CHAPTER 18

TWO-PORT NETWORKS

Research is to see what everybody else has seen, and think what nobody has thought.

—Albert Szent-Gyorgyi

Enhancing Your Career

Career in Education While two thirds of all engineers work in private industry, some work in academia and prepare students for engineering careers. The course on circuit analysis you are studying is an important part of the preparation process. If you enjoy teaching others, you may want to consider becoming an engineering educator.

Engineering professors work on state-of-the-art research projects, teach courses at graduate and undergraduate levels, and provide services to their professional societies and the community at large. They are expected to make original contributions in their areas of specialty. This requires a broad-based education in the fundamentals of electrical engineering and a mastery of the skills necessary for communicating their efforts to others.

If you like to do research, to work at the frontiers of engineering, to make contributions to technological advancement, to invent, consult, and/or teach, consider a career in engineering education. The best way to start is by talking with your professors and benefiting from their experience.

A solid understanding of mathematics and physics at the undergraduate level is vital to your success as an engineering professor. If you are having difficulty in solving your engineering textbook problems, start correcting any weaknesses you have in your mathematics and physics fundamentals.

Most universities these days require that engineering professors have a Ph.D. degree. In addition, some universities require that they be actively involved in research leading to publications in reputable journals. To prepare yourself for a career in engineering education, get as broad an education as possible, because electrical engineering is changing rapidly and becoming interdisciplinary. Without doubt, engineering education is a rewarding career. Professors get a sense of satisfaction and fulfillment as they see their students graduate, become leaders in the professions, and contribute significantly to the betterment of humanity.

The lecture method is still regarded as the most effective mode of teaching because of the personal contact with instructor and opportunity to ask questions. Source: ©PhotoDisc, Inc. Copyright 1999.

18.1 INTRODUCTION

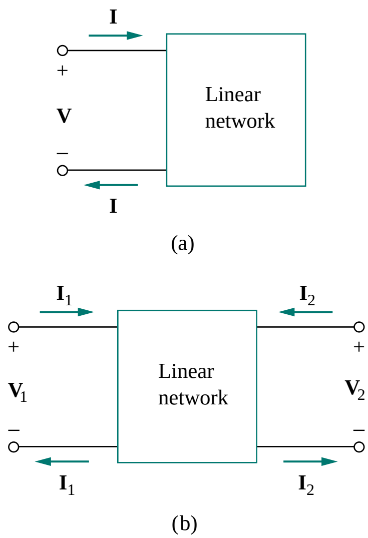

Figure 18.1 (a) One-port network, (b) two-port network.

A pair of terminals through which a current may enter or leave a network is known as a *port*. Two-terminal devices or elements (such as resistors, capacitors, and inductors) result in one-port networks. Most of the circuits we have dealt with so far are two-terminal or one-port circuits, represented in Fig. 18.1(a). We have considered the voltage across or current through a single pair of terminals—such as the two terminals of a resistor, a capacitor, or an inductor. We have also studied four-terminal or two-port circuits involving op amps, transistors, and transformers, as shown in Fig. 18.1(b). In general, a network may have *n* ports. A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.

In this chapter, we are mainly concerned with *two-port* networks (or, simply, *two-ports*).

> A two-port network is an electrical network with two separate ports for input and output.

Thus, a two-port network has two terminal pairs acting as access points. As shown in Fig. 18.1(b), the current entering one terminal of a pair leaves the other terminal in the pair. Three-terminal devices such as transistors can be configured into two-port networks.

Our study of two-port networks is for at least two reasons. First, such networks are useful in communications, control systems, power systems, and electronics. For example, they are used in electronics to model transistors and to facilitate cascaded design. Second, knowing the parameters of a two-port network enables us to treat it as a "black box" when embedded within a larger network.

To characterize a two-port network requires that we relate the terminal quantities $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, and $\mathbf{I}_2$ in Fig. 18.1(b), out of which two are independent. The various terms that relate these voltages and currents are called *parameters*. Our goal in this chapter is to derive six sets of these parameters. We will show the relationship between these parameters and how two-port networks can be connected in series, parallel, or cascade. As with op amps, we are only interested in the terminal behavior of the circuits. And we will assume that the two-port circuits contain no independent sources, although they can contain dependent sources. Finally, we will apply some of the concepts developed in this chapter to the analysis of transistor circuits and synthesis of ladder networks.

18.2 IMPEDANCE PARAMETERS

Impedance and admittance parameters are commonly used in the synthesis of filters. They are also useful in the design and analysis of impedance-matching networks and power distribution networks. We discuss impedance parameters in this section and admittance parameters in the next section.

A two-port network may be voltage-driven as in Fig. 18.2(a) or current-driven as in Fig. 18.2(b). From either Fig. 18.2(a) or (b), the terminal voltages can be related to the terminal currents as

$$\boxed{\begin{aligned} \mathbf{V}_1 &= \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{aligned}} \tag{18.1}$$

or in matrix form as

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} \tag{18.2}$$

Reminder: Only two of the four variables ($\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, and $\mathbf{I}_2$) are independent. The other two can be found using Eq. (18.1).

where the $\mathbf{z}$ terms are called the *impedance parameters*, or simply z *parameters*, and have units of ohms.

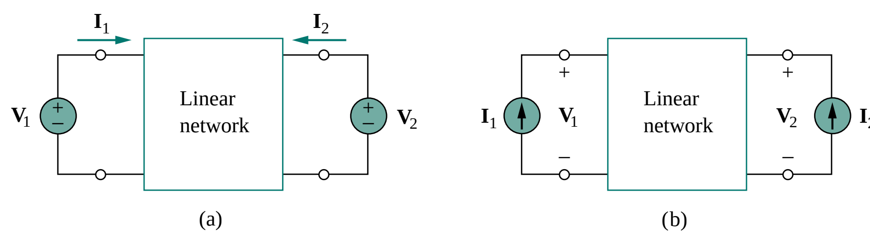

Figure 18.2 The linear two-port network: (a) driven by voltage sources, (b) driven by current sources.

The values of the parameters can be evaluated by setting $\mathbf{I}_1 = 0$ (input port open-circuited) or $\mathbf{I}_2 = 0$ (output port open-circuited). Thus,

$$\boxed{\begin{aligned} \mathbf{z}_{11} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad & \mathbf{z}_{12} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \\ \mathbf{z}_{21} &= \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad & \mathbf{z}_{22} &= \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \end{aligned}} \tag{18.3}$$

Since the z parameters are obtained by open-circuiting the input or output port, they are also called the *open-circuit impedance parameters*. Specifically,

$$\begin{aligned} \mathbf{z}_{11} &= \text{Open-circuit input impedance} \\ \mathbf{z}_{12} &= \text{Open-circuit transfer impedance from port 1 to port 2} \\ \mathbf{z}_{21} &= \text{Open-circuit transfer impedance from port 2 to port 1} \\ \mathbf{z}_{22} &= \text{Open-circuit output impedance} \end{aligned} \tag{18.4}$$

According to Eq. (18.3), we obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$ by connecting a voltage $\mathbf{V}_1$ (or a current source $\mathbf{I}_1$) to port 1 with port 2 open-circuited as in Fig. 18.3(a) and finding $\mathbf{I}_1$ and $\mathbf{V}_2$; we then get

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}, \qquad \mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} \tag{18.5}$$

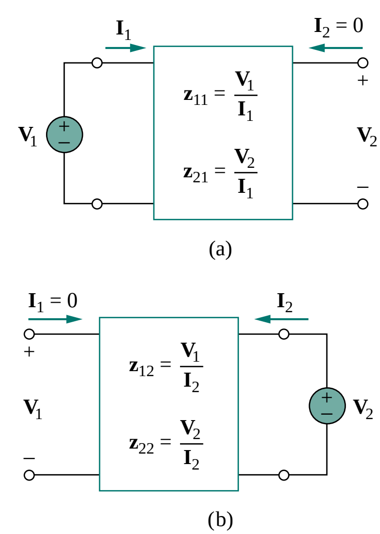

Figure 18.3 Determination of the z parameters: (a) finding $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, (b) finding $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$.

