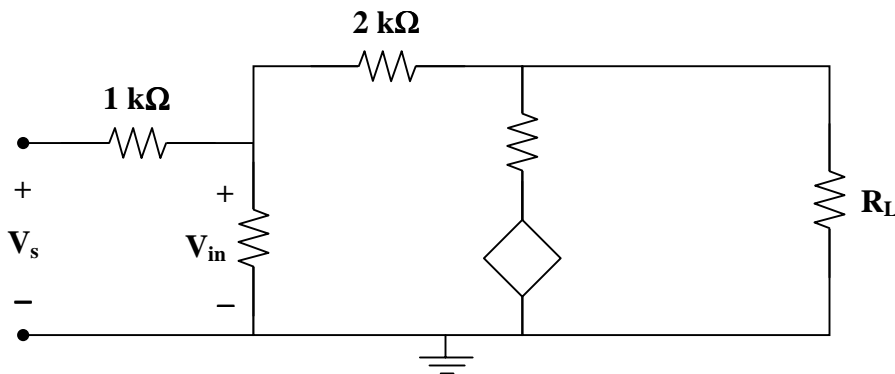# CHAPTER 5 - OPERATIONAL AMPLIFIER

List of topics for this chapter :
> Operational Amplifiers
> Ideal Operational Amplifier
> Inverting Amplifier
> Noninverting Amplifier
> Summing Amplifier
> Difference Amplifier
> Cascaded Operational Amplifier Circuits
> Operational Amplifier Circuits with PSpice
> Applications

OPERATIONAL AMPLIFIERS

Problem 5.1   Calculate V_{out} for $R_L = 1\,\Omega, 100\,\Omega, 1\,k\Omega, 10\,k\Omega, 100\,k\Omega$, given the

circuit in Figure 5.1.

Figure 5.1

> **Carefully DEFINE the problem.**
> Each component is labeled completely. The problem is clear.

> **PRESENT everything you know about the problem.**
> The value of the load resistor, R_L, is varied over a wide range of values.
>
> The dependent voltage source (a voltage-controlled voltage source) has a large gain (100k).
>
> Obviously, the goal of the problem is to determine the output voltage in terms of the input (or source) voltage.

> **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**

V_{out} can be determined using nodal analysis, mesh analysis, or circuit analysis. Because we want to find a voltage, rather than a current, we will use nodal analysis.

➤ **ATTEMPT a problem solution.**
Use nodal analysis to find V_{out} in terms of V_S and R_L.

At the left node or V_{in} :
$$\frac{V_{in} - V_s}{1k} + \frac{V_{in} - 0}{100k} + \frac{V_{in} - V_{out}}{2k} = 0$$

At the right node or V_{out} :
$$\frac{V_{out} - V_{in}}{2k} + \frac{V_{out} + 100k \, V_{in}}{50} + \frac{V_{out} - 0}{R_L} = 0$$

Simplifying the left node equation,
$$(100)(V_{in} - V_s) + V_{in} + (50)(V_{in} - V_{out}) = 0$$
$$151 V_{in} - 100 V_s - 50 V_{out} = 0$$

Simplifying the right node equation,
$$(V_{out} - V_{in}) + (40)(V_{out} + 100k \, V_{in}) + \frac{2k}{R_L} V_{out} = 0$$

$$\left(41 + \frac{2k}{R_L}\right) V_{out} + (4M - 1) V_{in} = 0$$

$$V_{in} = \frac{-\left(41 + \frac{2k}{R_L}\right)}{4M - 1} V_{out}$$

Substituting the simplified right node equation into the simplified left node equation,

$$\frac{-(151)\left(41 + \frac{2k}{R_L}\right)}{4M - 1} V_{out} - 100 V_s - 50 V_{out} = 0$$

$$\frac{-(151)\left(41 + \frac{2k}{R_L}\right) - (50)(4M - 1)}{(4M - 1)} V_{out} = 100 V_s$$

$$V_{out} = \frac{(100)(4M - 1)}{-(151)\left(41 + \frac{2k}{R_L}\right) - (50)(4M - 1)} V_s$$

Substitute each value for R_L into this equation to find V_{out} in terms of V_s.

For $R_L = 1\,\Omega$, $V_{out} = -1.99692283\, V_s$

$$\text{For } R_L = 100\,\Omega, \qquad V_{out} = -1.99990789\,V_s$$
$$\text{For } R_L = 1\,k\Omega, \qquad V_{out} = -1.99993507\,V_s$$
$$\text{For } R_L = 10\,k\Omega, \qquad V_{out} = -1.99993779\,V_s$$
$$\text{For } R_L = 100\,k\Omega, \qquad V_{out} = -1.99993806\,V_s$$

➢ **EVALUATE the solution and check for accuracy.**
First, the answers appear reasonable with the gain of the entire circuit approaching 2 as R_L increases in size.

