{0.4} = -3.75$ degrees/inch

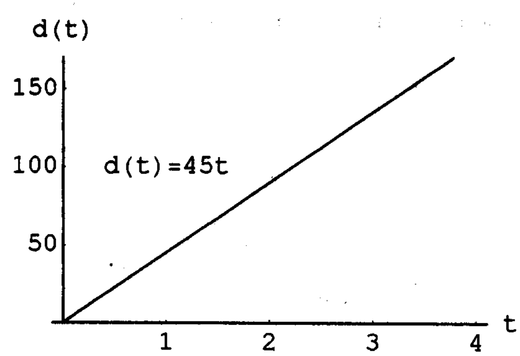
(b) $m \approx \frac{10 - 68}{4 - 0.4} = \frac{-58}{3.6} \approx -16.1$ degrees/inch

(c) $m \approx \frac{5 - 10}{4.7 - 4} = \frac{-5}{0.7} = -7.1$ degrees/inch

(d) Best insulator: Fiberglass insulation
Poorest insulator: Gypsum wallboard
The best insulator will have the largest temperature change per inch, because that will allow larger temperature differences on opposite sides of thinner layers.

27. Slope: $k = \frac{\Delta p}{\Delta d} = \frac{10.94 - 1}{100 - 0} = \frac{9.94}{100} = 0.0994$ atmospheres per meter
At 50 meters, the pressure is $p = 0.0994(50) + 1 = 5.97$ atmospheres.

28. (a) $d(t) = 45t$
(b)



(c) The slope is 45, which is the speed in miles per hour.
(d) Suppose the car has been traveling 45 mph for several hours when it is first observed at point P at time $t = 0$.
(e) The car starts at time $t = 0$ at a point 30 miles past P.

29. (a) Suppose x°F is the same as x°C.

$x = \frac{9}{5}x + 32$

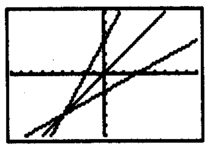
$\left(1 - \frac{9}{5}\right)x = 32$

$-\frac{4}{5}x = 32$

$x = -40$

Yes, −40°F is the same as −40°C.

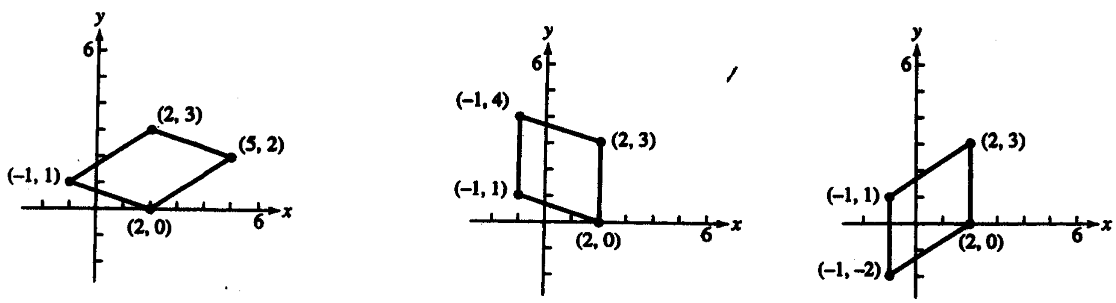
(b)



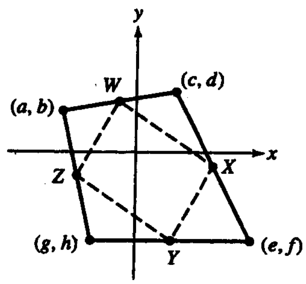
[−90, 90] by [−60, 60]

It is related because all three lines pass through the point $(-40, -40)$ where the Fahrenheit and Celsius temperatures are the same.

30. The coordinates of the three missing vertices are $(5, 2)$, $(-1, 4)$ and $(-1, -2)$, as shown below.



31.



Suppose that the vertices of the given quadrilateral are (a, b), (c, d), (e, f), and (g, h). Then the midpoints of the consecutive sides are $W\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$, $X\left(\frac{c+e}{2}, \frac{d+f}{2}\right)$, $Y\left(\frac{e+g}{2}, \frac{f+h}{2}\right)$, and $Z\left(\frac{g+a}{2}, \frac{h+b}{2}\right)$. When these four points are connected, the slopes of the sides of the resulting figure are:

WX: $\dfrac{\frac{d+f}{2}-\frac{b+d}{2}}{\frac{c+e}{2}-\frac{a+c}{2}} = \frac{f-b}{e-a}$

XY: $\dfrac{\frac{f+h}{2}-\frac{d+f}{2}}{\frac{e+g}{2}-\frac{c+e}{2}} = \frac{h-d}{g-c}$

ZY: $\dfrac{\frac{f+h}{2}-\frac{h+b}{2}}{\frac{e+g}{2}-\frac{g+a}{2}} = \frac{f-b}{e-a}$

WZ: $\dfrac{\frac{h+b}{2}-\frac{b+d}{2}}{\frac{g+a}{2}-\frac{a+c}{2}} = \frac{h-d}{g-c}$

Opposite sides have the same slope and are parallel.

32. The radius through (3, 4) has slope $\frac{4-0}{3-0} = \frac{4}{3}$.

The tangent line is perpendicular to this radius, so its slope is $\frac{-1}{4/3} = -\frac{3}{4}$. We seek the line of slope $-\frac{3}{4}$ that passes through (3, 4).

$y = -\frac{3}{4}(x-3)+4$

$y = -\frac{3}{4}x + \frac{9}{4} + 4$

$y = -\frac{3}{4}x + \frac{25}{4}$

33. (a) The equation for line L can be written as $y = -\frac{A}{B}x + \frac{C}{B}$, so its slope is $-\frac{A}{B}$. The perpendicular line has slope $\frac{-1}{-A/B} = \frac{B}{A}$ and passes through (a, b), so its equation is $y = \frac{B}{A}(x-a)+b$.

(b) Substituting $\frac{B}{A}(x-a)+b$ for y in the equation for line L gives:

$Ax + B\left[\frac{B}{A}(x-a)+b\right] = C$

$$A^2x + B^2(x-a) + ABb = AC$$
$$(A^2+B^2)x = B^2a + AC - ABb$$
$$x = \frac{B^2a + AC - ABb}{A^2+B^2}$$

Substituting the expression for x in the equation for line L gives:

$$A\left(\frac{B^2a + AC - ABb}{A^2+B^2}\right) + By = C$$

$$By = \frac{-A(B^2a + AC - ABb)}{A^2+B^2} + \frac{C(A^2+B^2)}{A^2+B^2}$$

$$By = \frac{-AB^2a - A^2C + A^2Bb + A^2C + B^2C}{A^2+B^2}$$

$$By = \frac{A^2Bb + B^2C - AB^2a}{A^2+B^2}$$

$$y = \frac{A^2b + BC - ABa}{A^2+B^2}$$

The coordinates of Q are $\left(\frac{B^2a + AC - ABb}{A^2+B^2}, \frac{A^2b + BC - ABa}{A^2+B^2}\right)$.

(c) Distance $= \sqrt{(x-a)^2 + (y-b)^2}$

$$= \sqrt{\left(\frac{B^2a + AC - ABb}{A^2+B^2} - a\right)^2 + \left(\frac{A^2b + BC - ABa}{A^2+B^2} - b\right)^2}$$

$$= \sqrt{\left(\frac{B^2a + AC - ABb - a(A^2+B^2)}{A^2+B^2}\right)^2 + \left(\frac{A^2b + BC - ABa - b(A^2+B^2)}{A^2+B^2}\right)^2}$$

$$= \sqrt{\left(\frac{AC - ABb - A^2a}{A^2+B^2}\right)^2 + \left(\frac{BC - ABa - B^2b}{A^2+B^2}\right)^2}$$

$$= \sqrt{\left(\frac{A(C - Bb - Aa)}{A^2+B^2}\right)^2 + \left(\frac{B(C - Aa - Bb)}{A^2+B^2}\right)^2}$$

$$= \sqrt{\frac{A^2(C - Aa - Bb)^2}{(A^2+B^2)^2} + \frac{B^2(C - Aa - Bb)^2}{(A^2+B^2)^2}}$$

$$= \sqrt{\frac{(A^2+B^2)(C - Aa - Bb)^2}{(A^2+B^2)^2}}$$

$$= \sqrt{\frac{(C - Aa - Bb)^2}{A^2+B^2}}$$

$$= \frac{|C - Aa - Bb|}{\sqrt{A^2 + B^2}}$$

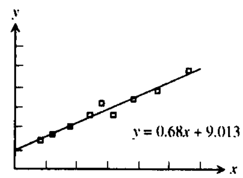
$$= \frac{|Aa + Bb - C|}{\sqrt{A^2 + B^2}}$$

34. The line of incidence passes through $(0,1)$ and $(1,0) \Rightarrow$ The line of reflection passes through $(1,0)$ and $(2,1)$ $\Rightarrow m = \frac{1-0}{2-1} = 1 \Rightarrow y - 0 = 1(x-1) \Rightarrow y = x - 1$ is the line of reflection.

35. $m = \frac{37.1}{100} = \frac{14}{\Delta x} \Rightarrow \Delta x = \frac{14}{.371}$. Therefore, distance between first and last rows is $\sqrt{(14)^2 + \left(\frac{14}{.371}\right)^2} \approx 40.25$ ft.

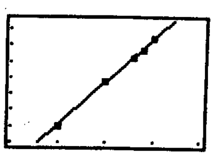
36. (a) $(-1,4)$ (b) $(3,-2)$ (c) $(5,2)$ (d) $(0,x)$
(e) $(-y,0)$ (f) $(-y,x)$ (g) $(3,-10)$

37. (a) $y = 0.680x + 9.013$
(b) The slope is 0.68. It represents the approximate average weight gain in pounds per month.
(c)



(d) When $x = 30$, $y \approx 0.680(30) + 9.013 = 29.413$.
She weighs about 29 pounds.

38. (a) $y = 1060.4233x - 2{,}077{,}548.669$
(b) The slope is 1060.4233. It represents the approximate rate of increase in earnings in dollars per year.
(c)



[1975, 1995] by [20,000, 35,000]

(d) When $x = 2000$, $y \approx 1060.4233(2000) - 2{,}077{,}548.669 \approx 43{,}298$.
In 2000, the construction workers' average annual compensation will be about $43,298.

39. (a) $y = 5632x - 11{,}080{,}280$
(b) The rate at which the median price is increasing in dollars per year
(c) $y = 2732x - 5{,}362{,}360$
(d) The median price is increasing at a rate of about $5632 per year in the Northeast, and about $2732 per year in the Midwest. It is increasing more rapidly in the Northeast.

P.2 FUNCTIONS AND GRAPHS

1. base = x; $(\text{height})^2 + \left(\frac{x}{2}\right)^2 = x^2 \Rightarrow \text{height} = \frac{\sqrt{3}}{2}x$; area is $a(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(x)\left(\frac{\sqrt{3}x}{2}\right) = \frac{\sqrt{3}}{4}x^2$; perimeter is $p(x) = x + x + x = 3x$.

2. s = side length $\Rightarrow s^2 + s^2 = d^2 \Rightarrow s = \frac{d}{\sqrt{2}}$; and area is $a = s^2 \Rightarrow a = \frac{1}{2}d^2$

3. Let D = diagonal of a face of the cube and ℓ = the length of an edge. Then $\ell^2 + D^2 = d^2$ and (by Exercise 2) $D^2 = 2\ell^2 \Rightarrow 3\ell^2 = d^2 \Rightarrow \ell = \frac{d}{\sqrt{3}}$. The surface area is $6\ell^2 = \frac{6d^2}{3} = 2d^2$ and the volume is $\ell^3 = \left(\frac{d}{\sqrt{3}}\right)^{3/2}$ $= \frac{d^3}{3\sqrt{3}}$.

4. The coordinates of P are $(x, \sqrt{x})$ so the slope of the line joining P to the origin is $m = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$ $(x > 0)$. Thus $\sqrt{x} = \frac{1}{m}$ and the x-coordinate of P is $x = \frac{1}{m^2}$; the y-coordinate of P is $y = \frac{1}{m}$.

5. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Is the graph of a function of x since any vertical line intersects the graph at most once.

6. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Not the graph of a function of x since it fails the vertical line test.

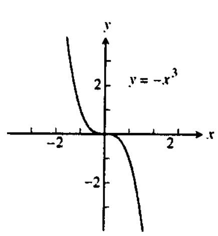
7. (a) domain = $(-\infty, \infty)$; range = $[1, \infty)$ (b) domain = $[0, \infty)$; range = $(-\infty, 1]$

8. (a) domain = $(0, \infty)$; y in range $\Rightarrow y = \frac{1}{\sqrt{t}}$, $t > 0 \Rightarrow y^2 = \frac{1}{t}$ and $y > 0 \Rightarrow$ y can be any positive real number $\Rightarrow$ range = $(0, \infty)$.
   (b) domain = $[0, \infty)$; y in range $\Rightarrow y = \frac{1}{1+\sqrt{t}}$, $t > 0$. If $t = 0$, then $y = 1$ and as t increases, y becomes a smaller and smaller positive real number $\Rightarrow$ range = $(0, 1]$.