Similarly, we obtain $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$ by connecting a voltage $\mathbf{V}_2$ (or a current source $\mathbf{I}_2$) to port 2 with port 1 open-circuited as in Fig. 18.3(b) and finding $\mathbf{I}_2$ and $\mathbf{V}_1$; we then get

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2}, \qquad \mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} \tag{18.6}$$

The above procedure provides us with a means of calculating or measuring the z parameters.

Sometimes $\mathbf{z}_{11}$ and $\mathbf{z}_{22}$ are called *driving-point impedances*, while $\mathbf{z}_{21}$ and $\mathbf{z}_{12}$ are called *transfer impedances*. A driving-point impedance is the input impedance of a two-terminal (one-port) device. Thus, $\mathbf{z}_{11}$ is the input driving-point impedance with the output port open-circuited, while $\mathbf{z}_{22}$ is the output driving-point impedance with the input port open-circuited.

When $\mathbf{z}_{11} = \mathbf{z}_{22}$, the two-port network is said to be *symmetrical*. This implies that the network has mirrorlike symmetry about some center line; that is, a line can be found that divides the network into two similar halves.

When the two-port network is linear and has no dependent sources, the transfer impedances are equal ($\mathbf{z}_{12} = \mathbf{z}_{21}$), and the two-port is said to be *reciprocal*. This means that if the points of excitation and response are interchanged, the transfer impedances remain the same. As illustrated in Fig. 18.4, a two-port is reciprocal if interchanging an ideal voltage source at one port with an ideal ammeter at the other port gives the same ammeter reading. The reciprocal network yields $\mathbf{V} = \mathbf{z}_{12}\mathbf{I}$ according to Eq. (18.1) when connected as in Fig. 18.4(a), but yields $\mathbf{V} = \mathbf{z}_{21}\mathbf{I}$ when connected as in Fig. 18.4(b). This is possible only if $\mathbf{z}_{12} = \mathbf{z}_{21}$. Any two-port that is made entirely of resistors, capacitors, and inductors must be reciprocal. For a reciprocal network, the T-equivalent circuit in Fig. 18.5(a) can be used. If the network is not reciprocal, a more general equivalent network is shown in Fig. 18.5(b); notice that this figure follows directly from Eq. (18.1).

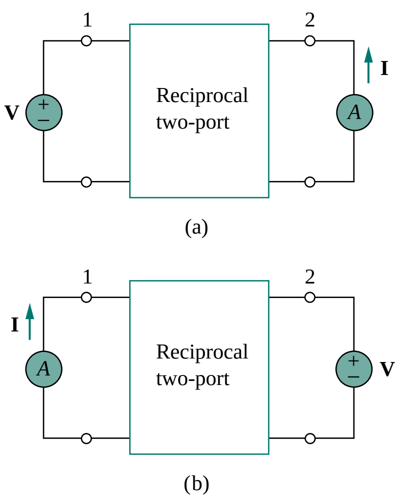

Figure 18.4 Interchanging a voltage source at one port with an ideal ammeter at the other port produces the same reading in a reciprocal two-port.

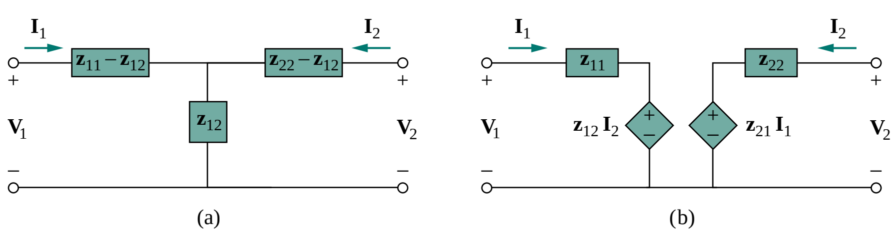

Figure 18.5 (a) T-equivalent circuit (for reciprocal case only), (b) general equivalent circuit.

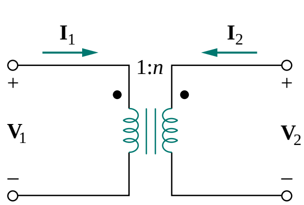

Figure 18.6 An ideal transformer has no z parameters.

It should be mentioned that for some two-port networks, the z parameters do not exist because they cannot be described by Eq. (18.1). As an example, consider the ideal transformer of Fig. 18.6. The defining equations for the two-port network are:

$$\mathbf{V}_1 = \frac{1}{n}\mathbf{V}_2, \qquad \mathbf{I}_1 = -n\mathbf{I}_2 \tag{18.7}$$

Observe that it is impossible to express the voltages in terms of the currents, and vice versa, as Eq. (18.1) requires. Thus, the ideal transformer has no z parameters. However, it does have hybrid parameters, as we shall see in Section 18.4.

EXAMPLE 18.1

Determine the z parameters for the circuit in Fig. 18.7.

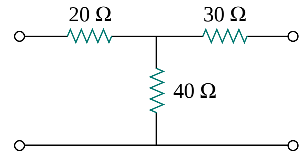

Figure 18.7 For Example 18.1.

Solution:

METHOD 1 To determine $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, we apply a voltage source $\mathbf{V}_1$ to the input port and leave the output port open as in Fig. 18.8(a). Then,

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{(20 + 40)\mathbf{I}_1}{\mathbf{I}_1} = 60\ \Omega$$

that is, $\mathbf{z}_{11}$ is the input impedance at port 1.

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{40\mathbf{I}_1}{\mathbf{I}_1} = 40\ \Omega$$

To find $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$, we apply a voltage source $\mathbf{V}_2$ to the output port and leave the input port open as in Fig. 18.8(b). Then,

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = \frac{40\mathbf{I}_2}{\mathbf{I}_2} = 40\ \Omega, \qquad \mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{(30 + 40)\mathbf{I}_2}{\mathbf{I}_2} = 70\ \Omega$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} 60\ \Omega & 40\ \Omega \\ 40\ \Omega & 70\ \Omega \end{bmatrix}$$

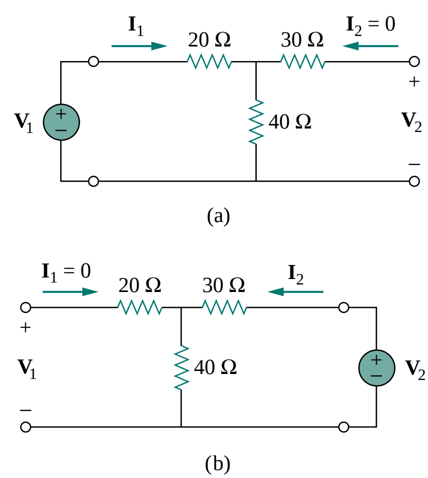

Figure 18.8 For Example 18.1: (a) finding $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, (b) finding $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$.

METHOD 2 Alternatively, since there is no dependent source in the given circuit, $\mathbf{z}_{12} = \mathbf{z}_{21}$ and we can use Fig. 18.5(a). Comparing Fig. 18.7 with Fig. 18.5(a), we get

$$\mathbf{z}_{12} = 40\ \Omega = \mathbf{z}_{21}$$

$$\mathbf{z}_{11} - \mathbf{z}_{12} = 20 \quad \Longrightarrow \quad \mathbf{z}_{11} = 20 + \mathbf{z}_{12} = 60\ \Omega$$

$$\mathbf{z}_{22} - \mathbf{z}_{12} = 30 \quad \Longrightarrow \quad \mathbf{z}_{22} = 30 + \mathbf{z}_{12} = 70\ \Omega$$

PRACTICE PROBLEM 18.1

Find the z parameters of the two-port network in Fig. 18.9.