In addition, even for $R_L = 1\,\Omega$, $V_{out} = 2\,V_s$ is a good approximation

Clearly, using an ideal op amp is reasonable.

➢ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

$$\text{For } R_L = 1\,\Omega, \qquad V_{out} = \mathbf{-1.99692283\,V_s}$$
$$\text{For } R_L = 100\,\Omega, \qquad V_{out} = \mathbf{-1.99990789\,V_s}$$
$$\text{For } R_L = 1\,k\Omega, \qquad V_{out} = \mathbf{-1.99993507\,V_s}$$
$$\text{For } R_L = 10\,k\Omega, \qquad V_{out} = \mathbf{-1.99993779\,V_s}$$
$$\text{For } R_L = 100\,k\Omega, \qquad V_{out} = \mathbf{-1.99993806\,V_s}$$

====

Problem 5.2   *[5.1]*   The equivalent model of a certain op amp is shown in Figure 5.1.
Determine:
      (a)   the input resistance,
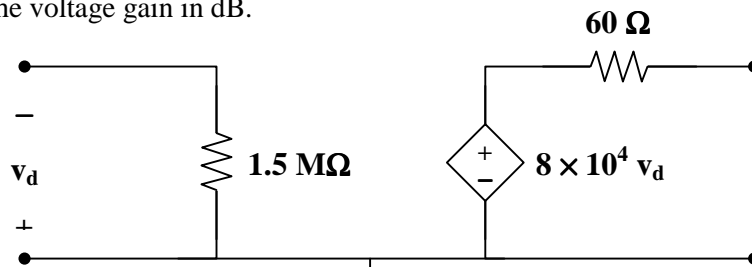   (b)   the output resistance,
      (c)   the voltage gain in dB.

Figure 5.1

(a) $R_{in} = \mathbf{1.5\,M\Omega}$

(b) $R_{out} = \mathbf{60\,\Omega}$

(c) $A = 8 \times 10^4$

$$A_{dB} = 20\log_{10}(8\times10^4)$$
$$A_{dB} = \underline{\textbf{98.06}}$$

Problem 5.3   Solve for V_{out} when $R_L = 1\,\Omega$, $1\text{ k}\Omega$, $100\text{ k}\Omega$, $1\text{ M}\Omega$, given the circuit in Figure 5.1.
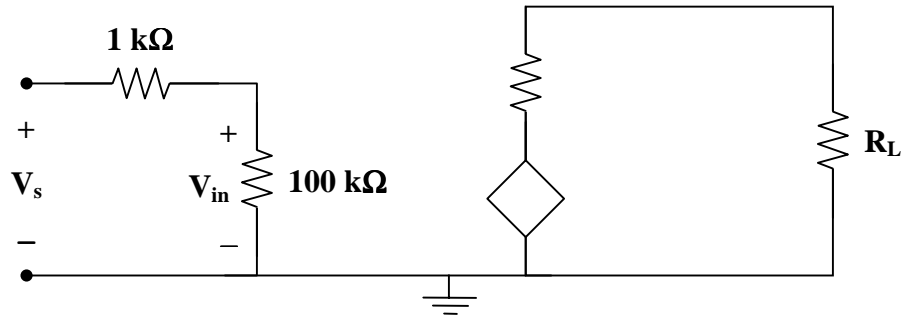

Figure 5.1

Use nodal analysis to find V_{out} in terms of V_S and R_L.

At the left node or V_{in} :
$$\frac{V_{in} - V_S}{1k} + \frac{V_{in} - 0}{100k} = 0$$

At the right node or V_{out} :
$$\frac{V_{out} + 100k\,V_{in}}{50} + \frac{V_{out} - 0}{R_L} = 0$$

Simplifying the left nodal equation,
$$(100)(V_{in} - V_S) + V_{in} = 0$$
$$101\,V_{in} = 100\,V_S$$
$$V_{in} = \frac{100}{101}V_S$$

Simplifying the right node equation, $\left(\dfrac{1}{50} + \dfrac{1}{R_L}\right)V_{out} + 2k\,V_{in} = 0$

$$\left(\frac{R_L + 50}{50\,R_L}\right)V_{out} = -2k\,V_{in}$$
$$V_{out} = \left(\frac{-100k\,R_L}{R_L + 50}\right)V_{in}$$

Substituting the simplified left node equation into the simplified right node equation,

$$V_{out} = \left(\frac{-100k\,R_L}{R_L + 50} \right)\left(\frac{100}{101} \right) V_S = \left(\frac{-10M\,R_L}{(101)(R_L + 50)} \right) V_S$$

Substitute each value for R_L into this equation to find V_{out} in terms of V_S.