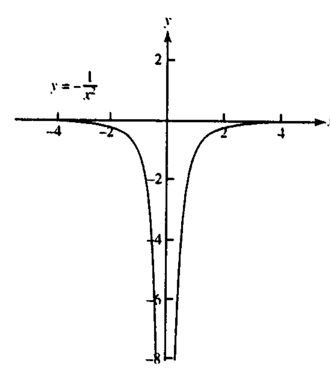
9. $4 - z^2 = (2 - z)(2 + z) \geq 0 \Leftrightarrow z \in [-2, 2]$ = domain. Largest value is $g(0) = \sqrt{4} = 2$ and smallest value is $g(-2) = g(2) = \sqrt{0} = 0 \Rightarrow$ range = $[0, 2]$.

10. domain = $(-\infty, \infty)$; range = $(-\infty, \infty)$

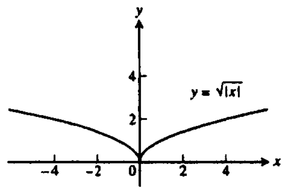
11. (a) Symmetric about the origin



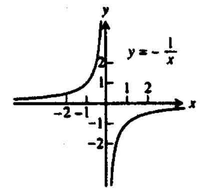
(b) Symmetric about the y-axis



12. (a) Symmetric about the y-axis

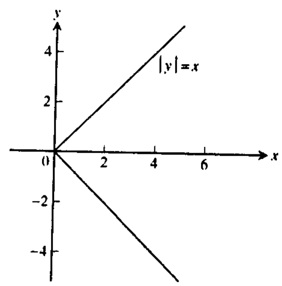


(b) Symmetric about the origin

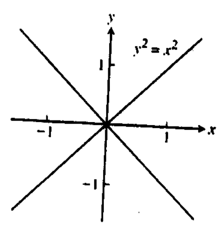


13. Neither graph passes the vertical line test

(a)

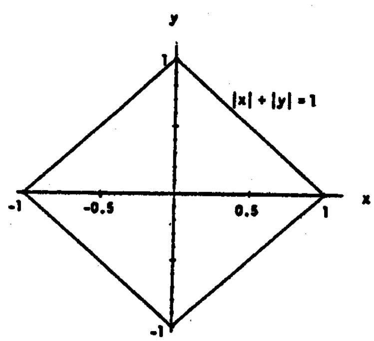


(b)

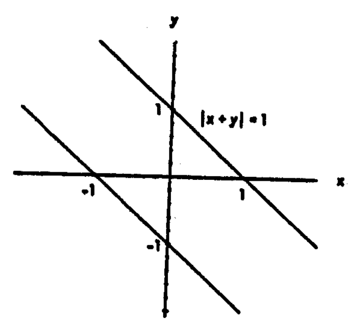


14. Neither graph passes the vertical line test

(a)



(b)



$$|x+y| = 1 \Leftrightarrow \left\{\begin{matrix} x+y=1 \\ \text{or} \\ x+y=-1 \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} y=1-x \\ \text{or} \\ y=-1-x \end{matrix}\right\}$$

15. (a) even
(b) odd

16. (a) even
(b) neither

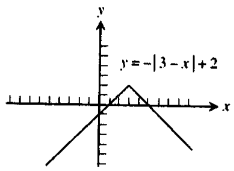
17. (a) odd
(b) even

18. (a) even
(b) odd

19. (a) neither
(b) even

20. (a) even
(b) even

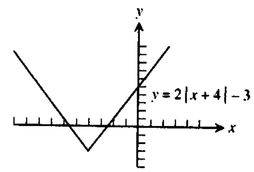
21. (a)



Note that $f(x) = -|x-3| + 2$, so its graph is the graph of the absolute value function reflected across the x-axis and then shifted 3 units right and 2 units upward.

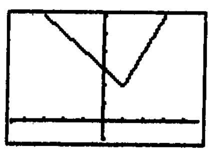
$(-\infty, \infty)$
$(-\infty, 2]$

(b) The graph of $f(x)$ is the graph of the absolute value function stretched vertically by a factor of 2 and then shifted 4 units to the left and 3 units downward



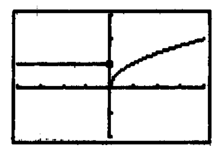
$(-\infty, \infty)$ or all real numbers
$[-3, \infty)$

22. (a)


[−4.7, 4.7] by [−1, 6]

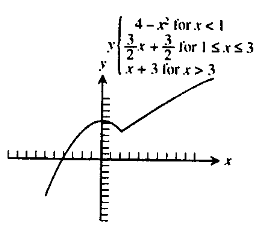
$(-\infty, \infty)$ or all real numbers
$[2, \infty)$

(b)


[−4, 4] by [−2, 3]

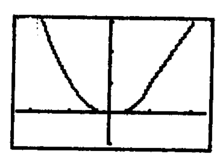
$(-\infty, \infty)$ or all real numbers
$[0, \infty)$

23. (a)



(b) $(-\infty, \infty)$ or all real numbers
(c) $(-\infty, \infty)$ or all real numbers

24. (a)


[−2.35, 2.35] by [−1, 3]

(b) $(-\infty, \infty)$ or all real numbers
(c) $[0, \infty)$

25. Because if the vertical line test holds, then for each x-coordinate, there is at most one y-coordinate giving a point on the curve. This y-coordinate corresponds to the value assigned to the x-coordinate. Since there is only one y-coordinate, the assignment is unique.

26. If the curve is not $y = 0$, there must be a point (x, y) on the curve where $y \neq 0$. That would mean that (x, y) and $(x, -y)$ are two different points on the curve and it is not the graph of a function, since it fails the vertical line test.

27. (a) Line through $(0,0)$ and $(1,1)$: $y = x$
Line through $(1,1)$ and $(2,0)$: $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x+2, & 1 < x \le 2 \end{cases}$$

(b) $$f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$$

(c) Line through $(0,2)$ and $(2,0)$: $y = -x + 2$

Line through $(2,1)$ and $(5,0)$: $m = \frac{0-1}{5-2} = \frac{-1}{3} = -\frac{1}{3}$, so $y = -\frac{1}{3}(x-2) + 1 = -\frac{1}{3}x + \frac{5}{3}$

$$f(x) = \begin{cases} -x+2, & 0 < x \le 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \le 5 \end{cases}$$

(d) Line through $(-1,0)$ and $(0,-3)$: $m = \frac{-3-0}{0-(-1)} = -3$, so $y = -3x - 3$

Line through $(0,3)$ and $(2,-1)$: $m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$, so $y = -2x + 3$

$$f(x) = \begin{cases} -3x-3, & -1 < x \le 0 \\ -2x+3, & 0 < x \le 2 \end{cases}$$

28. (a) Line through $(-1,1)$ and $(0,0)$: $y = -x$

Line through $(0,1)$ and $(1,1)$: $y = 1$

Line through $(1,1)$ and $(3,0)$: $m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2}$, so $y = -\frac{1}{2}(x-1) + 1 = -\frac{1}{2}x + \frac{3}{2}$

$$f(x) = \begin{cases} -x, & -1 \le x < 0 \\ 1, & 0 < x \le 1 \\ -\frac{1}{2}x + \frac{3}{2}, & 1 < x < 3 \end{cases}$$

(b) Line through $(-2,-1)$ and $(0,0)$: $y = \frac{1}{2}x$

Line through $(0,2)$ and $(1,0)$: $y = -2x + 2$

Line through $(1,-1)$ and $(3,-1)$: $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x, & -2 \le x \le 0 \\ -2x+2, & 0 < x \le 1 \\ -1, & 1 < x \le 3 \end{cases}$$

(c) Line through $\left(\frac{T}{2}, 0\right)$ and $(T, 1)$: $m = \frac{1-0}{T-(T/2)} = \frac{2}{T}$, so $y = \frac{2}{T}\left(x - \frac{T}{2}\right) + 0 = \frac{2}{T}x - 1$

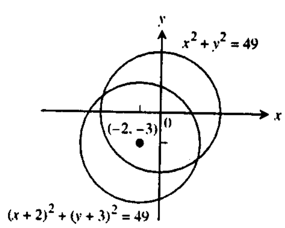
$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \le T \end{cases}$$

(d) $$f(x) = \begin{cases} A, & 0 \le x < \frac{T}{2} \\ -A, & \frac{T}{2} \le x < T \\ A, & T \le x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \le x \le 2T \end{cases}$$

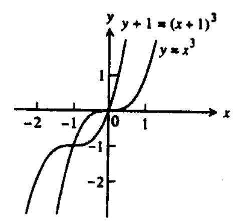
29. (a) Position 4 (b) Position 1 (c) Position 2 (d) Position 3

30. (a) $y = -(x-1)^2 + 4$ (b) $y = -(x+2)^2 + 3$ (c) $y = -(x+4)^2 - 1$ (d) $y = -(x-2)^2$

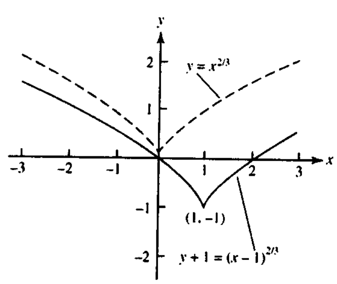
31.



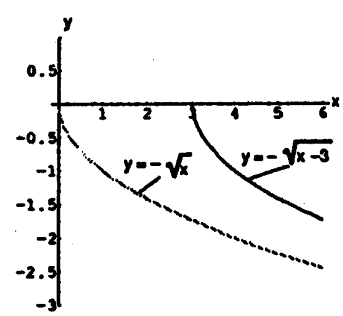
32.



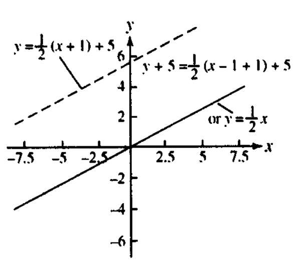
33.



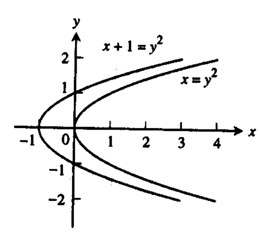
34.



35.



36.



37. (a) $f(g(0)) = f(-3) = 2$
(b) $g(f(0)) = g(5) = 22$
(c) $f(g(x)) = f(x^2 - 3) = x^2 - 3 + 5 = x^2 + 2$
(d) $g(f(x)) = g(x+5) = (x+5)^2 - 3 = x^2 + 10x + 22$
(e) $f(f(-5)) = f(0) = 5$
(f) $g(g(2)) = g(1) = -2$
(g) $f(f(x)) = f(x+5) = (x+5) + 5 = x + 10$
(h) $g(g(x)) = g(x^2 - 3) = (x^2 - 3)^2 - 3 = x^4 - 6x^2 + 6$

38. (a) $f\left(g\left(\frac{1}{2}\right)\right) = f\left(\frac{2}{3}\right) = -\frac{1}{3}$
(b) $g\left(f\left(\frac{1}{2}\right)\right) = g\left(-\frac{1}{2}\right) = 2$
(c) $f(g(x)) = f\left(\frac{1}{x+1}\right) = \frac{1}{x+1} - 1 = \frac{-x}{x+1}$
(d) $g(f(x)) = g(x-1) = \frac{1}{(x-1)+1} = \frac{1}{x}$
(e) $f(f(2)) = f(1) = 0$
(f) $g(g(2)) = g\left(\frac{1}{3}\right) = \frac{1}{\frac{4}{3}} = \frac{3}{4}$
(g) $f(f(x)) = f(x-1) = (x-1) - 1 = x - 2$
(h) $g(g(x)) = g\left(\frac{1}{x+1}\right) = \frac{1}{\frac{1}{x+1} + 1} = \frac{x+1}{x+2}$ $(x \neq -1 \text{ and } x \neq -2)$

39. (a) $u(v(f(x))) = u\left(v\left(\frac{1}{x}\right)\right) = u\left(\frac{1}{x^2}\right) = 4\left(\frac{1}{x}\right)^2 - 5 = \frac{4}{x^2} - 5$
(b) $u(f(v(x))) = u\left(f\left(x^2\right)\right) = u\left(\frac{1}{x^2}\right) = 4\left(\frac{1}{x^2}\right) - 5 = \frac{4}{x^2} - 5$
(c) $v(u(f(x))) = v\left(u\left(\frac{1}{x}\right)\right) = v\left(4\left(\frac{1}{x}\right) - 5\right) = \left(\frac{4}{x} - 5\right)^2$
(d) $v(f(u(x))) = v(f(4x-5)) = v\left(\frac{1}{4x-5}\right) = \left(\frac{1}{4x-5}\right)^2$
(e) $f(u(v(x))) = f\left(u\left(x^2\right)\right) = f\left(4\left(x^2\right) - 5\right) = \frac{1}{4x^2 - 5}$
(f) $f(v(u(x))) = f(v(4x-5)) = f\left((4x-5)^2\right) = \frac{1}{(4x-5)^2}$

40. (a) $h(g(f(x))) = h\left(g\left(\sqrt{x}\right)\right) = h\left(\frac{\sqrt{x}}{4}\right) = 4\left(\frac{\sqrt{x}}{4}\right) - 8 = \sqrt{x} - 8$
(b) $h(f(g(x))) = h\left(f\left(\frac{x}{4}\right)\right) = h\left(\sqrt{\frac{x}{4}}\right) = 4\sqrt{\frac{x}{4}} - 8 = 2\sqrt{x} - 8$

(c) $g(h(f(x))) = g(h(\sqrt{x})) = g(4\sqrt{x} - 8) = \frac{4\sqrt{x} - 8}{4} = \sqrt{x} - 2$

(d) $g(f(h(x))) = g(f(4x - 8)) = g(\sqrt{4x - 8}) = \frac{\sqrt{4x - 8}}{4} = \frac{\sqrt{x - 2}}{2}$

(e) $f(g(h(x))) = f(g(4x - 8)) = f\left(\frac{4x - 8}{4}\right) = f(x - 2) = \sqrt{x - 2}$

(f) $f(h(g(x))) = f\left(h\left(\frac{x}{4}\right)\right) = f\left(4\left(\frac{x}{4}\right) - 8\right) = f(x - 8) = \sqrt{x - 8}$

41. (a) $y = g(f(x))$ (b) $y = j(g(x))$
(c) $y = g(g(x))$ (d) $y = j(j(x))$
(e) $y = g(h(f(x)))$ (f) $y = h(j(f(x)))$

42. (a) $y = f(j(x))$ (b) $y = h(g(x)) = g(h(x))$
(c) $y = h(h(x))$ (d) $y = f(f(x))$
(e) $y = j(g(f(x)))$ (f) $y = g(f(h(x)))$

43. (a) Since $(f \circ g)(x) = \sqrt{g(x) - 5} = \sqrt{x^2 - 5}$, $g(x) = x^2$.