Answer: $\mathbf{z}_{11} = 14, \mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{z}_{22} = 6\ \Omega$.

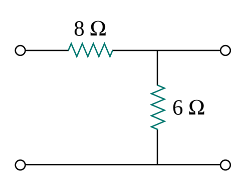

Figure 18.9 For Practice Prob. 18.1.

EXAMPLE 18.2

Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit in Fig. 18.10.

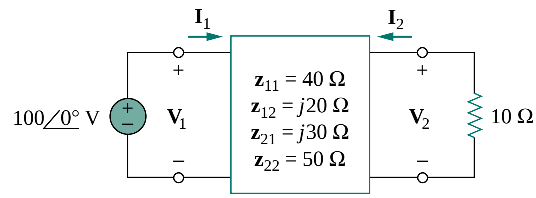

Figure 18.10 For Example 18.2.

Solution:

This is not a reciprocal network. We may use the equivalent circuit in Fig. 18.5(b) but we can also use Eq. (18.1) directly. Substituting the given z parameters into Eq. (18.1),

$$\mathbf{V}_1 = 40\mathbf{I}_1 + j20\mathbf{I}_2 \tag{18.2.1}$$

$$\mathbf{V}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2 \tag{18.2.2}$$

Since we are looking for $\mathbf{I}_1$ and $\mathbf{I}_2$, we substitute

$$\mathbf{V}_1 = 100\underline{/0^\circ}, \qquad \mathbf{V}_2 = -10\mathbf{I}_2$$

into Eqs. (18.2.1) and (18.2.2), which become

$$100 = 40\mathbf{I}_1 + j20\mathbf{I}_2 \tag{18.2.3}$$

$$-10\mathbf{I}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2 \quad \Longrightarrow \quad \mathbf{I}_1 = j2\mathbf{I}_2 \tag{18.2.4}$$

Substituting Eq. (18.2.4) into Eq. (18.2.3) gives

$$100 = j80\mathbf{I}_2 + j20\mathbf{I}_2 \quad \Longrightarrow \quad \mathbf{I}_2 = \frac{100}{j100} = -j$$

From Eq. (18.2.4), $\mathbf{I}_1 = j2(-j) = 2$. Thus,

$$\mathbf{I}_1 = 2\underline{/0^\circ}\text{ A}, \qquad \mathbf{I}_2 = 1\underline{/-90^\circ}\text{ A}$$

PRACTICE PROBLEM 18.2

Calculate $\mathbf{I}_1$ and $\mathbf{I}_2$ in the two-port of Fig. 18.11.

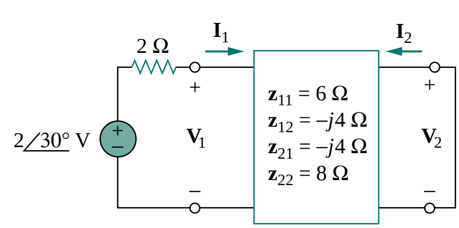

Figure 18.11 For Practice Prob. 18.2.

Answer: $2\underline{/20^\circ}$ A, $1\underline{/-60^\circ}$ A.

18.3 ADMITTANCE PARAMETERS

In the previous section we saw that impedance parameters may not exist for a two-port network. So there is a need for an alternative means of describing such a network. This need is met by the second set of parameters, which we obtain by expressing the terminal currents in terms of the terminal voltages. In either Fig. 18.12(a) or (b), the terminal currents can be expressed in terms of the terminal voltages as

$$\begin{aligned} \mathbf{I}_1 &= \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \end{aligned} \tag{18.8}$$

or in matrix form as

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{y}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} \tag{18.9}$$

The **y** terms are known as the *admittance parameters* (or, simply, *y parameters*) and have units of siemens.

The values of the parameters can be determined by setting $\mathbf{V}_1 = 0$ (input port short-circuited) or $\mathbf{V}_2 = 0$ (output port short-circuited). Thus,

$$\begin{aligned} \mathbf{y}_{11} &= \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} \\ \mathbf{y}_{21} &= \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} \end{aligned} \tag{18.10}$$

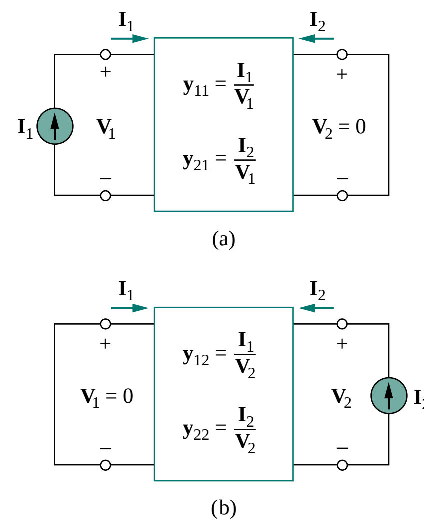

Figure 18.12 Determination of the y parameters: (a) finding y_{11} and y_{21}, (b) finding y_{12} and y_{22}.

Since the y parameters are obtained by short-circuiting the input or output port, they are also called the *short-circuit admittance parameters*. Specifically,

$$\begin{aligned} \mathbf{y}_{11} &= \text{Short-circuit input admittance} \\ \mathbf{y}_{12} &= \text{Short-circuit transfer admittance from port 2 to port 1} \\ \mathbf{y}_{21} &= \text{Short-circuit transfer admittance from port 1 to port 2} \\ \mathbf{y}_{22} &= \text{Short-circuit output admittance} \end{aligned} \tag{18.11}$$

Following Eq. (18.10), we obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by connecting a current $\mathbf{I}_1$ to port 1 and short-circuiting port 2 as in Fig. 18.12(a), finding $\mathbf{V}_1$ and $\mathbf{I}_2$, and then calculating

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1}, \qquad \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} \tag{18.12}$$

Similarly, we obtain $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$ by connecting a current source $\mathbf{I}_2$ to port 2 and short-circuiting port 1 as in Fig. 18.12(b), finding $\mathbf{I}_1$ and $\mathbf{V}_2$, and then getting

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2}, \qquad \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} \tag{18.13}$$

This procedure provides us with a means of calculating or measuring the y parameters. The impedance and admittance parameters are collectively referred to as *immittance* parameters.

For a two-port network that is linear and has no dependent sources, the transfer admittances are equal ($\mathbf{y}_{12} = \mathbf{y}_{21}$). This can be proved in the same way as for the z parameters. A reciprocal network ($\mathbf{y}_{12} = \mathbf{y}_{21}$) can be modeled by the Π-equivalent circuit in Fig. 18.13(a). If the network is not reciprocal, a more general equivalent network is shown in Fig. 18.13(b).

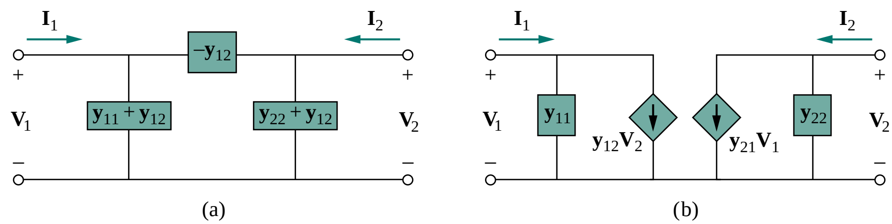

Figure 18.13 (a) Π-equivalent circuit (for reciprocal case only), (b) general equivalent circuit.