For $R_L = 1\,\Omega$, $V_{out} = \mathbf{-1{,}941.3706\,V_S}$

For $R_L = 1\,k\Omega$, $V_{out} = \mathbf{-94{,}295.1438\,V_S}$

For $R_L = 100\,k\Omega$, $V_{out} = \mathbf{-98{,}960.4208\,V_S}$

For $R_L = 1\,M\Omega$, $V_{out} = \mathbf{-99{,}004.9507\,V_S}$

IDEAL OPERATIONAL AMPLIFIER

An ideal op amp has infinite open-loop gain, infinite input resistance, and zero output resistance.

Problem 5.4   Looking at the circuit in Figure 5.1, what effect does R_L have on the value of V_{out}?
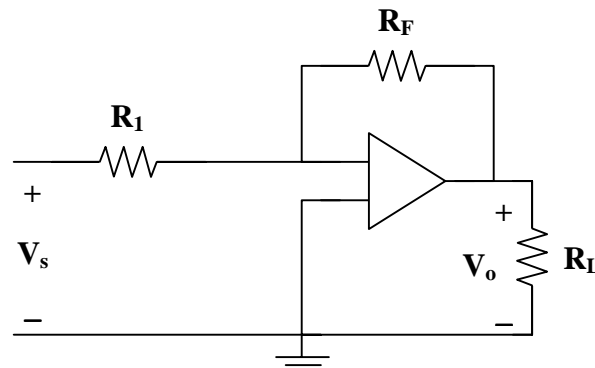

Figure 5.1

➤ **Carefully DEFINE the problem.**
Each component is labeled completely. The problem is clear.

➤ **PRESENT everything you know about the problem.**
Essentially, we are to determine if the value of R_L affects the output voltage in any way.

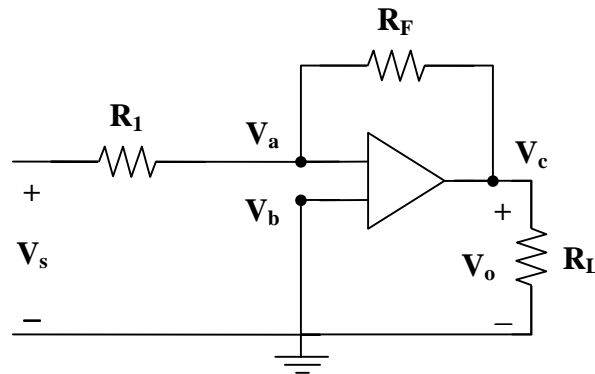Thus, the goal of the problem is to solve for V_o in terms of the other variables.

Treat the operational amplifier as ideal. Due to infinite input resistance, we know that the currents into both input terminals are zero. The voltage across the terminals is negligibly small or $V_a = V_b$.

➢ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
Because this is a circuit problem, we can use nodal analysis, mesh analysis, or basic circuit analysis. Nodal analysis typically works best for op amp circuits.

➢ **ATTEMPT a problem solution.**
Referring to the circuit below, there are three unknown nodes.

Write a node equation at node a. The node voltage at node b is already known, $V_b = 0$. Writing a node equation at node c will only introduce an additional unknown. This gives two equations with four unknowns. Solving for V_o in terms of V_s (and the resistances) implies that we need one equation with two unknowns. Hence, we need a constraint equation to reduce the number of unknowns.

At node a,
$$\frac{V_a - V_s}{R_1} + \frac{V_a - V_c}{R_F} + 0 = 0$$

At node b,
$$V_b = 0$$

The constraint equation comes from a characteristic of the ideal op amp.
$$V_a = V_b$$

Thus,
$$V_a = V_b = 0$$

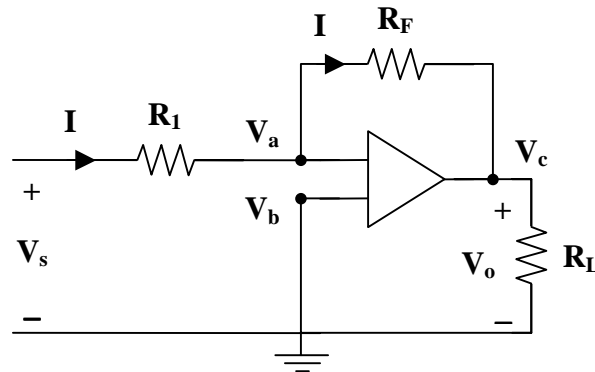Substitute the constraint into the node equation for node a to solve for V_o.
$$\frac{-V_s}{R_1} + \frac{-V_c}{R_F} = 0$$

Hence, $\qquad V_c = \dfrac{-R_F}{R_1} V_s$

Clearly, $\qquad V_o = V_c = \dfrac{-R_F}{R_1} V_s$

We have shown that the value of R_L has no effect on the value of V_o, assuming that R_L is finite and not equal to zero.