(b) Since $(f \circ g)(x) = 1 + \frac{1}{g(x)} = x$, we know that $\frac{1}{g(x)} = x - 1$, so $g(x) = \frac{1}{x - 1}$.

(c) Since $(f \circ g)(x) = f\left(\frac{1}{x}\right) = x$, $f(x) = \frac{1}{x}$.

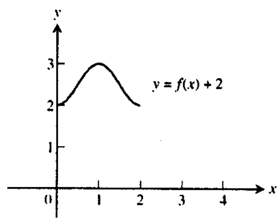
(d) Since $(f \circ g)(x) = f(\sqrt{x}) = |x|$, $f(x) = x^2$.
The completed table is shown. Note that the absolute value sign in part (d) is optional.

g(x)	f(x)	(f ∘ g)(x)
x^2	$\sqrt{x - 5}$	$\sqrt{x^2 - 5}$
$\frac{1}{x - 1}$	$1 + \frac{1}{x}$	$x, x \neq -1$
$\frac{1}{x}$	$\frac{1}{x}$	$x, x \neq 0$
$\sqrt{x}$	x^2	$\lvert x \rvert, x \geq 0$

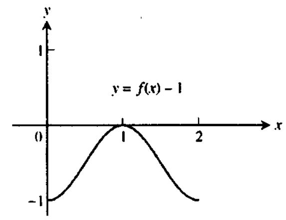
44.

	g(x)	f(x)	(f ∘ g)(x)
(a)	$x - 7$	$\sqrt{x}$	$\sqrt{x - 7}$
(b)	$x + 2$	$3x$	$3(x + 2) = 3x + 6$
(c)	x^2	$\sqrt{x - 5}$	$\sqrt{x^2 - 5}$
(d)	$\frac{x}{x - 1}$	$\frac{x}{x - 1}$	$\frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = \frac{x}{x - (x - 1)} = x$
(e)	$\frac{1}{x - 1}$	$1 + \frac{1}{x}$	$1 + \frac{1}{\frac{1}{x-1}} = 1 + (x - 1) = x$
(f)	$\frac{1}{x}$	$\frac{1}{x}$	$\frac{1}{\frac{1}{x}} = x$

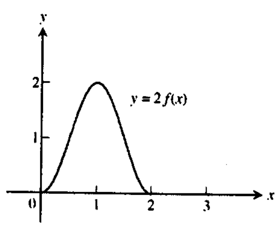
45. (a) domain: $[0, 2]$; range: $[2, 3]$



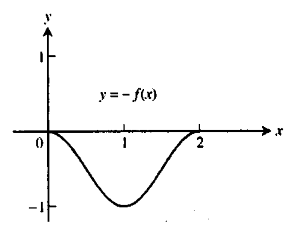
(b) domain: $[0, 2]$; range: $[-1, 0]$



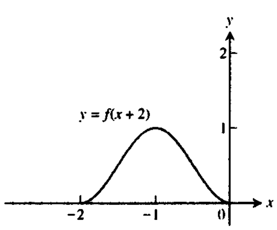
(c) domain: $[0, 2]$; range: $[0, 2]$



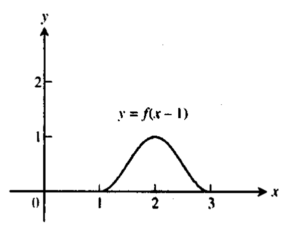
(d) domain: $[0, 2]$; range: $[-1, 0]$



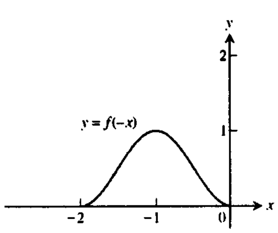
(e) domain: $[-2, 0]$; range: $[0, 1]$



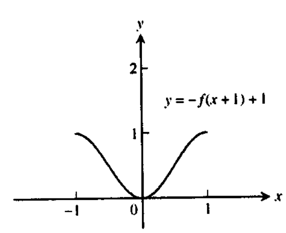
(f) domain: $[1, 3]$; range: $[0, 1]$



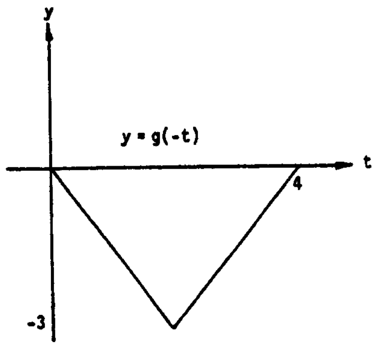
(g) domain: $[-2, 0]$; range: $[0, 1]$



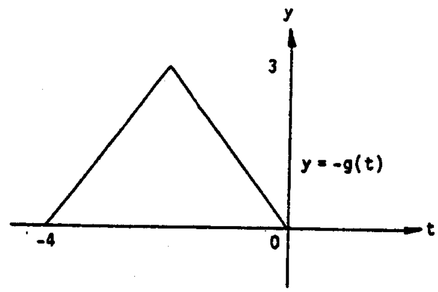
(h) domain: $[-1, 1]$; range: $[0, 1]$



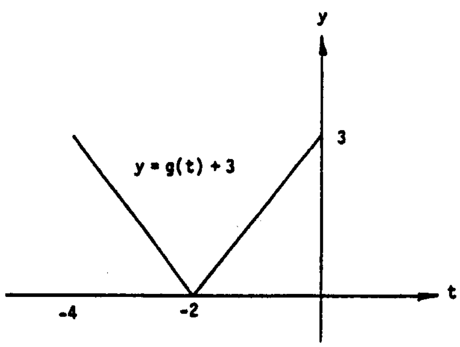
46. (a) domain: $[0,4]$; range: $[-3,0]$



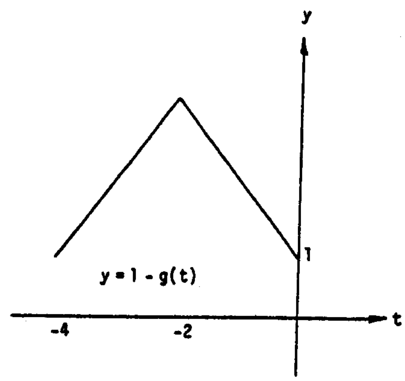
(b) domain: $[-4,0]$; range: $[0,3]$



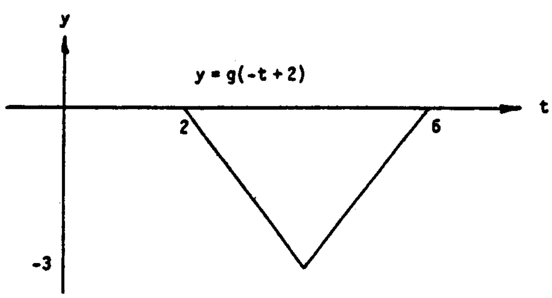
(c) domain: $[-4,0]$; range: $[0,3]$



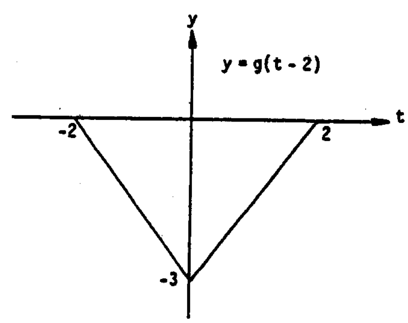
(d) domain: $[-4,0]$; range: $[1,4]$



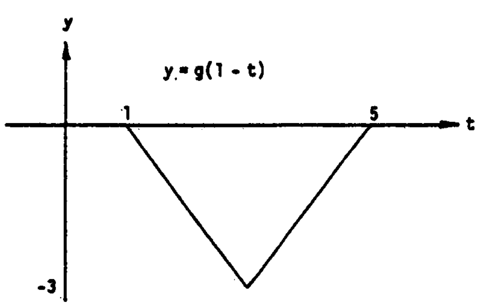
(e) domain: $[2,6]$; range: $[-3,0]$



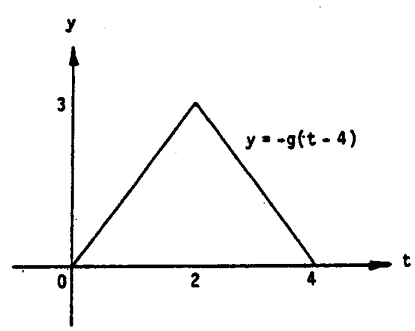
(f) domain: $[-2,2]$; range: $[-3,0]$



(g) domain: $[1,5]$; range: $[-3,0]$



(h) domain: $[0,4]$; range: $[0,3]$



47. (a) Because the circumference of the original circle was 8π and a piece of length x was removed.

(b) $r = \frac{8\pi - x}{2\pi} = 4 - \frac{x}{2\pi}$

(c) $h = \sqrt{16 - r^2} = \sqrt{16 - \left(4 - \frac{x}{2\pi}\right)} = \sqrt{16 - \left(16 - \frac{4x}{\pi} + \frac{x^2}{4\pi^2}\right)} = \sqrt{\frac{4x}{\pi} - \frac{x^2}{4\pi^2}} = \sqrt{\frac{16\pi x}{4\pi^2} - \frac{x^2}{4\pi^2}} = \frac{\sqrt{16\pi x - x^2}}{2\pi}$

(d) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{8\pi - x}{2\pi}\right)^2 \cdot \frac{\sqrt{16\pi x - x^2}}{2\pi} = \frac{(8\pi - x)^2\sqrt{16\pi x - x^2}}{24\pi^2}$

48. (a) Note that 2 mi = 10,560 ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at \$180 per foot and (10,560 − x) feet of land cable at \$100 per foot. The cost is $C(x) = 180\sqrt{800^2 + x^2} + 100(10{,}560 - x)$

(b) $C(0) = \$1{,}200{,}000$
$C(500) \approx \$1{,}175{,}812$
$C(1000) \approx \$1{,}186{,}512$
$C(1500) = \$1{,}212{,}000$
$C(2000) \approx \$1{,}243{,}732$
$C(2500) \approx \$1{,}278{,}479$
$C(3000) \approx \$1{,}314{,}870$
Values beyond this are all larger. It would appear that the least expensive location is less than 2000 ft from point P.

49. (a) Yes. Since $(f \cdot g)(-x) = f(-x) \cdot g(-x) = f(x) \cdot g(x) = (f \cdot g)(x)$, the function $(f \cdot g)(x)$ will also be even.

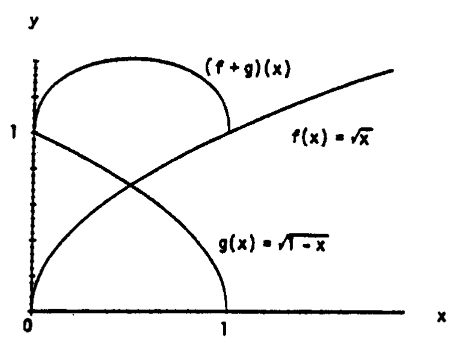
(b) The product will be even, since
$$\begin{aligned}(f \cdot g)(-x) &= f(-x) \cdot g(-x) \\ &= (-f(x)) \cdot (-g(x)) \\ &= f(x) \cdot g(x) \\ &= (f \cdot g)(x).\end{aligned}$$

(c) Yes, $f(x) = 0$ is both even and odd since $f(-x) = -f(x) = f(x)$

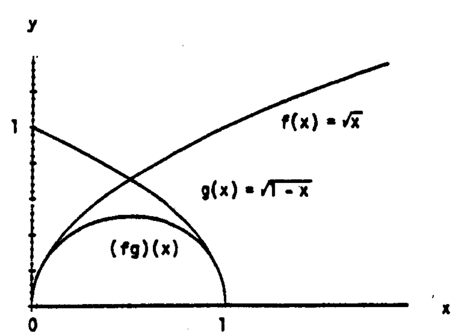
50. (a) Pick 11, for example: $11 + 5 = 16 \to 2 \cdot 16 = 32 \to 32 - 6 = 26 \to 26/2 = 13 \to 13 - 2 = 11$, the original number.

(b) $f(x) = \frac{2(x+5) - 6}{2} - 2 = x$, the number you started with.