EXAMPLE 18.3

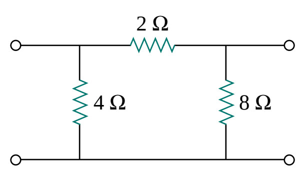

Figure 18.14 For Example 18.3.

Obtain the y parameters for the Π network shown in Fig. 18.14.

Solution:

METHOD 1 To find $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, short-circuit the output port and connect a current source $\mathbf{I}_1$ to the input port as in Fig. 18.15(a). Since the 8-Ω resistor is short-circuited, the 2-Ω resistor is in parallel with the 4-Ω resistor. Hence,

$$\mathbf{V}_1 = \mathbf{I}_1(4 \parallel 2) = \frac{4}{3}\mathbf{I}_1, \qquad \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{\mathbf{I}_1}{\frac{4}{3}\mathbf{I}_1} = 0.75 \text{ S}$$

By current division,

$$-\mathbf{I}_2 = \frac{4}{4+2}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1, \qquad \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-\frac{2}{3}\mathbf{I}_1}{\frac{4}{3}\mathbf{I}_1} = -0.5 \text{ S}$$

To get $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$, short-circuit the input port and connect a current source $\mathbf{I}_2$ to the output port as in Fig. 18.15(b). The 4-Ω resistor is short-circuited so that the 2-Ω and 8-Ω resistors are in parallel.

$$\mathbf{V}_2 = \mathbf{I}_2(8 \parallel 2) = \frac{8}{5}\mathbf{I}_2, \qquad \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{\mathbf{I}_2}{\frac{8}{5}\mathbf{I}_2} = \frac{5}{8} = 0.625 \text{ S}$$

By current division,

$$-\mathbf{I}_1 = \frac{8}{8+2}\mathbf{I}_2 = \frac{4}{5}\mathbf{I}_2, \qquad \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\frac{4}{5}\mathbf{I}_2}{\frac{8}{5}\mathbf{I}_2} = -0.5 \text{ S}$$

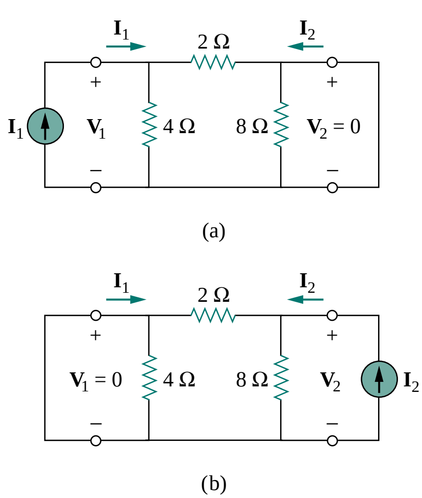

Figure 18.15 For Example 18.3: (a) finding $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, (b) finding $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$.

METHOD 2 Alternatively, comparing Fig. 18.14 with Fig. 18.13(a),

$$\mathbf{y}_{12} = -\frac{1}{2} \text{ S} = \mathbf{y}_{21}$$

$$\mathbf{y}_{11} + \mathbf{y}_{12} = \frac{1}{4} \qquad \Longrightarrow \qquad \mathbf{y}_{11} = \frac{1}{4} - \mathbf{y}_{12} = 0.75 \text{ S}$$

$$\mathbf{y}_{22} + \mathbf{y}_{12} = \frac{1}{8} \qquad \Longrightarrow \qquad \mathbf{y}_{22} = \frac{1}{8} - \mathbf{y}_{12} = 0.625 \text{ S}$$

as obtained previously.

PRACTICE PROBLEM 18.3

Obtain the y parameters for the T network shown in Fig. 18.16.

Answer: $\mathbf{y}_{11} = 0.2273$ S, $\mathbf{y}_{12} = \mathbf{y}_{21} = -0.0909$ S, $\mathbf{y}_{22} = 0.1364$ S.

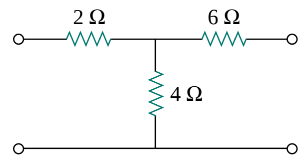

Figure 18.16 For Practice Prob. 18.3.

EXAMPLE 18.4

Determine the y parameters for the two-port shown in Fig. 18.17.

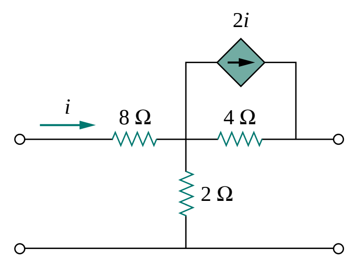

Figure 18.17 For Example 18.4.

Solution:

We follow the same procedure as in the previous example. To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, we use the circuit in Fig. 18.18(a), in which port 2 is short-circuited and a current source is applied to port 1. At node 1,

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{8} = 2\mathbf{I}_1 + \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - 0}{4}$$

But $\mathbf{I}_1 = \dfrac{\mathbf{V}_1 - \mathbf{V}_o}{8}$; therefore,

$$0 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{8} + \frac{3\mathbf{V}_o}{4}$$

$$0 = \mathbf{V}_1 - \mathbf{V}_o + 6\mathbf{V}_o \quad \Longrightarrow \quad \mathbf{V}_1 = -5\mathbf{V}_o$$

Hence,

$$\mathbf{I}_1 = \frac{-5\mathbf{V}_o - \mathbf{V}_o}{8} = -0.75\mathbf{V}_o$$

and

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{-0.75\mathbf{V}_o}{-5\mathbf{V}_o} = 0.15 \text{ S}$$

At node 2,

$$\frac{\mathbf{V}_o - 0}{4} + 2\mathbf{I}_1 + \mathbf{I}_2 = 0$$

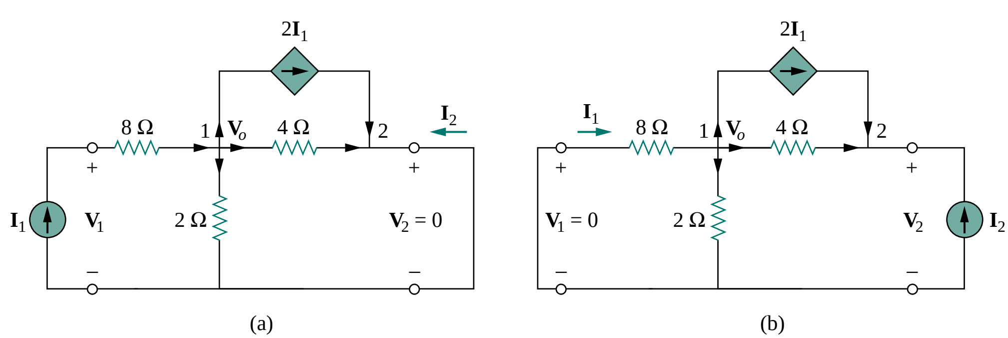

Figure 18.18 Solution of Example 18.4: (a) finding $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, (b) finding $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$.

or

$$-\mathbf{I}_2 = 0.25\mathbf{V}_o - 1.5\mathbf{V}_o = -1.25\mathbf{V}_o$$

Hence,

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{1.25\mathbf{V}_o}{-5\mathbf{V}_o} = -0.25 \text{ S}$$

Similarly, we get $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$ using Fig. 18.18(b). At node 1,

$$\frac{0 - \mathbf{V}_o}{8} = 2\mathbf{I}_1 + \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - \mathbf{V}_2}{4}$$

But $\mathbf{I}_1 = \dfrac{0 - \mathbf{V}_o}{8}$; therefore,

$$0 = -\frac{\mathbf{V}_o}{8} + \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - \mathbf{V}_2}{4}$$

or

$$0 = -\mathbf{V}_o + 4\mathbf{V}_o + 2\mathbf{V}_o - 2\mathbf{V}_2 \quad \Longrightarrow \quad \mathbf{V}_2 = 2.5\mathbf{V}_o$$

Hence,

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\mathbf{V}_o/8}{2.5\mathbf{V}_o} = -0.05 \text{ S}$$

At node 2,

$$\frac{\mathbf{V}_o - \mathbf{V}_2}{4} + 2\mathbf{I}_1 + \mathbf{I}_2 = 0$$

or

$$-\mathbf{I}_2 = 0.25\mathbf{V}_o - \frac{1}{4}(2.5\mathbf{V}_o) - \frac{2\mathbf{V}_o}{8} = -0.625\mathbf{V}_o$$

Thus,

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{0.625\mathbf{V}_o}{2.5\mathbf{V}_o} = 0.25 \text{ S}$$

Notice that $\mathbf{y}_{12} \neq \mathbf{y}_{21}$ in this case, since the network is not reciprocal.