➢ **EVALUATE the solution and check for accuracy.**
Consider the following circuit.

Using Ohm's law, $I = \dfrac{V_s - V_a}{R_1}$

From a characteristic of the ideal op amp, $V_a = V_b$

But $\qquad V_b = 0 \longrightarrow V_a = 0$

So, $\qquad I = \dfrac{V_s}{R_1}$

Also, $\qquad I = \dfrac{V_a - V_c}{R_F} = \dfrac{-V_c}{R_F}$

But $\qquad V_c = V_o$

So, $\qquad I = \dfrac{-V_o}{R_F}$

Thus,

$$\dfrac{V_s}{R_1} = \dfrac{-V_o}{R_F} \longrightarrow V_o = \dfrac{-R_F}{R_1} V_s$$

Our check for accuracy was successful.

➤ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

The value of R_L has no effect on the value of V_o, assuming that R_L is finite and not zero.

==

Problem 5.5   *[5.11]*   Find v_o and i_o in the circuit in Figure 5.1.
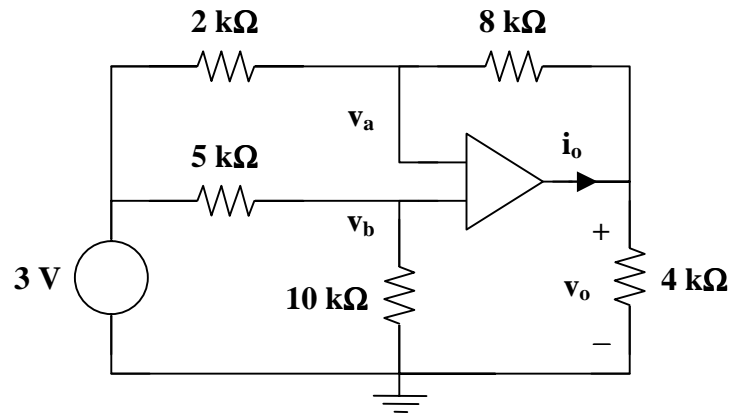

Figure 5.1

At node b, $v_b = \left(\dfrac{10}{10+5} \right)(3) = 2 \text{ V}$

At node a, $\dfrac{3 - v_a}{2} = \dfrac{v_a - v_o}{8} \longrightarrow 12 = 5 v_a - v_o$

But $v_a = v_b = 2 \text{ V}$

So, $12 = (5)(2) - v_o$
         $v_o = \textbf{-2 V}$

$i_o = \dfrac{v_o - v_a}{8k} + \dfrac{v_o - 0}{4k} = \dfrac{-2-2}{8k} + \dfrac{-2}{4k} = -0.5m - 0.5m$
$i_o = \textbf{-1 mA}$

INVERTING AMPLIFIER

An inverting amplifier reverses the polarity of the input signal while amplifying it.

Problem 5.6 *[5.19]* Using the circuit in Figure 5.1, calculate v_o if $v_s = 0$.
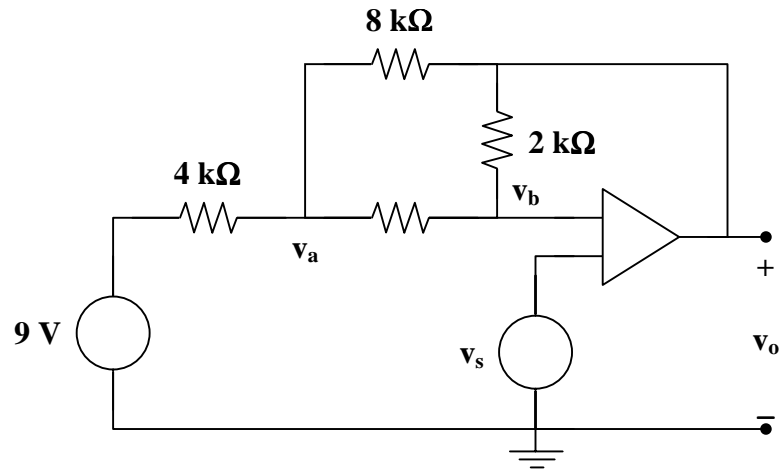

Figure 5.1

At node a,

$$\frac{9 - v_a}{4k} = \frac{v_a - v_o}{8k} + \frac{v_a - v_b}{4k}$$

$$18 = 5v_a - v_o - 2v_b \qquad (1)$$

At node b,

$$\frac{v_a - v_b}{4k} = \frac{v_b - v_o}{2k}$$

$$v_a = 3v_b - 2v_o \qquad (2)$$

But $\qquad v_b = v_s = 0$

Hence, (2) becomes $\qquad v_a = -2v_o$

and (1) becomes $\qquad 18 = -10v_o - v_o = -11v_o$ and $v_o = \underline{\textbf{- 1.6364 V}}$