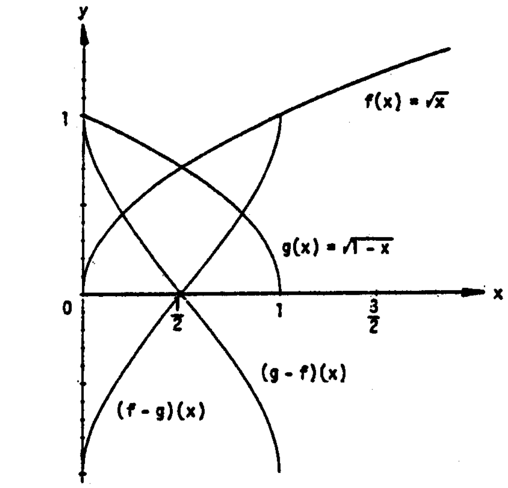
51. (a)



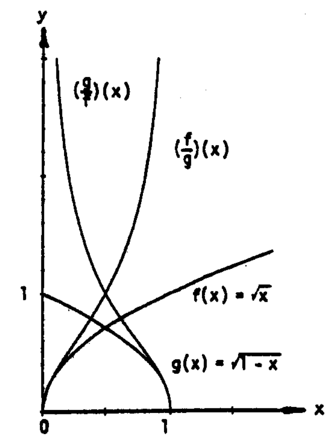
(b)



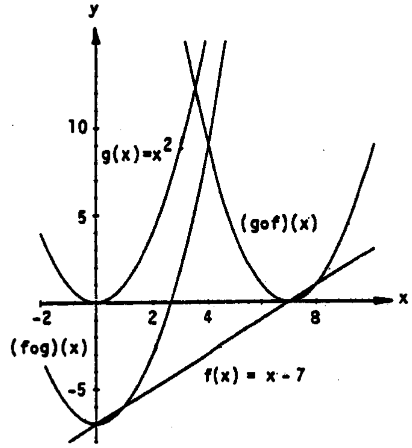
(c)



(d)

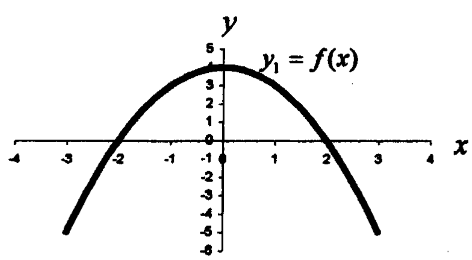


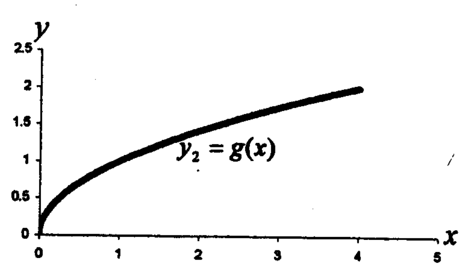
52.

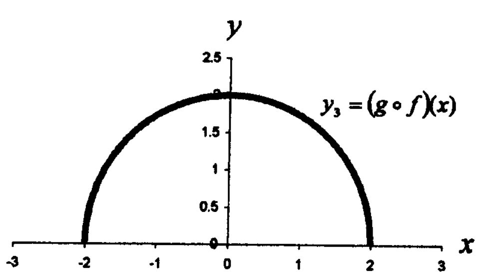


53. (a) $y_4 = (f \circ g)(x)$; $y_3 = (g \circ f)(x)$

(b)

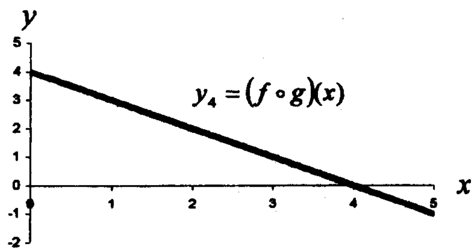






$D(g \circ f) = [-2, 2]$; The domain of $g \circ f$ is the set of all values of x in the domain of f for which the values $y_1 = f(x)$ are in the domain of g.
$R(g \circ f) = [0, 2]$; The range of $g \circ f$ is the subset of the range of g that includes all the values of g(x) evaluated at the values from the range of f where g(x) is defined.

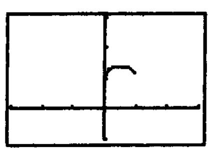
(c) The graphs of $y_1 = f(x)$ and $y_2 = g(x)$ are shown in part (a).



$D(f \circ g) = [0, \infty)$; The domain of $f \circ g$ is the set of all values of x in the domain of g for which the values $y_2 = g(x)$ are in the domain of f.
$R(f \circ g) = (-\infty, 4]$; The range of $f \circ g$ is the subset of the range of f that includes all the values of f(x) evaluated at the values from the range of g where f(x) is defined.

(d) $(g \circ f)(x) = \sqrt{4 - x^2}$; $D(g \circ f) = [-2, 2]$; $R(g \circ f) = [0, 2]$
$(f \circ g)(x) = 4 - (\sqrt{x})^2 = 4 - x$ for $x \geq 0$; $D(f \circ g) = [0, \infty)$; $R(f \circ g) = (-\infty, 4]$

54. (a)



$[-3, 3]$ by $[-1, 3]$

(b) Domain of y_1: $[0, \infty)$
Domain of y_2: $(-\infty, 1]$
Domain of y_3: $[0, 1]$

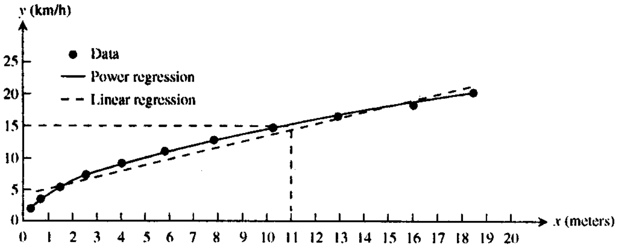
(c) The functions $y_1 - y_2$, $y_2 - y_1$, and $y_1 \cdot y_2$ all have domain $[0, 1]$, the same as the domain of $y_1 + y_2$ found in part (b).

Domain of $\frac{y_1}{y_2}$: $[0, 1)$

Domain of $\frac{y_2}{y_1}$: $(0, 1]$

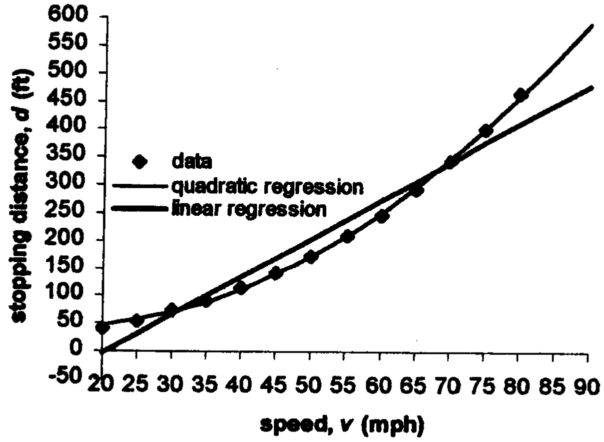
(d) The domain of a sum, difference, or product of two functions is the intersection of their domains.
The domain of a quotient of two functions is the intersection of their domains with any zeros of the denominator removed.

55. (a) The power regression function on the TI-92 Plus calculator gives $y = 4.44647x^{0.511414}$
(b)



(c) 15.2 km/h
(d) The linear regression function on the TI-92 Plus calculator gives $y = 0.913675x + 4.189976$ and it is shown on the graph in part (b). The linear regression function gives a speed of 14.2 km/h when $y = 11\text{m}$. The power regression curve in part (a) better fits the data.

56. (a) Let v represent the speed in miles per hour and d the stopping distance in feet. The quadratic regression function on the TI-92 Plus calculator gives $d = 0.0886v^2 - 1.97v + 50.1$.
(b)



(c) From the graph in part (b), the stopping distance is about 370 feet when the vehicle speed is 72 mph and it is about 525 feet when the speed is 85 mph.
Algebraically: $d_{\text{quadratic}}(72) = 0.0886(72)^2 - 1.97(72) + 50.1 = 367.6$ ft.

$$d_{\text{quadratic}}(85) = 0.0886(85)^2 - 1.97(85) + 50.1 = 522.8 \text{ ft.}$$

(d) The linear regression function on the TI-92 Plus calculator gives $d = 6.89v - 140.4 \Rightarrow d_{\text{linear}}(72)$ $= 6.89(72) - 140.4 = 355.7$ ft and $d_{\text{linear}}(85) = 6.89(85) - 140.4 = 445.2$ ft. The linear regression line is shown on the graph in part (b). The quadratic regression curve clearly gives the better fit.

P.3 EXPONENTIAL FUNCTIONS

1. The graph of $y = 2^x$ is increasing from left to right and has the negative x-axis as an asymptote. (a)

2. The graph of $y = 3^{-x}$ or, equivalently, $y = \left(\frac{1}{3}\right)^x$, is decreasing from left to right and has the positive x-axis as an asymptote. (d)

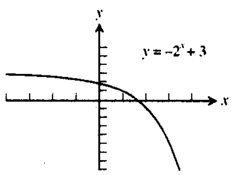
3. The graph of $y = -3^{-x}$ is the reflection about the x-axis of the graph in Exercise 2. (e)

4. The graph of $y = -0.5^{-x}$ or, equivalently, $y = -2^{x}$, is the reflection about the x-axis of the graph in Exercise 1. (c)

5. The graph of $y = 2^{-x} - 2$ is decreasing from left to right and has the line $y = -2$ as an asymptote. (b)

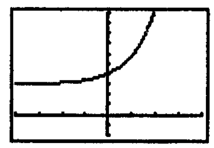
6. The graph of $y = 1.5^{x} - 2$ is increasing from left to right and has the line $y = -2$ as an asymptote. (f)

7.



Domain: $(-\infty, \infty)$
Range: $(-\infty, 3)$
x-intercept: ≈ 1.585
y-intercept: 2

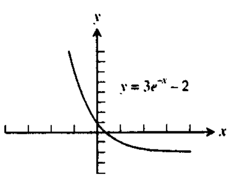
8.



[−4, 4] by [−2, 10]

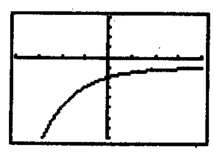
Domain: $(-\infty, \infty)$
Range: $(3, \infty)$
x-intercept: None
y-intercept: 4

9.



Domain: $(-\infty, \infty)$
Range: $(-2, \infty)$
x-intercept: ≈ 0.405
y-intercept: 1

10.



[−4, 4] by [−8, 4]

Domain: $(-\infty, \infty)$
Range: $(-\infty, -1)$
x-intercept: None
y-intercept: −2

11. $9^{2x} = (3^2)^{2x} = 3^{4x}$

12. $16^{3x} = (2^4)^{3x} = 2^{12x}$

13. $\left(\frac{1}{8}\right)^{2x} = (2^{-3})^{2x} = 2^{-6x}$

14. $\left(\frac{1}{27}\right)^{x} = (3^{-3})^{x} = 3^{-3x}$

15.

x	y	Δy
1	−1	
		2
2	1	
		2
3	3	
		2
4	5	

16.

x	y	Δy
1	1	
		−3
2	−2	
		−3
3	−5	
		−3
4	−8	

17.

x	y	Δy
1	1	
		3
2	4	
		5
3	9	
		7
4	16	

18.

x	y	ratio
1	8.155	
		2.718
2	22.167	
		2.718
3	60.257	
		2.718
4	163.79	

19. The slope of a straight line is $m = \frac{\Delta y}{\Delta x} \rightarrow \Delta y = m(\Delta x)$. In Exercise 15, each $\Delta x = 1$ and $m = 2 \rightarrow$ each $\Delta y = 2$, and in problem 16, each $\Delta x = 1$ and $m = -3 \rightarrow$ each $\Delta y = -3$. If the changes in x are constant for a linear function, say $\Delta x = c$, then the changes in y are also constant, specifically, $\Delta y = mc$.

20. From the table in Exercise 17, it can be seen that $\Delta y = 2x + 1$. Some examples are: $\Delta y = 9 - 4 = 5 = 2(2) + 1 = 2x + 1$ and $\Delta y = 16 - 9 = 7 = 2(3) + 1 = 2x + 1$. As x changes from $x = 1000$ to $x = 1001$, the change in y is $\Delta y = 2(1000) + 1 = 2001$. As x changes from n to $n + 1$, where n is an arbitrary positive integer, the change in y is $\Delta y = 2n + 1$.

21. Since $f(1) = 4.5$ we have $ka = 4.5$, and since $f(-1) = 0.5$ we have $ka^{-1} = 0.5$.
Dividing, we have

$\frac{ka}{ka^{-1}} = \frac{4.5}{0.5}$

$a^2 = 9$

$a = \pm 3$

Since $f(x) = k \cdot a^x$ is an exponential function, we require $a > 0$, so $a = 3$. Then $ka = 4.5$ gives $3k = 4.5$, so $k = 1.5$. The values are $a = 3$ and $k = 1.5$.

22. Since $f(1) = 1.5$ we have $ka = 1.5$, and since $f(-1) = 6$ we have $ka^{-1} = 6$.
Dividing, we have

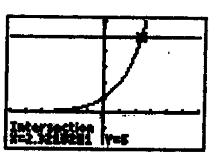
$\frac{ka}{ka^{-1}} = \frac{1.5}{6}$

$a^2 = 0.25$

$a = \pm 0.5$

Since $f(x) = k \cdot a^x$ is an exponential function, we require $a > 0$, so $a = 0.5$. Then $ka = 1.5$ gives $0.5k = 1.5$, so $k = 3$. The values are $a = 0.5$ and $k = 3$.