PRACTICE PROBLEM 18.4

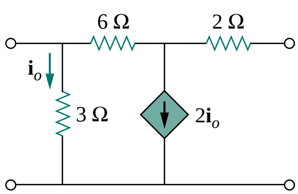

Figure 18.19 For Practice Prob. 18.4.

Obtain the y parameters for the circuit in Fig. 18.19.

Answer: $\mathbf{y}_{11} = 0.625$ S, $\mathbf{y}_{12} = -0.125$ S, $\mathbf{y}_{21} = 0.375$ S, $\mathbf{y}_{22} = 0.125$ S.

18.4 HYBRID PARAMETERS

The z and y parameters of a two-port network do not always exist. So there is a need for developing another set of parameters. This third set of parameters is based on making $\mathbf{V}_1$ and $\mathbf{I}_2$ the dependent variables. Thus, we obtain

$$\begin{aligned}\mathbf{V}_1 &= \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2\end{aligned} \tag{18.14}$$

or in matrix form,

$$\begin{bmatrix}\mathbf{V}_1 \\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix}\mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{I}_1 \\ \mathbf{V}_2\end{bmatrix} = [\mathbf{h}]\begin{bmatrix}\mathbf{I}_1 \\ \mathbf{V}_2\end{bmatrix} \tag{18.15}$$

The **h** terms are known as the *hybrid parameters* (or, simply, *h parameters*) because they are a hybrid combination of ratios. They are very useful for describing electronic devices such as transistors (see Section 18.9); it is much easier to measure experimentally the h parameters of such devices than to measure their z or y parameters. In fact, we have seen that the ideal transformer in Fig. 18.6, described by Eq. (18.7), does not have z parameters. The ideal transformer can be described by the hybrid parameters, because Eq. (18.7) conforms with Eq. (18.14).

The values of the parameters are determined as

$$\begin{aligned}\mathbf{h}_{11} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, & \mathbf{h}_{12} &= \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} \\ \mathbf{h}_{21} &= \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, & \mathbf{h}_{22} &= \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}\end{aligned} \tag{18.16}$$

It is evident from Eq. (18.16) that the parameters $\mathbf{h}_{11}$, $\mathbf{h}_{12}$, $\mathbf{h}_{21}$, and $\mathbf{h}_{22}$ represent an impedance, a voltage gain, a current gain, and an admittance, respectively. This is why they are called the hybrid parameters. To be specific,

$$\begin{aligned}\mathbf{h}_{11} &= \text{Short-circuit input impedance} \\ \mathbf{h}_{12} &= \text{Open-circuit reverse voltage gain} \\ \mathbf{h}_{21} &= \text{Short-circuit forward current gain} \\ \mathbf{h}_{22} &= \text{Open-circuit output admittance}\end{aligned} \tag{18.17}$$

The procedure for calculating the h parameters is similar to that used for the z or y parameters. We apply a voltage or current source to the appropriate port, short-circuit or open-circuit the other port, depending on the parameter of interest, and perform regular circuit analysis. For reciprocal networks, $\mathbf{h}_{12} = -\mathbf{h}_{21}$. This can be proved in the same way as we proved that $\mathbf{z}_{12} = \mathbf{z}_{21}$. Figure 18.20 shows the hybrid model of a two-port network.

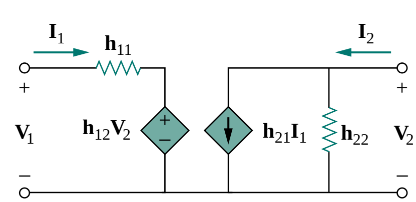

Figure 18.20 The h-parameter equivalent network of a two-port network.

A set of parameters closely related to the h parameters are the g *parameters* or *inverse hybrid parameters*. These are used to describe the terminal currents and voltages as

$$\begin{aligned}\mathbf{I}_1 &= \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2\end{aligned} \tag{18.18}$$

or

$$\begin{bmatrix}\mathbf{I}_1 \\ \mathbf{V}_2\end{bmatrix} = \begin{bmatrix}\mathbf{g}_{11} & \mathbf{g}_{12} \\ \mathbf{g}_{21} & \mathbf{g}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1 \\ \mathbf{I}_2\end{bmatrix} = [\mathbf{g}]\begin{bmatrix}\mathbf{V}_1 \\ \mathbf{I}_2\end{bmatrix} \tag{18.19}$$

The values of the g parameters are determined as

$$\mathbf{g}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{g}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_1=0}$$
$$\mathbf{g}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{g}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{V}_1=0} \tag{18.20}$$

Thus, the inverse hybrid parameters are specifically called

$$\begin{aligned} \mathbf{g}_{11} &= \text{Open-circuit input admittance} \\ \mathbf{g}_{12} &= \text{Short-circuit reverse current gain} \\ \mathbf{g}_{21} &= \text{Open-circuit forward voltage gain} \\ \mathbf{g}_{22} &= \text{Short-circuit output impedance} \end{aligned} \tag{18.21}$$

Figure 18.21 shows the inverse hybrid model of a two-port network.

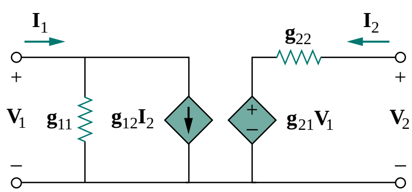

Figure 18.21 The g-parameter model of a two-port network.

EXAMPLE 18.5

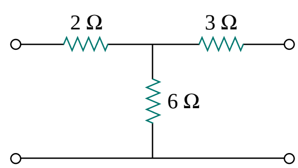

Figure 18.22 For Example 18.5.

Find the hybrid parameters for the two-port network of Fig. 18.22.

Solution:

To find $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, we short-circuit the output port and connect a current source $\mathbf{I}_1$ to the input port as shown in Fig. 18.23(a). From Fig. 18.23(a),

$$\mathbf{V}_1 = \mathbf{I}_1(2 + 3 \parallel 6) = 4\mathbf{I}_1$$

Hence,

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 4\ \Omega$$

Also, from Fig. 18.23(a) we obtain, by current division,

$$-\mathbf{I}_2 = \frac{6}{6+3}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1$$

Hence,

$$\mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{2}{3}$$

To obtain $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$, we open-circuit the input port and connect a voltage source $\mathbf{V}_2$ to the output port as in Fig. 18.23(b). By voltage division,

$$\mathbf{V}_1 = \frac{6}{6+3}\mathbf{V}_2 = \frac{2}{3}\mathbf{V}_2$$

Hence,

$$\mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{2}{3}$$

Also,

$$\mathbf{V}_2 = (3+6)\mathbf{I}_2 = 9\mathbf{I}_2$$

Thus,

$$\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{9}\ \text{S}$$

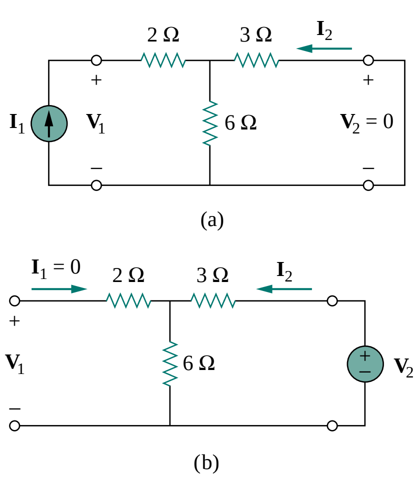

Figure 18.23 For Example 18.5: (a) computing $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, (b) computing $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$.