Problem 5.7   Express V_o in terms of V_s for the circuit shown in Figure 5.1.
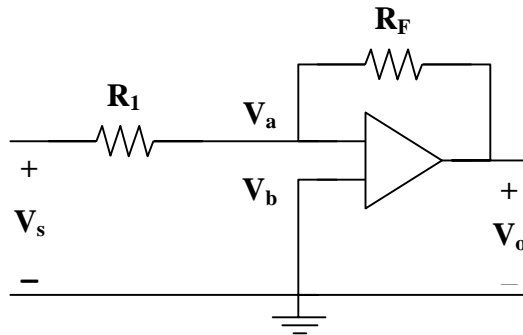

Figure 5.1

Using nodal analysis,

$$\frac{V_a - V_s}{R_1} + \frac{V_a - V_o}{R_F} = 0$$

where $V_a = V_b = 0$ is the constraint equation.

$$\frac{-V_s}{R_1} + \frac{-V_o}{R_F} = 0$$

$$V_o = \frac{-R_F}{R_1} V_s$$

NONINVERTING AMPLIFIER

Problem 5.8   How does the circuit in Figure 5.1 differ from the circuit in Figure 5.1?
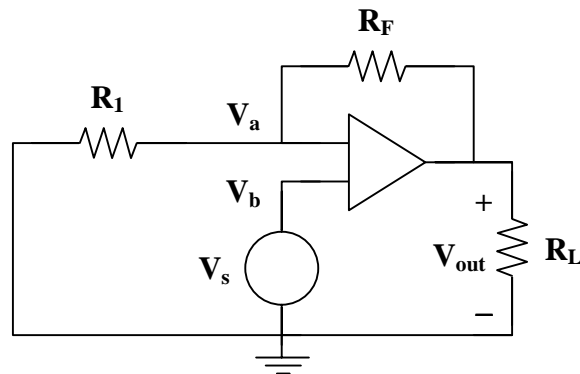

Figure 5.1

Using nodal analysis,

$$\frac{V_a - 0}{R_1} + \frac{V_a - V_{out}}{R_F} = 0$$

where $V_a = V_b = V_s$ is the constraint equation.

Simplifying,

$$\frac{V_s}{R_1} + \frac{V_s - V_{out}}{R_F} = 0$$

$$\frac{1}{R_F}V_{out} = \left(\frac{1}{R_1} + \frac{1}{R_F}\right)V_s$$

$$V_{out} = \left(\frac{R_F}{R_1} + 1\right)V_s$$

The significant difference between the two circuits is that the voltage gain for this circuit is positive. In addition it should be noted that in the circuit of Figure 5.1, the relationship between the output voltage, V_o, and the input voltage, V_s, is a simple ratio of R_F and R_1. For the circuit in Figure 5.1, however, the gain can never be less than one. Since there is rarely a case where the gain is less than one, this is not normally a problem.

SUMMING AMPLIFIER

A summing amplifier combines several inputs and produces an output that is the weighted sum of the inputs.

Problem 5.9   *[5.33]*   A four-input summing amplifier has $R_1 = R_2 = R_3 = R_4 = 12\ k\Omega$. What value of feedback resistor is needed to make it an averaging amplifier?

In order for $v_o = \frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3 + \frac{R_f}{R_4}v_4 = \frac{R_f}{12k}(v_1 + v_2 + v_3 + v_4)$

to become $v_o = \frac{-1}{4}(v_1 + v_2 + v_3 + v_4) \longrightarrow \frac{1}{4} = \frac{R_f}{12k}$

$$R_f = \frac{12k}{4} = \underline{\mathbf{3\ k\Omega}}$$

Problem 5.10   Express V_{out} in terms of V_1 and V_2 for the circuit shown in Figure 5.1. What have we done here?
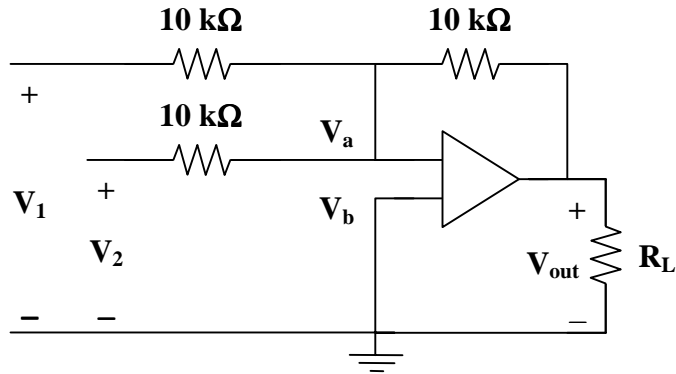