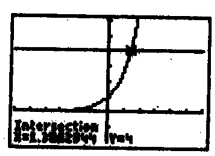
23.



[−6, 6] by [−2, 6]

$x \approx 2.3219$

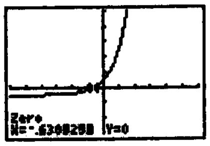
24.



[−6, 6] by [−2, 6]

$x \approx 1.3863$

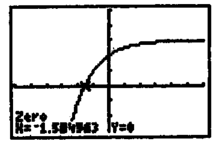
25.



[−6, 6] by [−3, 5]

$x \approx -0.6309$

26.



[−6, 6] by [−3, 5]

$x \approx -1.5850$

27. $5422(1.018)^{19} \approx 7609.7$ million

28. (a) When $t = 0$, $B = 100e^0 = 100$. There were 100 bacteria present initially.
   (b) When $t = 6$, $B = 100e^{0.693(6)} \approx 6394.351$. After 6 hours, there are about 6394 bacteria.
   (c) Solving $100e^{0.693t} = 200$ graphically, we find that $t \approx 1.000$. The population will be 200 after about 1 hour. Since the population doubles (from 100 to 200) in about 1 hour, the doubling time is about 1 hour.

29. Let t be the number of years. Solving $500{,}000(1.0375)^t = 1{,}000{,}000$ graphically, we find that $t \approx 18.828$. The population will reach 1 million in about 19 years.

30. (a) The population is given by $P(t) = 6250(1.0275)^t$, where t is the number of years after 1890.
   Population in 1915: $P(25) \approx 12{,}315$
   Population in 1940: $P(50) \approx 24{,}265$
   (b) Solving $P(t) = 50{,}000$ graphically, we find that $t \approx 76.651$. The population reached 50,000 about 77 years after 1890, in 1967.

31. (a) $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$
   (b) Solving $A(t) = 1$ graphically, we find that $t \approx 38$. There will be 1 gram remaining after about 38.1145 days.

32. Let t be the number of years. Solving $2300(1.06)^t = 4150$ graphically, we find that $t \approx 10.129$. It will take about 10.129 years. (If the interest is not credited to the account until the end of each year, it will take 11 years.)

33. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A(1.0625)^t = 2A$, which is equivalent to $1.0625^t = 2$. Solving graphically, we find that $t \approx 11.433$. It will take about 11.433 years. (If the interest is credited at the end of each year, it will take 12 years.)

34. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A\left(1 + \frac{0.0625}{12}\right)^{12t} = 2A$, which is equivalent to $\left(1 + \frac{0.0625}{12}\right)^{12t} = 2$. Solving graphically, we find that $t \approx 11.119$. It will take about 11.119 years. (If the interest is credited at the end of each month, it will take 11 years 2 months.)

35. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $Ae^{0.0625t} = 2A$, which is equivalent to $e^{0.0625t} = 2$. Solving graphically, we find that $t \approx 11.090$. It will take about 11.090 years.

36. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A(1.0575)^t = 3A$, which is equivalent to $1.0575^t = 3$. Solving graphically, we find that $t \approx 19.650$. It will take about 19.650 years. (If the interest is credited at the end of each year, it will take 20 years.)

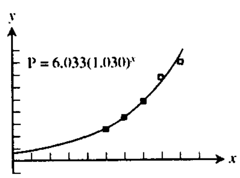
37. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A\left(1+\frac{0.0575}{365}\right)^{365t} = 3A$, which is equivalent to $\left(1+\frac{0.0575}{365}\right)^{365t} = 3$. Solving graphically, we find that $t \approx 19.108$. It will take about 19.108 years.

38. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $Ae^{0.0575t} = 3A$, which is equivalent to $e^{0.0575t} = 3$. Solving graphically, we find that $t \approx 19.106$. It will take about 19.106 years.

39. After t hours, the population is $P(t) = 2^{t/0.5}$ or, equivalently, $P(t) = 2^{2t}$. After 24 hours, the population is $P(24) = 2^{48} \approx 2.815 \times 10^{14}$ bacteria.

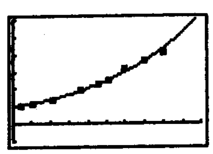
40. (a) Each year, the number of cases is $100\% - 20\% = 80\%$ of the previous year's number of cases. After t years, the number of cases will be $C(t) = 10{,}000(0.8)^t$. Solving $C(t) = 1000$ graphically, we find that $t \approx 10.319$. It will take about 10.319 years.
    (b) Solving $C(t) = 1$ graphically, we find that $t \approx 41.275$. It will take about 41.275 years.

41. (a) Let $x = 0$ represent 1900, $x = 1$ represent 1901, and so on. The regression equation is $P(x) = 6.033(1.030)^x$.



    (b) The regression equation gives an estimate of $P(0) \approx 6.03$ million, which is not very close to the actual population.
    (c) Since the equation is of the form $P(x) = P(0) \cdot 1.030^x$, the annual rate of growth is about 3%.

42. (a) The regression equation is $P(x) = 4.831(1.019)^x$.



[0, 100] by [−5, 30]

    (b) $P(90) \approx 26.3$ million
    (c) Since the equation is of the form $P(x) = P(0) \cdot 1.019^x$, the annual rate of growth is approximately 1.9%.

P.4 FUNCTIONS AND LOGARITHMS

1. Yes one-to-one, the graph passes the horizontal test.

2. Not one-to-one, the graph fails the horizontal test.

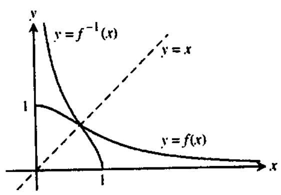
3. Not one-to-one since (for example) the horizontal line $y = 2$ intersects the graph twice.

4. Not one-to-one, the graph fails the horizontal test

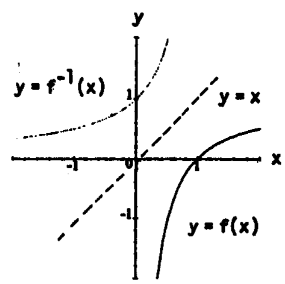
5. Yes one-to-one, the graph passes the horizontal test

6. Yes one-to-one, the graph passes the horizontal test

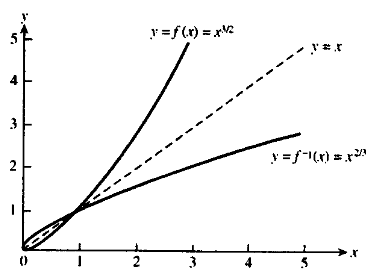
7. Domain: $0 < x \leq 1$, Range: $y \geq 0$



8. Domain: $x < 1$, Range: $y > 0$

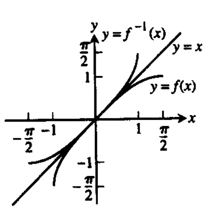


9.



Domain: $x \geq 0$, Range: $y \geq 0$

10. Domain: $-1 \leq x \leq 1$, Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



11. Step 1: $y = x^2 + 1 \Rightarrow x^2 = y - 1 \Rightarrow x = \sqrt{y-1}$
Step 2: $y = \sqrt{x-1} = f^{-1}(x)$

12. Step 1: $y = x^2 \Rightarrow x = -\sqrt{y}$
Step 2: $y = -\sqrt{x} = f^{-1}(x)$

13. Step 1: $y = x^3 - 1 \Rightarrow x^3 = y + 1 \Rightarrow x = (y+1)^{1/3}$
Step 2: $y = \sqrt[3]{x+1} = f^{-1}(x)$

14. Step 1: $y = x^2 - 2x + 1 \Rightarrow y = (x-1)^2 \Rightarrow \sqrt{y} = x - 1 \Rightarrow x = \sqrt{y} + 1$
Step 2: $y = 1 + \sqrt{x} = f^{-1}(x)$

15. Step 1: $y = (x+1)^2 \Rightarrow \sqrt{y} = x + 1 \Rightarrow x = \sqrt{y} - 1$
Step 2: $y = \sqrt{x} - 1 = f^{-1}(x)$

16. Step 1: $y = x^{2/3} \Rightarrow x = y^{3/2}$
Step 2: $y = x^{3/2} = f^{-1}(x)$

17. $y = 2x + 3 \rightarrow y - 3 = 2x \rightarrow \frac{y-3}{2} = x$. Interchange x and y: $\frac{x-3}{2} = y \rightarrow f^{-1}(x) = \frac{x-3}{2}$
Verify.
$(f \circ f^{-1})(x) = f\left(\frac{x-3}{2}\right) = 2\left(\frac{x-3}{2}\right) + 3 = (x-3) + 3 = x$
$(f^{-1} \circ f)(x) = f^{-1}(2x+3) = \frac{(2x+3)-3}{2} = \frac{2x}{2} = x$

18. $y = 5 - 4x \rightarrow 4x = 5 - y \rightarrow x = \frac{5-y}{4}$. Interchange x and y: $y = \frac{5-x}{4} \rightarrow f^{-1}(x) = \frac{5-x}{4}$
Verify.
$(f \circ f^{-1})(x) = f\left(\frac{5-x}{4}\right) = 5 - 4\left(\frac{5-x}{4}\right) = 5 - (5-x) = x$
$(f^{-1} \circ f)(x) = f^{-1}(5-4x) = \frac{5-(5-4x)}{4} = \frac{4x}{4} = x$

19. $y = x^3 - 1 \rightarrow y + 1 = x^3 \rightarrow (y+1)^{1/3} = x$. Interchange x and y: $(x+1)^{1/3} = y$
$\rightarrow f^{-1}(x) = (x+1)^{1/3}$ or $\sqrt[3]{x+1}$
Verify.
$(f \circ f^{-1})(x) = f(\sqrt[3]{x+1}) = (\sqrt[3]{x+1})^3 - 1 = (x+1) - 1 = x$
$(f^{-1} \circ f)(x) = f^{-1}(x^3 - 1) = \sqrt[3]{(x^3-1)+1} = \sqrt[3]{x^3} = x$

20. $y = x^2 + 1, x \geq 0 \rightarrow y - 1 = x^2, x \geq 0 \rightarrow \sqrt{y-1} = x$.
Interchange x and y: $\sqrt{x-1} = y \rightarrow f^{-1}(x) = \sqrt{x-1}$ or $(x-1)^{1/2}$
Verify. For $x \geq 1$ (the domain of f^{-1}),
$(f \circ f^{-1})(x) = f(\sqrt{x-1}) = (\sqrt{x-1})^2 + 1 = (x-1) + 1 = x$
For $x > 0$ (the domain of f),
$(f^{-1} \circ f)(x) = f^{-1}(x^2+1) = \sqrt{(x^2+1)-1} = \sqrt{x^2} = |x| = x$

21. $y = x^2,\ x \le 0 \to x = -\sqrt{y}$. Interchange x and y: $y = -\sqrt{x} \to f^{-1}(x) = -\sqrt{x}$ or $-x^{1/2}$

Verify.

For $x \ge 0$ (the domain of f^{-1}), $(f \circ f^{-1})(x) = f(-\sqrt{x}) = (-\sqrt{x})^2 = x$

For $x \le 0$ (the domain of f), $(f^{-1} \circ f)(x) = f^{-1}(x^2) = -\sqrt{x^2} = -|x| = x$

22. $y = x^{2/3},\ x \ge 0 \to y^{3/2} = (x^{2/3})^{3/2},\ x \ge 0 \to y^{3/2} = x$

Interchange x and y: $x^{3/2} = y \to f^{-1}(x) = x^{3/2}$

Verify.

For $x \ge 0$ (the domain of f^{-1}), $(f \circ f^{-1})(x) = f(x^{3/2}) = (x^{3/2})^{2/3} = x$

For $x \ge 0$ (the domain of f), $(f^{-1} \circ f)(x) = f^{-1}(x^{2/3}) = (x^{2/3})^{3/2} = |x| = x$

23. $y = -(x-2)^2,\ x \le 2 \to (x-2)^2 = -y,\ x \le 2 \to x - 2 = -\sqrt{-y} \to x = 2 - \sqrt{-y}$.

Interchange x and y: $y = 2 - \sqrt{-x} \to f^{-1}(x) = 2 - \sqrt{-x}$ or $2 - (-x)^{1/2}$

Verify.

For $x \le 0$ (the domain of f^{-1})

$(f \circ f^{-1})(x) = f(2 - \sqrt{-x}) = -[(2 - \sqrt{-x}) - 2]^2 = -(-\sqrt{-x})^2 = -|x| = x$

For $x \le 2$ (the domain of f),

$(f^{-1} \circ f)(x) = f^{-1}(-(x-2)^2) = 2 - \sqrt{(x-2)^2} = 2 - |x-2| = 2 + (x-2) = x$

24. $y = (x^2 + 2x + 1),\ x \ge -1 \to y = (x+1)^2,\ x \ge -1 \to \sqrt{y} = x + 1 \to \sqrt{y} - 1 = x$.

Interchange x and y: $\sqrt{x} - 1 = y \to f^{-1}(x) = \sqrt{x} - 1$ or $x^{1/2} - 1$

Verify.