PRACTICE PROBLEM 18.5

Determine the h parameters for the circuit in Fig. 18.24.

Answer: $\mathbf{h}_{11} = 1.2\ \Omega$, $\mathbf{h}_{12} = 0.4$, $\mathbf{h}_{21} = -0.4$, $\mathbf{h}_{22} = 0.4$ S.

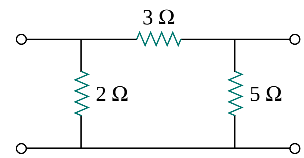

Figure 18.24 For Practice Prob. 18.5.

EXAMPLE 18.6

Determine the Thevenin equivalent at the output port of the circuit in Fig. 18.25.

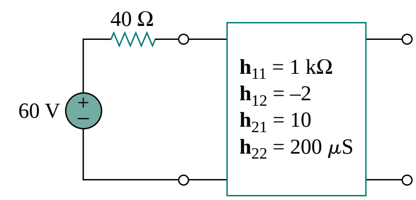

Figure 18.25 For Example 18.6.

Solution:

To find $\mathbf{Z}_{\text{Th}}$ and $\mathbf{V}_{\text{Th}}$, we apply the normal procedure, keeping in mind the formulas relating the input and output ports of the h model. To obtain $\mathbf{Z}_{\text{Th}}$, remove the 60-V voltage source at the input port and apply a 1-V voltage source at the output port, as shown in Fig. 18.26(a). From Eq. (18.14),

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \tag{18.6.1}$$

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \tag{18.6.2}$$

But $\mathbf{V}_2 = 1$, and $\mathbf{V}_1 = -40\mathbf{I}_1$. Substituting these into Eqs. (18.6.1) and (18.6.2), we get

$$-40\mathbf{I}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12} \quad \Longrightarrow \quad \mathbf{I}_1 = -\frac{\mathbf{h}_{12}}{40 + \mathbf{h}_{11}} \tag{18.6.3}$$

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22} \tag{18.6.4}$$

Substituting Eq. (18.6.3) into Eq. (18.6.4) gives

$$\mathbf{I}_2 = \mathbf{h}_{22} - \frac{\mathbf{h}_{21}\mathbf{h}_{12}}{\mathbf{h}_{11} + 40} = \frac{\mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{21}\mathbf{h}_{12} + \mathbf{h}_{22}40}{\mathbf{h}_{11} + 40}$$

Therefore,

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{\mathbf{I}_2} = \frac{\mathbf{h}_{11} + 40}{\mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{21}\mathbf{h}_{12} + \mathbf{h}_{22}40}$$

Substituting the values of the h parameters,

$$\mathbf{Z}_{\text{Th}} = \frac{1000 + 40}{10^3 \times 200 \times 10^{-6} + 20 + 40 \times 200 \times 10^{-6}} = \frac{1040}{20.21} = 51.46\ \Omega$$

To get $\mathbf{V}_{\text{Th}}$, we find the open-circuit voltage $\mathbf{V}_2$ in Fig. 18.26(b). At the input port,

$$-60 + 40\mathbf{I}_1 + \mathbf{V}_1 = 0 \quad \Longrightarrow \quad \mathbf{V}_1 = 60 - 40\mathbf{I}_1 \tag{18.6.5}$$

At the output,

$$\mathbf{I}_2 = 0 \tag{18.6.6}$$

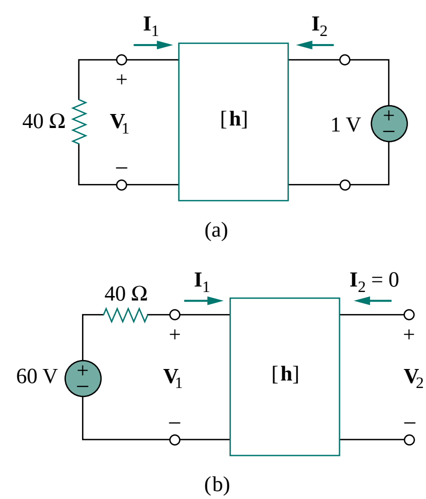

Figure 18.26 For Example 18.6: (a) finding $\mathbf{Z}_{\text{Th}}$, (b) finding $\mathbf{V}_{\text{Th}}$.

Substituting Eqs. (18.6.5) and (18.6.6) into Eqs. (18.6.1) and (18.6.2), we obtain

$$60 - 40\mathbf{I}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$

or

$$60 = (\mathbf{h}_{11} + 40)\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \tag{18.6.7}$$

and

$$0 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \qquad \Longrightarrow \qquad \mathbf{I}_1 = -\frac{\mathbf{h}_{22}}{\mathbf{h}_{21}}\mathbf{V}_2 \tag{18.6.8}$$

Now substituting Eq. (18.6.8) into Eq. (18.6.7) gives

$$60 = \left[-(\mathbf{h}_{11} + 40)\frac{\mathbf{h}_{22}}{\mathbf{h}_{21}} + \mathbf{h}_{12}\right]\mathbf{V}_2$$

or

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_2 = \frac{60}{-(\mathbf{h}_{11} + 40)\mathbf{h}_{22}/\mathbf{h}_{21} + \mathbf{h}_{12}} = \frac{60\mathbf{h}_{21}}{\mathbf{h}_{12}\mathbf{h}_{21} - \mathbf{h}_{11}\mathbf{h}_{22} - 40\mathbf{h}_{22}}$$

Substituting the values of the h parameters,

$$\mathbf{V}_{\text{Th}} = \frac{60 \times 10}{-20.21} = -29.69 \text{ V}$$

PRACTICE PROBLEM 18.6

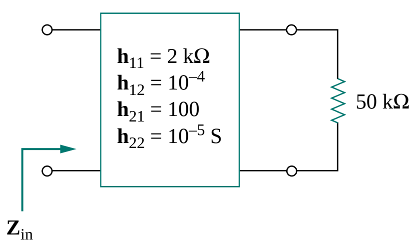

Figure 18.27 For Practice Prob. 18.6.

Find the impedance at the input port of the circuit in Fig. 18.27.

Answer: 1667 Ω.

EXAMPLE 18.7

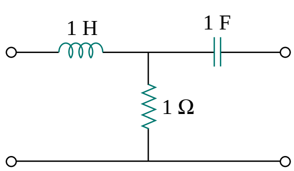

Figure 18.28 For Example 18.7.

Find the g parameters as functions of s for the circuit in Fig. 18.28.