Figure 5.1

Using nodal analysis,

$$\frac{V_a - V_1}{10k} + \frac{V_a - V_2}{10k} + \frac{V_a - V_{out}}{10k} = 0$$

where $V_a = V_b = 0$ is the constraint equation.

$$\frac{-V_1}{10k} + \frac{-V_2}{10k} + \frac{-V_{out}}{10k} = 0$$
$$V_{out} = -V_1 - V_2$$
$$V_{out} = \underline{-(V_1 + V_2)}$$

<u>**We have constructed an inverting, summing amplifier**</u>.

DIFFERENCE AMPLIFIER

A difference amplifier amplifies the difference between two inputs but rejects any signals common to the two inputs.

Problem 5.11   Using an operational amplifier, can we construct a circuit where $V_{out} = V_2 - V_1$?

<u>**Yes**</u>, we want to construct what is called a difference amplifier. We can do this using the circuit shown in Figure 5.1.
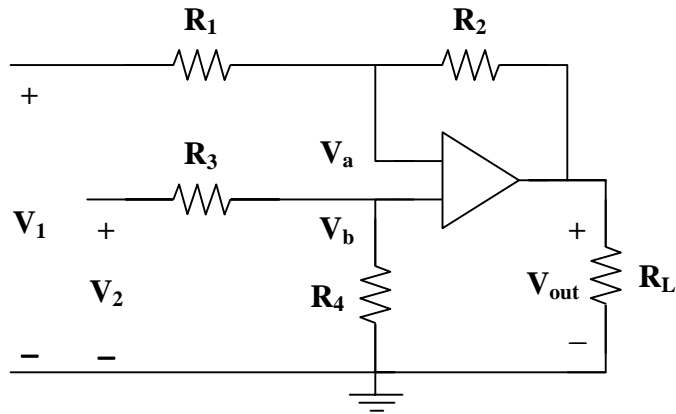
Figure 5.1

Now, verify that this circuit will amplify the difference of the two inputs.

Using nodal analysis,

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_{out}}{R_2} = 0 \qquad \text{and} \qquad \frac{V_b - V_2}{R_3} + \frac{V_b - 0}{R_4} = 0$$

where $V_a = V_b$ is the constraint equation.

Simplifying,

$$\left(\frac{R_2}{R_1} + 1\right)V_a - \frac{R_2}{R_1}V_1 = V_{out} \qquad \text{and} \qquad V_b = \frac{R_4}{R_3 + R_4}V_2$$

Using the constraint equation to combine the two equations yields

$$V_{out} = \left(\frac{R_2}{R_1} + 1\right)\left(\frac{R_4}{R_3 + R_4}\right)V_2 - \frac{R_2}{R_1}V_1$$

or

$$V_{out} = \frac{R_2}{R_1}\left(\frac{1 + R_1/R_2}{1 + R_3/R_4}\right)V_2 - \frac{R_2}{R_1}V_1$$

When $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$, $\qquad\qquad V_{out} = \dfrac{R_2}{R_1}(V_2 - V_1)$

If $R_1 = R_2$ and $R_3 = R_4$,

$$V_{out} = V_2 - V_1$$

This was the desired case.

Problem 5.12   *[5.39]*   Design a difference amplifier to have a gain of 2 and a common mode input resistance of 10 kΩ at each input.

The input resistances are
$$R_1 = R_3 = 10 \text{ k}\Omega$$

For a gain of 2,
$$\frac{R_2}{R_1} = 2 \quad \longrightarrow \quad R_2 = 2R_1 = 20 \text{ k}\Omega$$

A property of difference amplifiers is
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Thus,
$$R_4 = R_2 = 20 \text{ k}\Omega$$

Now, verify the results,
$$v_o = \frac{R_2}{R_1} \frac{(1 + R_1/R_2)}{(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$$
$$v_o = \left(\frac{20k}{10k}\right)\left(\frac{1 + 0.5}{1 + 0.5}\right)v_2 - \frac{20k}{10k} v_1$$
$$v_o = 2(v_2 - v_1)$$

This is the desired result. Therefore,
$$R_1 = R_3 = \underline{\textbf{10 k}\Omega} \qquad\qquad R_2 = R_4 = \underline{\textbf{20 k}\Omega}$$

CASCADED OPERATIONAL AMPLIFIER CIRCUITS

A cascade connection is a head-to-tail arrangement of two or more op amp circuits such that the output of one op amp circuit is the input to the next op amp circuit.