For $x \ge 0$ (the domain of f^{-1}),

$(f \circ f^{-1})(x) = f(\sqrt{x} - 1) = \left[(\sqrt{x}-1)^2 + 2(\sqrt{x}-1) + 1\right] = (\sqrt{x})^2 - 2\sqrt{x} + 1 + 2\sqrt{x} - 2 + 1 = (\sqrt{x})^2 = x$

For $x \ge -1$ (the domain of f),

$(f^{-1} \circ f)(x) = f^{-1}(x^2 + 2x + 1) = \sqrt{x^2 + 2x + 1} - 1 = \sqrt{(x+1)^2} - 1 = |x+1| - 1 = (x+1) - 1 = x$

25. $y = \frac{1}{x^2},\ x > 0 \to x^2 = \frac{1}{y},\ x > 0 \to x = \sqrt{\frac{1}{y}} = \frac{1}{\sqrt{y}}$.

Interchange x and y: $y = \frac{1}{\sqrt{x}} \to f^{-1}(x) = \frac{1}{\sqrt{x}}$ or $\frac{1}{x^{1/2}}$

Verify.

For $x > 0$ (the domain of f^{-1}), $(f \circ f^{-1})(x) = f\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{(1/\sqrt{x})^2} = x$

For $x > 0$ (the domain of f), $(f^{-1} \circ f)(x) = f^{-1}\left(\frac{1}{x^2}\right) = \frac{1}{\sqrt{1/x^2}} = \sqrt{x^2} = |x| = x$

26. $y = \frac{1}{x^3} \to x^3 = \frac{1}{y} \to x = \sqrt[3]{\frac{1}{y}} = \frac{1}{\sqrt[3]{y}}.$

Interchange x and y: $y = \frac{1}{\sqrt[3]{x}} \to f^{-1}(x) = \frac{1}{\sqrt[3]{x}}$ or $\frac{1}{x^{1/3}}$

Verify.

$$(f \circ f^{-1})(x) = f\left(\frac{1}{\sqrt[3]{x}}\right) = \frac{1}{\left(1/\sqrt[3]{x}\right)^3} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{1}{x^3}\right) = \frac{1}{\sqrt[3]{1/x^3}} = x$$

27. $y = \frac{2x+1}{x+3} \to xy + 3y = 2x + 1 \to xy - 2x = 1 - 3y \to (y-2)x = 1 - 3y \to x = \frac{1-3y}{y-2}.$

Interchange x and y: $y = \frac{1-3x}{x-2} \to f^{-1}(x) = \frac{1-3x}{x-2}$

Verify.

$$(f \circ f^{-1})(x) = f\left(\frac{1-3x}{x-2}\right) = \frac{2\left(\frac{1-3x}{x-2}\right)+1}{\frac{1-3x}{x-2}+3} = \frac{2(1-3x)+(x-2)}{(1-3x)+3(x-2)} = \frac{-5x}{-5} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{2x+1}{x+3}\right) = \frac{1-3\left(\frac{2x+1}{x+3}\right)}{\frac{2x+1}{x+3}-2} = \frac{(x+3)-3(2x+1)}{(2x+1)-2(x+3)} = \frac{-5x}{-5} = x$$

28. $y = \frac{x+3}{x-2} \to xy - 2y = x + 3 \to xy - x = 2y + 3 \to x(y-1) = 2y + 3 \to x = \frac{2y+3}{y-1}.$

Interchange x and y: $y = \frac{2x+3}{x-1} \to f^{-1}(x) = \frac{2x+3}{x-1}$

Verify.

$$(f \circ f^{-1})(x) = f\left(\frac{2x+3}{x-1}\right) = \frac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2} = \frac{(2x+3)+3(x-1)}{(2x+3)-2(x-1)} = \frac{5x}{5} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{x+3}{x-2}\right) = \frac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1} = \frac{2(x+3)+3(x-2)}{(x+3)-(x-2)} = \frac{5x}{5} = x$$

29. $y = \left(e^a\right)^x - 1 \to e^a = 3 \to a = \ln 3 \to y = e^{x \ln 3} - 1$

(a) $D = (-\infty, \infty)$

(b) $R = (-1, \infty)$

30. $y = \left(e^a\right)^{x+1} \to e^a = 4 \to a = \ln 4 \to y = e^{(x+1)\ln 4} = e^{x \ln 4} e^{\ln 4} = 4e^{x \ln 4}$

(a) $D = (-\infty, \infty)$

(b) $R = (0, \infty)$

31. $y = 1 - (\ln 3) \log_3 x = 1 - (\ln 3)\,\frac{\ln x}{\ln 3} = 1 - \ln x$

(a) $D = (0, \infty)$

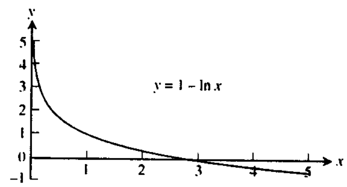
(b) $R = (-\infty, \infty)$

32. $y = (\ln 10) \log(x+2) = (\ln 10)\frac{\ln(x+2)}{\ln 10} = \ln(x+2)$

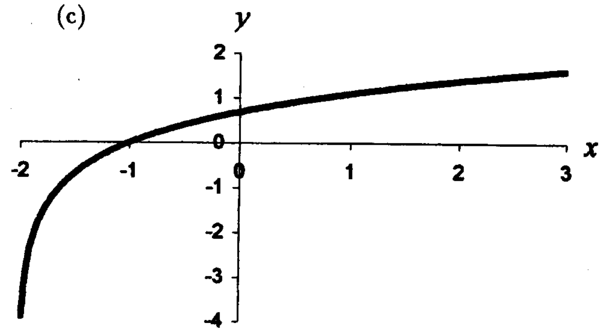
(a) $D = (-2, \infty)$

(b) $R = (-\infty, \infty)$

(c)



(c)



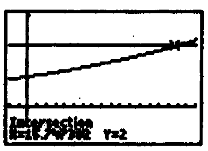
33. $(1.045)^t = 2$

$\ln(1.045)^t = \ln 2$

$t \ln 1.04 = \ln 2$

$t = \dfrac{\ln 2}{\ln 1.045} \approx 15.75$

Graphical support:



[−2, 18] by [−1, 3]

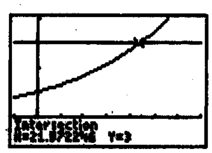
34. $e^{0.05t} = 3$

$\ln e^{0.05t} = \ln 3$

$0.05t = \ln 3$

$t = \dfrac{\ln 3}{0.05} = 20 \ln 3 \approx 21.97$

Graphical support:



[−5, 35] by [−1, 4]

35. $e^x + e^{-x} = 3$

$e^x - 3 + e^{-x} = 0$

$e^x(e^x - 3 + e^{-x}) = e^x(0)$

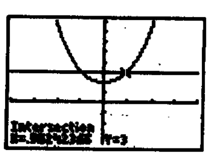
$(e^x)^2 - 3e^x + 1 = 0$

$e^x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$

$e^x = \dfrac{3 \pm \sqrt{5}}{2}$

$x = \ln\left(\dfrac{3 \pm \sqrt{5}}{2}\right) \approx -0.96$ or 0.96

Graphical support:



[−4, 4] by [−4, 8]

36. $2^x + 2^{-x} = 5$

$2^x - 5 + 2^{-x} = 0$

$2^x(2^x - 5 + 2^{-x}) = 2^x(0)$

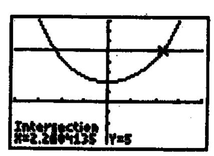
$(2^x)^2 - 5(2^x) + 1 = 0$

$2^x = \dfrac{5 \pm \sqrt{(-5)^2 - 4(1)(1)}}{2(1)}$

$2^x = \dfrac{5 \pm \sqrt{21}}{2}$

$x = \log_2\left(\dfrac{5 \pm \sqrt{21}}{2}\right) \approx -2.26$ or 2.26

Graphical support:



[−4, 4] by [−4, 8]

37. $\ln y = 2t + 4 \to e^{\ln y} = e^{2t+4} \to y = e^{2t+4}$

38. $\ln(y-1) - \ln 2 = x + \ln x \to \ln(y-1) = x + \ln x + \ln 2 \to e^{\ln(y-1)} = e^{x + \ln x + \ln 2} \to y - 1 = e^x(x)(2)$
$\to y = 2xe^x + 1$

39. (a) $y = \frac{100}{1+2^{-x}} \to 1 + 2^{-x} = \frac{100}{y} \to 2^{-x} = \frac{100}{y} - 1 \to \log_2(2^{-x}) = \log_2\left(\frac{100}{y} - 1\right) \to -x = \log_2\left(\frac{100}{y} - 1\right)$
$\to x = -\log_2\left(\frac{100}{y} - 1\right) = -\log_2\left(\frac{100-y}{y}\right) = \log_2\left(\frac{y}{100-y}\right).$

Interchange x and y: $y = \log_2\left(\frac{x}{100-x}\right) \to f^{-1}(x) = \log_2\left(\frac{x}{100-x}\right)$

Verify.

$$(f \circ f^{-1})(x) = f\left(\log_2\left(\frac{x}{100-x}\right)\right) = \frac{100}{1 + 2^{-\log_2\left(\frac{x}{100-x}\right)}} = \frac{100}{1 + 2^{\log_2\left(\frac{100-x}{x}\right)}} = \frac{100}{1 + \frac{100-x}{x}}$$

$$= \frac{100x}{x + (100-x)} = \frac{100x}{100} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{100}{1+2^{-x}}\right) = \log_2\left(\frac{\frac{100}{1+2^{-x}}}{100 - \frac{100}{1+2^{-x}}}\right) = \log_2\left(\frac{100}{100(1+2^{-x}) - 100}\right)$$

$$= \log_2\left(\frac{1}{2^{-x}}\right) = \log_2(2^x) = x$$

(b) $y = \frac{50}{1+1.1^{-x}} \to 1 + 1.1^{-x} = \frac{50}{y} \to 1.1^{-x} = \frac{50}{y} - 1 \to \log_{1.1}(1.1^{-x}) = \log_{1.1}\left(\frac{50}{y} - 1\right) \to -x = \log_{1.1}\left(\frac{50}{y} - 1\right)$
$\to x = -\log_{1.1}\left(\frac{50}{y} - 1\right) = -\log_{1.1}\left(\frac{50-y}{y}\right) = \log_{1.1}\left(\frac{y}{50-y}\right).$

Interchange x and y: $y = \log_{1.1}\left(\frac{x}{50-x}\right) \to f^{-1}(x) = \log_{1.1}\left(\frac{x}{50-x}\right)$

Verify.

$$(f \circ f^{-1})(x) = f\left(\log_{1.1}\left(\frac{x}{50-x}\right)\right) = \frac{50}{1 + 1.1^{-\log_{1.1}\left(\frac{x}{50-x}\right)}} = \frac{50}{1 + 1.1^{\log_{1.1}\left(\frac{50-x}{x}\right)}} = \frac{50}{1 + \frac{50-x}{x}}$$

$$= \frac{50x}{x + (50-x)} = \frac{50x}{50} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{50}{1+1.1^{-x}}\right) = \log_{1.1}\left(\frac{\frac{50}{1+1.1^{-x}}}{50 - \frac{50}{1+1.1^{-x}}}\right) = \log_{1.1}\left(\frac{50}{50(1+1.1^{-x}) - 50}\right)$$

$$= \log_{1.1}\left(\frac{1}{1.1^{-x}}\right) = \log_{1.1}(1.1^x) = x$$

40. (a) Suppose that $f(x_1) = f(x_2)$. Then $mx_1 + b = mx_2 + b$ so $mx_1 = mx_2$. Since $m \neq 0$, this gives $x_1 = x_2$.

(b) $y = mx + b \to y - b = mx \to \frac{y-b}{m} = x.$

Interchange x and y: $\frac{x-b}{m} = y \rightarrow f^{-1}(x) = \frac{x-b}{m}$

The slopes are reciprocals.

(c) If the original functions both have slope m, each of the inverse functions will have slope $\frac{1}{m}$. The graphs of the inverses will be parallel lines with nonzero slope.

(d) If the original functions have slopes m and $-\frac{1}{m}$, respectively, then the inverse functions will have slopes $\frac{1}{m}$ and $-m$, respectively. Since each of $\frac{1}{m}$ and $-m$ is the negative reciprocal of the other, the graphs of the inverses will be perpendicular lines with nonzero slopes.

41. (a) Amount $= 8\left(\frac{1}{2}\right)^{t/12}$

(b) $8\left(\frac{1}{2}\right)^{t/12} = 1 \rightarrow \left(\frac{1}{2}\right)^{t/12} = \frac{1}{8} \rightarrow \left(\frac{1}{2}\right)^{t/12} = \left(\frac{1}{2}\right)^3 \rightarrow \frac{t}{12} = 3 \rightarrow t = 36$

There will be 1 gram remaining after 36 hours.

42. $500(1.0475)^t = 1000 \rightarrow 1.0475^t = 2 \rightarrow \ln(1.0475^t) = \ln 2 \rightarrow t \ln 1.0475 = \ln 2 \rightarrow t = \frac{\ln 2}{\ln 1.0475} \approx 14.936$

It will take about 14.936 years. (If the interest is paid at the end of each year, it will take 15 years.)

43. $375{,}000(1.0225)^t = 1{,}000{,}000 \rightarrow 1.0225^t = \frac{8}{3} \rightarrow \ln(1.0225^t) = \ln\left(\frac{8}{3}\right) \rightarrow t \ln 1.0225 = \ln\left(\frac{8}{3}\right)$

$\rightarrow t = \frac{\ln(8/3)}{\ln 1.0225} \approx 44.081$

It will take about 44.081 years.