Solution:

In the s domain,

$$1 \text{ H} \quad \Longrightarrow \quad sL = s, \qquad 1 \text{ F} \quad \Longrightarrow \quad \frac{1}{sC} = \frac{1}{s}$$

To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, we open-circuit the output port and connect a voltage source $\mathbf{V}_1$ to the input port as in Fig. 18.29(a). From the figure,

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{s + 1}$$

or

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{s + 1}$$

By voltage division,

$$\mathbf{V}_2 = \frac{1}{s+1}\mathbf{V}_1$$

or

$$\mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{s+1}$$

To obtain $\mathbf{g}_{12}$ and $\mathbf{g}_{22}$, we short-circuit the input port and connect a current source $\mathbf{I}_2$ to the output port as in Fig. 18.29(b). By current division,

$$\mathbf{I}_1 = -\frac{1}{s+1}\mathbf{I}_2$$

or

$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = -\frac{1}{s+1}$$

Also,

$$\mathbf{V}_2 = \mathbf{I}_2\left(\frac{1}{s} + s \parallel 1\right)$$

or

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{s} + \frac{s}{s+1} = \frac{s^2+s+1}{s(s+1)}$$

Thus,

$$[\mathbf{g}] = \begin{bmatrix} \dfrac{1}{s+1} & -\dfrac{1}{s+1} \\ \dfrac{1}{s+1} & \dfrac{s^2+s+1}{s(s+1)} \end{bmatrix}$$

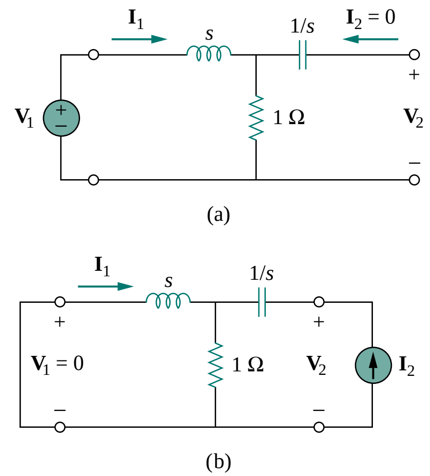

Figure 18.29 Determining the g parameters in the s domain for the circuit in Fig. 18.28.

PRACTICE PROBLEM 18.7

For the ladder network in Fig. 18.30, determine the g parameters in the s domain.

Answer: $[\mathbf{g}] = \begin{bmatrix} \dfrac{s+2}{s^2+3s+1} & -\dfrac{1}{s^2+3s+1} \\ \dfrac{1}{s^2+3s+1} & \dfrac{s(s+2)}{s^2+3s+1} \end{bmatrix}$.

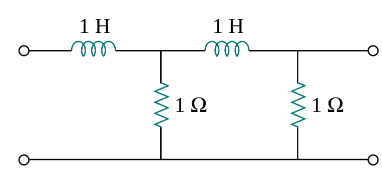

Figure 18.30 For Practice Prob. 18.7.

18.5 TRANSMISSION PARAMETERS

Since there are no restrictions on which terminal voltages and currents should be considered independent and which should be dependent variables, we expect to be able to generate many sets of parameters. Another set of parameters relates the variables at the input port to those at the output port. Thus,

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2 \\ \mathbf{I}_1 &= \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2 \end{aligned} \tag{18.22}$$

or

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix} = [\mathbf{T}] \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix} \tag{18.23}$$

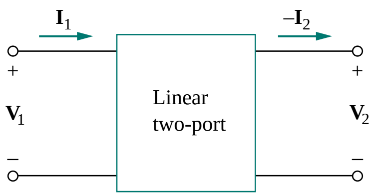

Figure 18.31 Terminal variables used to define the **ABCD** parameters.

Equations (18.22) and (18.23) relate the input variables ($\mathbf{V}_1$ and $\mathbf{I}_1$) to the output variables ($\mathbf{V}_2$ and $-\mathbf{I}_2$). Notice that in computing the transmission parameters, $-\mathbf{I}_2$ is used rather than $\mathbf{I}_2$, because the current is considered to be leaving the network, as shown in Fig. 18.31, as opposed to entering the network as in Fig. 18.1(b). This is done merely for conventional reasons; when you cascade two-ports (output to input), it is most logical to think of $\mathbf{I}_2$ as leaving the two-port. It is also customary in the power industry to consider $\mathbf{I}_2$ as leaving the two-port.

The two-port parameters in Eqs. (18.22) and (18.23) provide a measure of how a circuit transmits voltage and current from a source to a load. They are useful in the analysis of transmission lines (such as cable and fiber) because they express sending-end variables ($\mathbf{V}_1$ and $\mathbf{I}_1$) in terms of the receiving-end variables ($\mathbf{V}_2$ and $-\mathbf{I}_2$). For this reason, they are called *transmission parameters*. They are also known as **ABCD** parameters. They are used in the design of telephone systems, microwave networks, and radars.

The transmission parameters are determined as

$$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{B} = -\left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$
$$\mathbf{C} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{D} = -\left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0} \tag{18.24}$$

Thus, the transmission parameters are called, specifically,

$$\begin{aligned} \mathbf{A} &= \text{Open-circuit voltage ratio} \\ \mathbf{B} &= \text{Negative short-circuit transfer impedance} \\ \mathbf{C} &= \text{Open-circuit transfer admittance} \\ \mathbf{D} &= \text{Negative short-circuit current ratio} \end{aligned} \tag{18.25}$$

A and **D** are dimensionless, **B** is in ohms, and **C** is in siemens. Since the transmission parameters provide a direct relationship between input and output variables, they are very useful in cascaded networks.

Our last set of parameters may be defined by expressing the variables at the output port in terms of the variables at the input port. We obtain

$$\begin{aligned} \mathbf{V}_2 &= \mathbf{a}\mathbf{V}_1 - \mathbf{b}\mathbf{I}_1 \\ \mathbf{I}_2 &= \mathbf{c}\mathbf{V}_1 - \mathbf{d}\mathbf{I}_1 \end{aligned} \tag{18.26}$$

or

$$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ -\mathbf{I}_1 \end{bmatrix} = [\mathbf{t}] \begin{bmatrix} \mathbf{V}_1 \\ -\mathbf{I}_1 \end{bmatrix} \tag{18.27}$$

The parameters **a**, **b**, **c**, and **d** are called the *inverse transmission parameters*. They are determined as follows:

$$\mathbf{a} = \left.\frac{\mathbf{V}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_1=0}, \qquad \mathbf{b} = -\left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_1=0}$$
$$\mathbf{c} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_1=0}, \qquad \mathbf{d} = -\left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_1=0} \tag{18.28}$$

From Eq. (18.28) and from our experience so far, it is evident that these parameters are known individually as

$$\begin{aligned} \mathbf{a} &= \text{Open-circuit voltage gain} \\ \mathbf{b} &= \text{Negative short-circuit transfer impedance} \\ \mathbf{c} &= \text{Open-circuit transfer admittance} \\ \mathbf{d} &= \text{Negative short-circuit current gain} \end{aligned} \tag{18.29}$$

While **a** and **d** are dimensionless, **b** and **c** are in ohms and siemens, respectively. In terms of the transmission or inverse transmission parameters, a network is reciprocal if

$$\mathbf{AD} - \mathbf{BC} = 1, \qquad \mathbf{ad} - \mathbf{bc} = 1 \tag{18.30}$$

These relations can be proved in the same way as the transfer impedance relations for the *z* parameters. Alternatively, we will be able to use Table 18.1 a little later to derive Eq. (18.30) from the fact that $\mathbf{z}_{12} = \mathbf{z}_{21}$ for reciprocal networks.

EXAMPLE 18.8

Find the transmission parameters for the two-port network in Fig. 18.32.

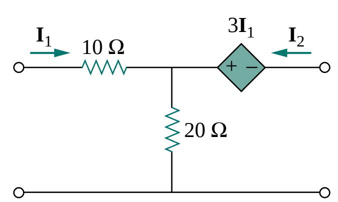

Figure 18.32 For Example 18.8.