Problem 5.13   *[5.45]*   Refer to the circuit in Figure 5.1. Calculate i_o if:

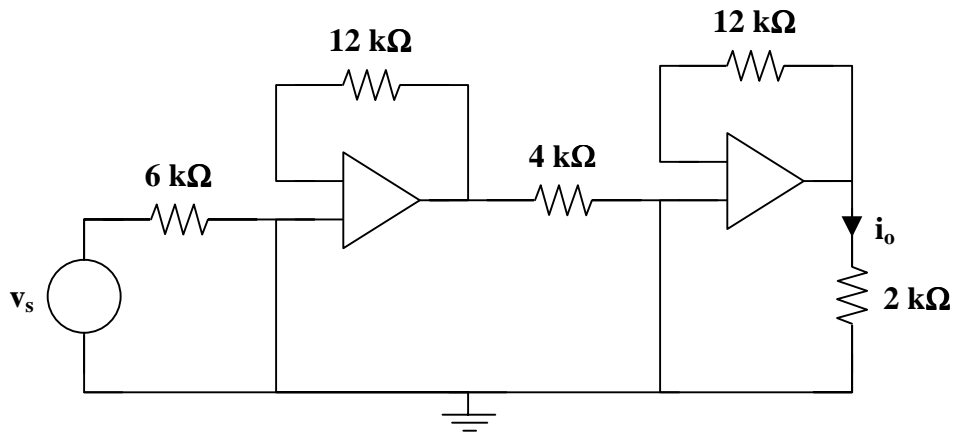   (a) $v_s = 12$ mV

    (b) $v_s = 10\cos(377t)$ mV

Figure 5.1

This is a cascading system of two inverting amplifiers.

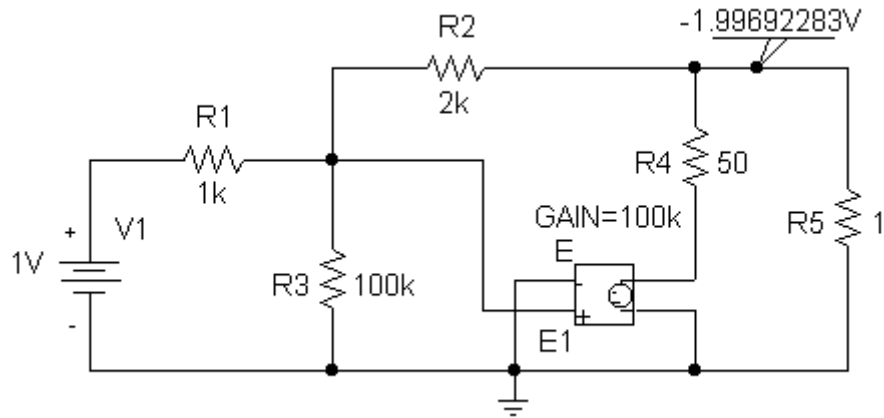$$v_o = \left(\frac{-12}{4}\right)\left(\frac{-12}{6}\right)v_s = 6\,v_s$$

$$i_o = \frac{v_s}{2\times10^3} = 3\times10^{-3}\,v_s$$

(a)   When $v_s = 12\,\text{mV}$,
$$i_o = \underline{\mathbf{36\;\mu A}}$$

(b)   When $v_s = 10\cos(377t)\,\text{mV}$
$$i_o = \underline{\mathbf{30\cos(377t)\;\mu A}}$$

OPERATIONAL AMPLIFIER CIRCUITS WITH PSPICE

Problem 5.14   Solve Problem 5.1 using PSpice.

PSpice does not perform symbolic simulations. So, let $V_s = 1\,\text{V}$. Add a VIEWPOINT to the circuit to indicate the output voltage. Set the load resistor to the desired value, save the schematic and simulate. With the repetition of setting the load resistor and simulating the circuit, the output voltage for each load resistor in Problem 5.1 can be verified.

From the schematic,

for $R_L = 1\,\Omega$, $V_{out} = \underline{\textbf{-1.99692283 V}}$

By changing the load resistor and simulating the circuit, it can be shown that

for $R_L = 100\,\Omega$, $V_{out} = \underline{\textbf{-1.99990789 V}}$

for $R_L = 1\,k\Omega$, $V_{out} = \underline{\textbf{-1.99993507 V}}$

for $R_L = 10\,k\Omega$, $V_{out} = \underline{\textbf{-1.99993779 V}}$

for $R_L = 100\,k\Omega$, $V_{out} = \underline{\textbf{-1.99993806 V}}$

This matches the answers obtained in Problem 5.1 when $V_s = 1\,V$.

Problem 5.15   Solve Problem 5.7 using PSpice.

Consider the following schematic.

Because PSpice does not perform symbolic simulations, let $V_s = 1\,V$. We also need to choose values for the resistors.