44. Let O = original sound level $= 10 \log_{10}(I \times 10^{12})$ db from Equation (1) in the text. Solving $O + 10 = 10 \log_{10}(kI \times 10^{12})$ for $k \Rightarrow 10 \log_{10}(I \times 10^{12}) + 10 = 10 \log_{10}(kI \times 10^{12})$

$\Rightarrow \log_{10}(I \times 10^{12}) + 1 = \log_{10}(kI \times 10^{12}) \Rightarrow \log_{10}(I \times 10^{12}) + 1 = \log_{10} k + \log_{10}(I \times 10^{12})$

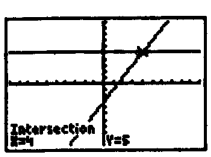
$\Rightarrow 1 = \log_{10} k \Rightarrow 1 = \frac{\ln k}{\ln 10} \Rightarrow \ln k = \ln 10 \Rightarrow k = 10$

45. Sound level with intensity = 10I is $10 \log_{10}(10I \times 10^{12}) = 10\left[\log_{10} 10 + \log_{10}(I \times 10^{12})\right]$

$= 10 + 10 \log_{10}(I \times 10^{12}) =$ original sound level $+ 10 \Rightarrow$ an increase of 10 db

46. $y = y_0 e^{-0.18t}$ represents the decay equation; solving $(0.9)y_0 = y_0 e^{-0.18t} \Rightarrow t = \frac{\ln(0.9)}{-0.18} \approx 0.585$ days

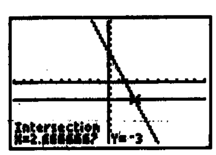
47.



[−10, 10] by [−10, 10]

(4, 5)

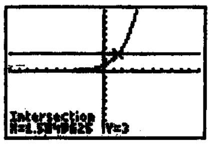
48.



[−10, 10] by [−10, 10]

$\left(\frac{8}{3}, -3\right) \approx (2.67, -3)$

49. (a)

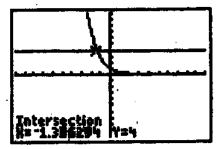


[−10, 10] by [−10, 10]

(1.58, 3)

(b) No points of intersection, since $2^x > 0$ for all values of x.

50. (a)

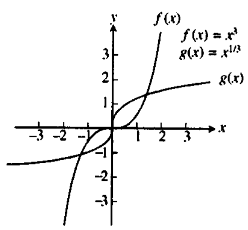


[−10, 10] by [−10, 10]

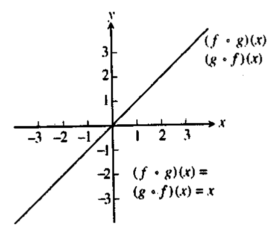
(−1.39, 4)

(b) No points of intersection, since $e^{-x} > 0$ for all values of x.

51. (a)

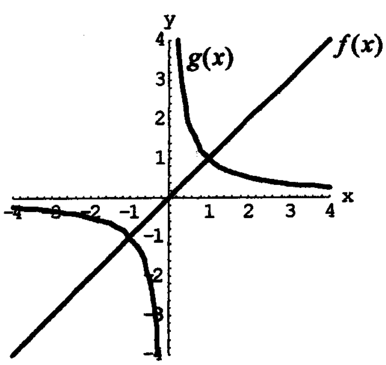


(b) and (c)

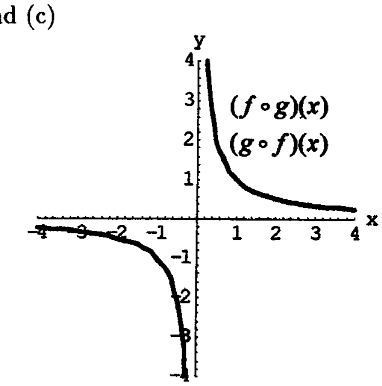


We conclude that f and g are inverses of each other because $(f \circ g)(x) = (g \circ f)(x) = x$, the identity function.

52. (a)

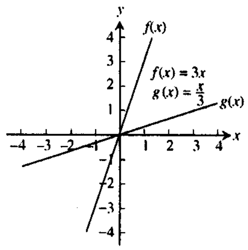


(b) and (c)

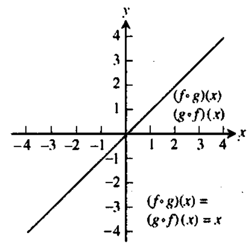


We conclude that f is the identity function because $(f \circ g)(x) = (g \circ f)(x) = \frac{1}{x} = g(x)$

53. (a)

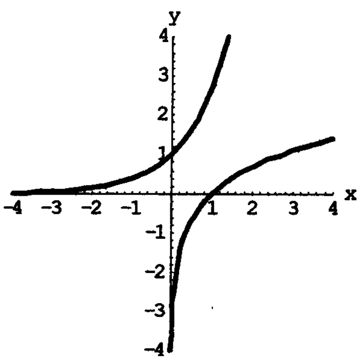


(b) and (c)

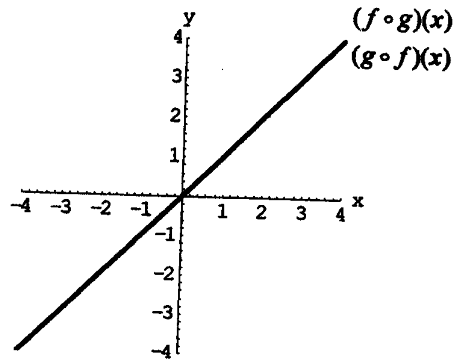


We conclude that f and g are inverses of each other because $(f \circ g)(x) = (g \circ f)(x) = x$, the identity function.

54. (a)

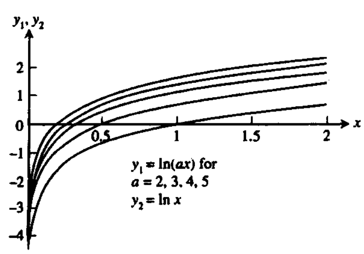


(b) and (c)



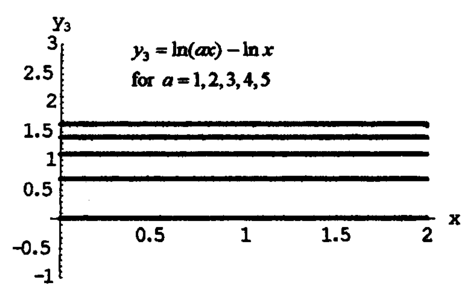
We conclude that f and g are inverses of each other because $(f \circ g)(x) = (g \circ f)(x) = x$, the identity function.

55. (a)



The graphs of y_1 appear to be vertical translates of y_2

(b)



The graphs of $y_1 - y_2$ support the finding in part (a).

(c) $y_3 = y_1 - y_2 = \ln ax - \ln x = (\ln a + \ln x) - \ln x = \ln a$, a constant.

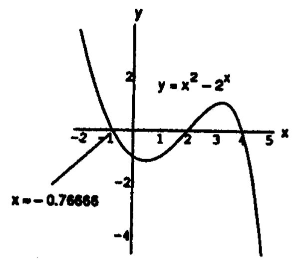
56. (a) y_2 is a vertical shift (upward) of y_1, although it's difficult to see that near the vertical asymptote at $x = 0$. One might use "trace" or "table" to verify this.

(b) Each graph of y_3 is a horizontal line.

(c) The graphs of y_4 and $y = a$ are the same.

(d) $e^{y_2 - y_1} = a$, $\ln\left(e^{y_2 - y_1}\right) = \ln a$, $y_2 - y_1 = \ln a$, $y_1 = y_2 - \ln a = \ln x - \ln a$

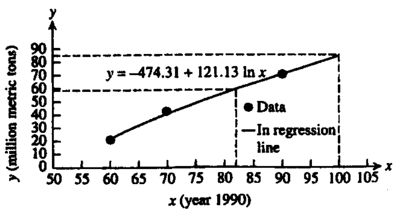
57. From zooming in on the graph at the right, we estimate the third root to be $x \approx -0.76666$



58. The functions $f(x) = x^{\ln 2}$ and $g(x) = 2^{\ln x}$ appear to have identical graphs for $x > 0$. This is no accident, because $x^{\ln 2} = e^{\ln 2 \cdot \ln x} = \left(e^{\ln 2}\right)^{\ln x} = 2^{\ln x}$.

59. (a) The LnReg command on the TI-92 Plus calculator gives $y(x) = -474.31 + 121.13 \ln x$ $\Rightarrow y(82) = -474.31 + 121.13 \ln(82) = 59.48$ million metric tons produced in 1982 and $y(100) = -474.31 + 121.13 \ln(100) = 83.51$ million metric tons produced in 2000.

(b)



(c) From the graph in part (b), $y(82) \approx 59$ and $y(100) \approx 84$.

60. (a) $y = -2539.852 + 636.896 \ln x$

(b) When $x = 75$, $y \approx 209.94$. About 209.94 million metric tons were produced.

(c) $-2539.852 + 636.896 \ln x = 400$

$$636.896 \ln x = 2939.852$$
$$\ln x = \frac{2939.852}{636.896}$$
$$x = e^{\frac{2939.852}{636.896}} \approx 101.08$$

According to the regression equation, Saudi Arabian oil production will reach 400 million metric tons when $x \approx 101.08$, in about 2001.

P.5 TRIGONOMETRIC FUNCTIONS AND THEIR INVERSES

1. (a) $s = r\theta = (10)\left(\frac{4\pi}{5}\right) = 8\pi$ m (b) $s = r\theta = (10)(110^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{110\pi}{18} = \frac{55\pi}{9}$ m

2. $\theta = \frac{s}{r} = \frac{10\pi}{8} = \frac{5\pi}{4}$ radians and $\frac{5\pi}{4}\left(\frac{180^\circ}{\pi}\right) = 225^\circ$

3.

θ	$-\pi$	$-\frac{2\pi}{3}$	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin\theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos\theta$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	0	$\sqrt{3}$	0	und.	-1
$\cot\theta$	und.	$\frac{1}{\sqrt{3}}$	und.	0	-1
$\sec\theta$	-1	-2	1	und.	$-\sqrt{2}$
$\csc\theta$	und.	$-\frac{2}{\sqrt{3}}$	und.	1	$\sqrt{2}$

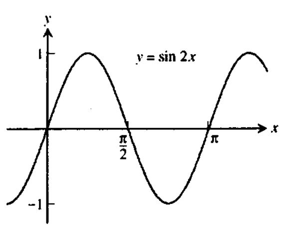
4.

θ	$-\frac{3\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin\theta$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$
$\tan\theta$	und.	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$
$\cot\theta$	0	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	1	$-\sqrt{3}$
$\sec\theta$	und.	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$\csc\theta$	1	$-\frac{2}{\sqrt{3}}$	-2	$\sqrt{2}$	2

5. (a) $\cos x = -\frac{4}{5}$, $\tan x = -\frac{3}{4}$ (b) $\sin x = -\frac{2\sqrt{2}}{3}$, $\tan x = -2\sqrt{2}$

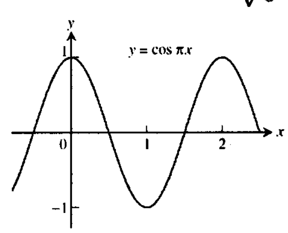
6. (a) $\sin x = -\frac{1}{\sqrt{5}}$, $\cos x = -\frac{2}{\sqrt{5}}$ (b) $\cos x = -\frac{\sqrt{3}}{2}$, $\tan x = \frac{1}{\sqrt{3}}$

7. (a)



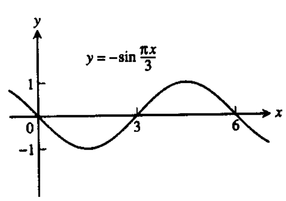
period $= \pi$

(b)



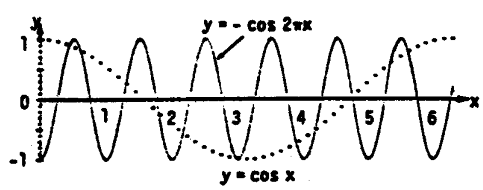
period $= 2$

8. (a)



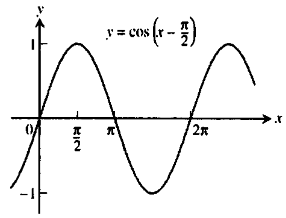
period $= 6$

(b)



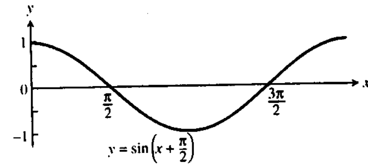
period $= 1$

9. (a)



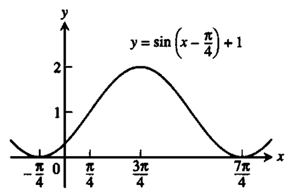
period $= 2\pi$

(b)



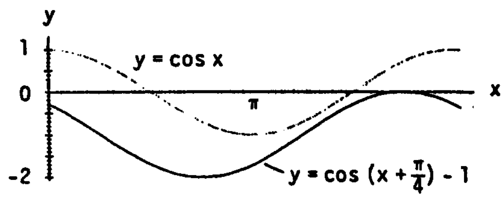
period $= 2\pi$

10. (a)