For the circuit above, let

$$V_s = \underline{1\,V}, \qquad R_1 = \underline{1\,k\Omega}, \quad \text{and} \quad R_F = \underline{10\,k\Omega}$$

which produces,

$$V_o = \frac{-R_F}{R_1} V_s = \frac{-10k}{1k}(1) = \underline{-10\,V}$$

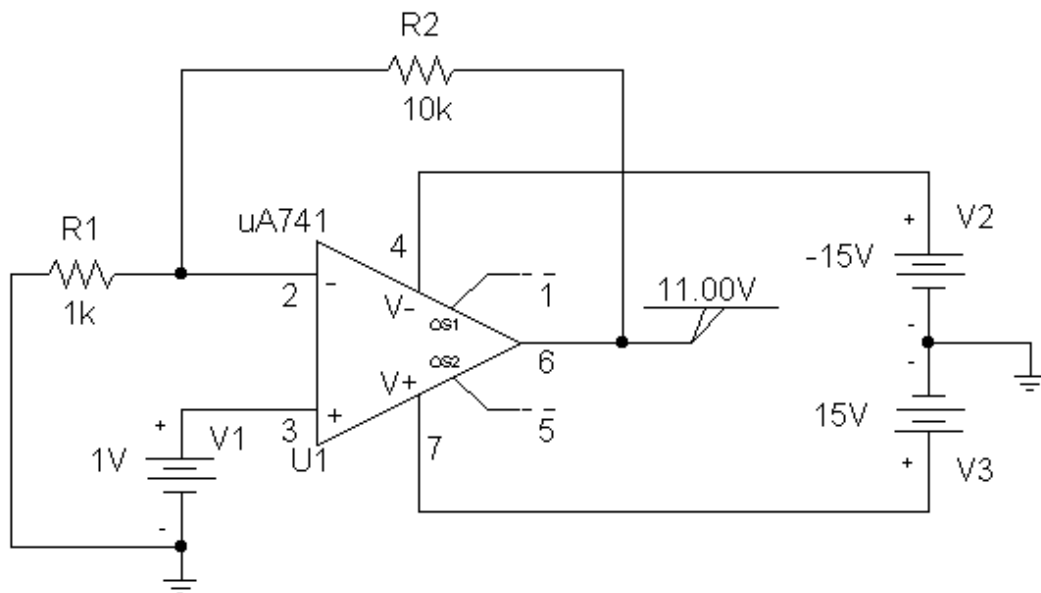This verifies the answer obtained in Problem 5.7.

One must realize that Problem 5.7 was performed assuming an ideal op amp; PSpice does not simulate an ideal op amp. Thus, the output voltage may not be an integer value even though the calculations from Problem 5.7 would predict an integer value.

Also, the output voltage cannot be greater than V+ or less than V−, where V+ and V− are the power supply voltages of the op amp.

There are three modes in which real op amps can operate. The most desirable is to have them give the desired output. The second mode is when the op amp goes into saturation, reaching its maximum output voltage and remaining there. The third mode is that the op amp can act like an oscillator; its output voltage can be some type of periodic signal such as a sine wave.

Problem 5.16   Solve Problem 5.8 using PSpice.

Consider the following schematic.

For the circuit above,
$$V_s = \underline{1\ V}, \qquad R_1 = \underline{1\ k\Omega}, \quad \text{and} \quad R_F = \underline{10\ k\Omega}$$
and
$$V_o = \left(\frac{R_F}{R_1} + 1\right)V_s = \left(\frac{10k}{1k} + 1\right)(1) = \underline{\underline{11\ V}}$$
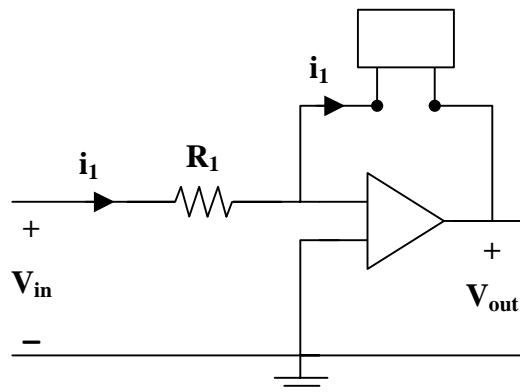
This verifies the answer obtained in Problem 5.8

Also, see the comments made concerning ideal versus real op amps in Problem 5.15.

APPLICATIONS

Problem 5.17   Use an operational amplifier to change an ideal voltage source to an ideal current source.

Consider the following circuit.

Clearly,
$$i_1 = \frac{V_{in}}{R_1}$$

In the circuit above, the op amp will maintain the current through the black box in the feedback path at i_1. Thus, the op amp is working like an ideal current source.