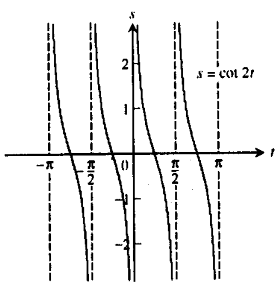
period $= 2\pi$

(b)

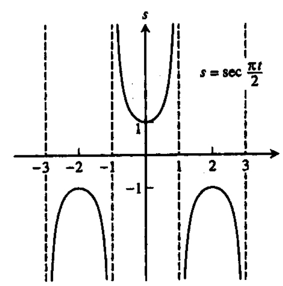


period $= 2\pi$

11. period $= \frac{\pi}{2}$, symmetric about the origin



12. period $= 4$, symmetric about the y-axis



13. (a) $\cos(\pi + x) = \cos \pi \cos x - \sin \pi \sin x = (-1)(\cos x) - (0)(\sin x) = -\cos x$
(b) $\sin(2\pi - x) = \sin 2\pi \cos(-x) + \cos(2\pi) \sin(-x) = (0)(\cos(-x)) + (1)(\sin(-x)) = -\sin x$

14. (a) $\sin\left(\frac{3\pi}{2} - x\right) = \sin\left(\frac{3\pi}{2}\right)\cos(-x) + \cos\left(\frac{3\pi}{2}\right)\sin(-x) = (-1)(\cos x) + (0)(\sin(-x)) = -\cos x$
(b) $\cos\left(\frac{3\pi}{2} + x\right) = \cos\left(\frac{3\pi}{2}\right)\cos x - \sin\left(\frac{3\pi}{2}\right)\sin x = (0)(\cos x) - (-1)(\sin x) = \sin x$

15. (a) $\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos\left(-\frac{\pi}{2}\right) - \sin x \sin\left(-\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(-1) = \sin x$

$\cos(A - B) = \cos(A + (-B)) = \cos A \cos(-B) - \sin A \sin(-B) = \cos A \cos B - \sin A(-\sin B)$
$= \cos A \cos B + \sin A \sin B$

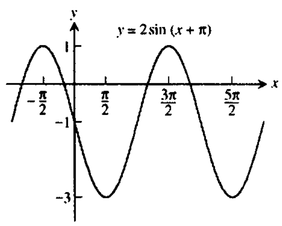
16. (a) $\sin\left(x + \frac{\pi}{2}\right) = \sin x \cos\left(\frac{\pi}{2}\right) + \cos x \sin\left(\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(1) = \cos x$

(b) $\sin(A - B) = \sin(A + (-B)) = \sin A \cos(-B) + \cos A \sin(-B) = \sin A \cos B + \cos A(-\sin B)$
$= \sin A \cos B - \cos A \sin B$

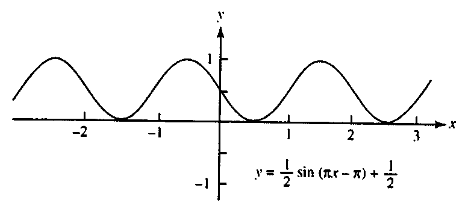
17. If $B = A$, $A - B = 0 \Rightarrow \cos(A - B) = \cos 0 = 1$. Also $\cos(A - B) = \cos(A - A) = \cos A \cos A + \sin A \sin A$ $= \cos^2 A + \sin^2 A$. Therefore, $\cos^2 A + \sin^2 A = 1$.

18. If $B = 2\pi$, then $\cos(A + 2\pi) = \cos A \cos 2\pi - \sin A \sin 2\pi = (\cos A)(1) - (\sin A)(0) = \cos A$ and $\sin(A + 2\pi) = \sin A \cos 2\pi + \cos A \sin 2\pi = (\sin A)(1) + (\cos A)(0) = \sin A$. The result agrees with the fact that the cosine and sine functions have period 2π.

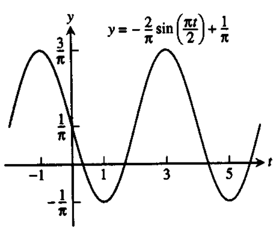
19. (a) $A = 2$, $B = 2\pi$, $C = -\pi$, $D = -1$



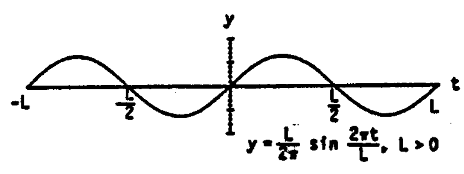
(b) $A = \frac{1}{2}$, $B = 2$, $C = 1$, $D = \frac{1}{2}$



20. (a) $A = -\frac{2}{\pi}$, $B = 4$, $C = 0$, $D = \frac{1}{\pi}$



(b) $A = \frac{L}{2\pi}$, $B = L$, $C = 0$, $D = 0$



21. (a) amplitude $= |A| = 37$
(b) period $= |B| = 365$
(c) right horizontal shift $= C = 101$
(d) upward vertical shift $= D = 25$

22. (a) It is highest when the value of the sine is 1 at $f(101) = 37 \sin(0) + 25 = 62^\circ$F.
The lowest mean daily temp. is $37(-1) + 25 = -12^\circ$F.
(b) The average of the highest and lowest mean daily temperatures $= \frac{62^\circ + (-12)^\circ}{2} = 25^\circ$F.
The average of the sine function is its horizontal axis, $y = 25$.

23. (a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{6}$

24. (a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $-\frac{\pi}{3}$

25. (a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$

26. (a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$

27. The angle α is the large angle between the wall and the right end of the blackboard minus the small angle between the left end of the blackboard and the wall $\Rightarrow \alpha = \cot^{-1}\left(\frac{x}{15}\right) - \cot^{-1}\left(\frac{x}{3}\right)$.

28. $65^\circ + (90^\circ - \beta) + (90^\circ - \alpha) = 180^\circ \Rightarrow \alpha = 65^\circ - \beta = 65^\circ - \tan^{-1}\left(\frac{21}{50}\right) \approx 65^\circ - 22.78^\circ \approx 42.22^\circ$

29. According to the figure in the text, we have the following: By the law of cosines, $c^2 = a^2 + b^2 - 2ab \cos\theta$ $= 1^2 + 1^2 - 2\cos(A - B) = 2 - 2\cos(A - B)$. By distance formula, $c^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$

$= \cos^2 A - 2 \cos A \cos B + \cos^2 B + \sin^2 A - 2 \sin A \sin B + \sin^2 B = 2 - 2(\cos A \cos B + \sin A \sin B)$. Thus $c^2 = 2 - 2 \cos(A - B) = 2 - 2(\cos A \cos B + \sin A \sin B) \Rightarrow \cos(A - B) = \cos A \cos B + \sin A \sin B$.

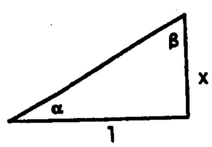
30. Consider the figure where $\theta = A + B$ is the sum of two angles. By the law of cosines, $c^2 = a^2 + b^2 - 2ab \cos\theta$
$= 1^2 + 1^2 - 2 \cos(A + B) = 2 - 2 \cos(A + B)$.
Also, by the distance formula,
$c^2 = (\cos A - \cos B)^2 + (\sin A + \sin B)^2$
$= \cos^2 A - 2 \cos A \cos B + \cos^2 B + \sin^2 A + 2 \sin A \sin B + \sin^2 B$
$= 2 - 2(\cos A \cos B - \sin A \sin B)$. Thus,
$2 - 2 \cos(A + B) = 2 - 2(\cos A \cos B - \sin A \sin B)$
$\Rightarrow \cos(A + B) = \cos A \cos B - \sin A \sin B$.

31. Take each square as a unit square. From the diagram we have the following: the smallest angle α has a tangent of $1 \Rightarrow \alpha = \tan^{-1} 1$; the middle angle β has a tangent of $2 \Rightarrow \beta = \tan^{-1} 2$; and the largest angle γ has a tangent of $3 \Rightarrow \gamma = \tan^{-1} 3$. The sum of these three angles is $\pi \Rightarrow \alpha + \beta + \gamma = \pi$
$\Rightarrow \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$.

32. (a) From the symmetry of the diagram, we see that $\pi - \sec^{-1} x$ is the vertical distance from the graph of $y = \sec^{-1} x$ to the line $y = \pi$ and this distance is the same as the height of $y = \sec^{-1} x$ above the x-axis at $-x$; i.e., $\pi - \sec^{-1} x = \sec^{-1}(-x)$.

(b) $\cos^{-1}(-x) = \pi - \cos^{-1} x$, where $-1 \le x \le 1 \Rightarrow \cos^{-1}\left(-\frac{1}{x}\right) = \pi - \cos^{-1}\left(\frac{1}{x}\right)$, where $x \ge 1$ or $x \le -1$
$\Rightarrow \sec^{-1}(-x) = \pi - \sec^{-1} x$

33. $\sin^{-1}(1) + \cos^{-1}(1) = \frac{\pi}{2} + 0 = \frac{\pi}{2}$; $\sin^{-1}(0) + \cos^{-1}(0) = 0 + \frac{\pi}{2} = \frac{\pi}{2}$; and $\sin^{-1}(-1) + \cos^{-1}(-1) = -\frac{\pi}{2} + \pi = \frac{\pi}{2}$.
If $x \in (-1, 0)$ and $x = -a$, then $\sin^{-1}(x) + \cos^{-1}(x) = \sin^{-1}(-a) + \cos^{-1}(-a) = -\sin^{-1} a + (\pi - \cos^{-1} a)$
$= \pi - (\sin^{-1} a + \cos^{-1} a) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$ from Equations (7) and (9) in the text.

34. 

$\Rightarrow \tan\alpha = x$ and $\tan\beta = \frac{1}{x} \Rightarrow \frac{\pi}{2} = \alpha + \beta = \tan^{-1} x + \tan^{-1} \frac{1}{x}$.

35. From the figures in the text, we see that $\sin B = \frac{h}{c}$. If C is an acute angle, then $\sin C = \frac{h}{b}$. On the other hand, if C is obtuse (as in the figure on the right), then $\sin C = \sin(\pi - C) = \frac{h}{b}$. Thus, in either case, $h = b \sin C = c \sin B \Rightarrow ah = ab \sin C = ac \sin B$.

By the law of cosines, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ and $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$. Moreover, since the sum of the interior angles of a triangle is π, we have $\sin A = \sin(\pi - (B + C)) = \sin(B + C) = \sin B \cos C + \cos B \sin C$
$= \left(\frac{h}{c}\right)\left[\frac{a^2 + b^2 - c^2}{2ab}\right] + \left[\frac{a^2 + c^2 - b^2}{2ac}\right]\left(\frac{h}{b}\right) = \left(\frac{h}{2abc}\right)(2a^2 + b^2 - c^2 + c^2 - b^2) = \frac{ah}{bc} \Rightarrow ah = bc \sin A$.

Combining our results we have ah = ab sin C, ah = ac sin B, and ah = bc sin A. Dividing by abc gives $\frac{h}{bc} = \underbrace{\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}}_{\text{law of sines}}$.

36. $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \cos B}{\cos A \cos B - \sin A \sin B} = \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}} = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

37. (a) $c^2 = a^2 + b^2 - 2ab\cos C = 2^2 + 3^2 - 2(2)(3)\cos(60^\circ) = 4 + 9 - 12\cos(60^\circ) = 13 - 12\left(\frac{1}{2}\right) = 7$.
Thus, $c = \sqrt{7} \approx 2.65$.

(b) $c^2 = a^2 + b^2 - 2ab\cos C = 2^2 + 3^2 - 2(2)(3)\cos(40^\circ) = 13 - 12\cos(40^\circ)$. Thus,
$c = \sqrt{13 - 12\cos 40^\circ} \approx 1.951$.

38. (a) By the law of sines, $\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sqrt{3}/2}{c}$. By Exercise 55 we know that $c = \sqrt{7}$.
Thus $\sin B = \frac{3\sqrt{3}}{2\sqrt{7}} \simeq 0.982$.

(a) From the figure at the right and the law of cosines,
$b^2 = a^2 + 2^2 - 2(2a)\cos B = a^2 + 4 - 4a\left(\frac{1}{2}\right) = a^2 - 2a + 4$.

Applying the law of sines to the figure, $\frac{\sin A}{a} = \frac{\sin B}{b}$
$\Rightarrow \frac{\sqrt{2}/2}{a} = \frac{\sqrt{3}/2}{b} \Rightarrow b = \sqrt{\frac{3}{2}}a$. Thus, combining results,
$a^2 - 2a + 4 = b^2 = \frac{3}{2}a^2 \Rightarrow 0 = \frac{1}{2}a^2 + 2a - 4$
$\Rightarrow 0 = a^2 + 4a - 8$. From the quadratic formula and the
fact that $a > 0$, we have $a = \frac{-4 + \sqrt{4^2 - 4(1)(-8)}}{2} = \frac{4\sqrt{3} - 4}{2} \simeq 1.464$.

39. (a) The graphs of y = sin x and y = x nearly coincide when x is near the origin (when the calculator is in radians mode).

(b) In degree mode, when x is near zero degrees the sine of x is much closer to zero than x itself. The curves look like intersecting straight lines near the origin when the calculator is in degree mode.

40. (a) Cos x and sec x are positive in QI and QIV and negative in QII and QIII. Sec x is undefined when cos x is 0. The range of sec x is $(-\infty, -1] \cup [1, \infty)$; the range of cos x is $[-1, 1]$.