Solution:

To determine **A** and **C**, we leave the output port open as in Fig. 18.33(a) so that $\mathbf{I}_2 = 0$ and place a voltage source $\mathbf{V}_1$ at the input port. We have

$$\mathbf{V}_1 = (10 + 20)\mathbf{I}_1 = 30\mathbf{I}_1 \qquad \text{and} \qquad \mathbf{V}_2 = 20\mathbf{I}_1 - 3\mathbf{I}_1 = 17\mathbf{I}_1$$

Thus,

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{30\mathbf{I}_1}{17\mathbf{I}_1} = 1.765, \qquad \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{\mathbf{I}_1}{17\mathbf{I}_1} = 0.0588 \text{ S}$$

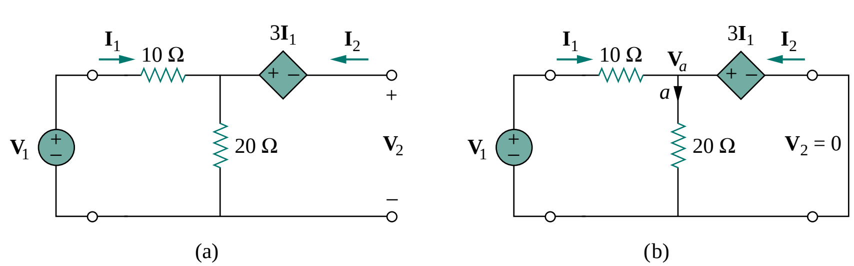

Figure 18.33 For Example 18.8: (a) finding **A** and **C**, (b) finding **B** and **D**.

To obtain **B** and **D**, we short-circuit the output port so that $\mathbf{V}_2 = 0$ as shown in Fig. 18.33(b) and place a voltage source $\mathbf{V}_1$ at the input port. At node a in the circuit of Fig. 18.33(b), KCL gives

$$\frac{\mathbf{V}_1 - \mathbf{V}_a}{10} - \frac{\mathbf{V}_a}{20} + \mathbf{I}_2 = 0 \tag{18.8.1}$$

But $\mathbf{V}_a = 3\mathbf{I}_1$ and $\mathbf{I}_1 = (\mathbf{V}_1 - \mathbf{V}_a)/10$. Combining these gives

$$\mathbf{V}_1 = 13\mathbf{I}_1 \tag{18.8.2}$$

Substituting Eq. (18.8.2) into Eq. (18.8.1) and replacing the first term with $\mathbf{I}_1$,

$$\mathbf{I}_1 - \frac{3\mathbf{I}_1}{20} + \mathbf{I}_2 = 0 \qquad \Longrightarrow \qquad \frac{17}{20}\mathbf{I}_1 = -\mathbf{I}_2$$

Therefore,

$$\mathbf{D} = -\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{20}{17} = 1.176, \qquad \mathbf{B} = -\frac{\mathbf{V}_1}{\mathbf{I}_2} = \frac{-13\mathbf{I}_1}{(-17/20)\,\mathbf{I}_1} = 15.29\ \Omega$$

PRACTICE PROBLEM 18.8

Find the transmission parameters for the circuit in Fig. 18.16 (see Practice Prob. 18.3).

Answer: $\mathbf{A} = 1.5$, $\mathbf{B} = 11\ \Omega$, $\mathbf{C} = 0.25$ S, $\mathbf{D} = 2.5$.

EXAMPLE 18.9

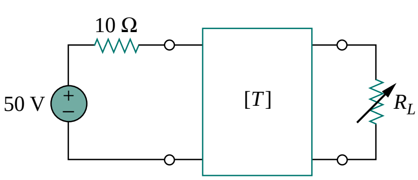

Figure 18.34 For Example 18.9.

The **ABCD** parameters of the two-port network in Fig. 18.34 are

$$\begin{bmatrix} 4 & 20\ \Omega \\ 0.1\text{ S} & 2 \end{bmatrix}$$

The output port is connected to a variable load for maximum power transfer. Find R_L and the maximum power transferred.

Solution:

What we need is to find the Thevenin equivalent ($\mathbf{Z}_{\text{Th}}$ and $\mathbf{V}_{\text{Th}}$) at the load or output port. We find $\mathbf{Z}_{\text{Th}}$ using the circuit in Fig. 18.35(a). Our goal is to get $\mathbf{Z}_{\text{Th}} = \mathbf{V}_2/\mathbf{I}_2$. Substituting the given **ABCD** parameters into Eq. (18.22), we obtain

$$\mathbf{V}_1 = 4\mathbf{V}_2 - 20\mathbf{I}_2 \tag{18.9.1}$$

$$\mathbf{I}_1 = 0.1\mathbf{V}_2 - 2\mathbf{I}_2 \tag{18.9.2}$$

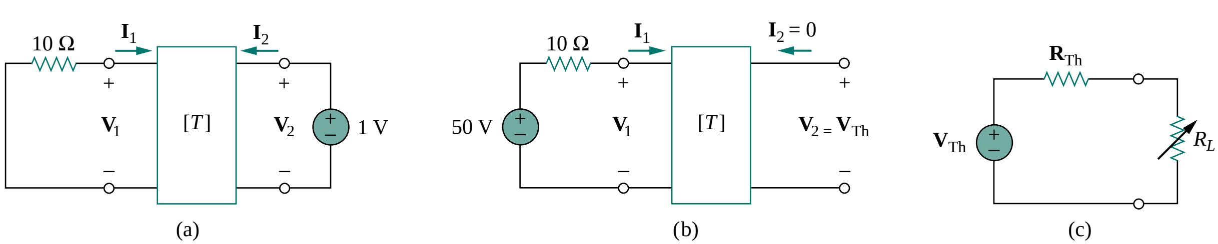

Figure 18.35 Solution of Example 18.9: (a) finding $\mathbf{Z}_{\text{Th}}$, (b) finding $\mathbf{V}_{\text{Th}}$, (c) finding R_L for maximum power transfer.

At the input port, $\mathbf{V}_1 = -10\mathbf{I}_1$. Substituting this into Eq. (18.9.1) gives

$$-10\mathbf{I}_1 = 4\mathbf{V}_2 - 20\mathbf{I}_2$$

or

$$\mathbf{I}_1 = -0.4\mathbf{V}_2 + 2\mathbf{I}_2 \tag{18.9.3}$$

Setting the right-hand sides of Eqs. (18.9.2) and (18.9.3) equal,

$$0.1\mathbf{V}_2 - 2\mathbf{I}_2 = -0.4\mathbf{V}_2 + 2\mathbf{I}_2 \quad \Longrightarrow \quad 0.5\mathbf{V}_2 = 4\mathbf{I}_2$$

Hence,

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{4}{0.5} = 8\ \Omega$$

To find $\mathbf{V}_{\text{Th}}$, we use the circuit in Fig. 18.35(b). At the output port $\mathbf{I}_2 = 0$ and at the input port $\mathbf{V}_1 = 50 - 10\mathbf{I}_1$. Substituting these into Eqs. (18.9.1) and (18.9.2),

$$50 - 10\mathbf{I}_1 = 4\mathbf{V}_2 \tag{18.9.4}$$

$$\mathbf{I}_1 = 0.1\mathbf{V}_2 \tag{18.9.5}$$

Substituting Eq. (18.9.5) into Eq. (18.9.4),

$$50 - \mathbf{V}_2 = 4\mathbf{V}_2 \quad \Longrightarrow \quad \mathbf{V}_2 = 10$$

Thus,

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_2 = 10\ \text{V}$$

The equivalent circuit is shown in Fig. 18.35(c). For maximum power transfer,

$$R_L = \mathbf{Z}_{\text{Th}} = 8\ \Omega$$

From Eq. (4.24), the maximum power is

$$P = I^2 R_L = \left(\frac{\mathbf{V}_{\text{Th}}}{2R_L}\right)^2 R_L = \frac{\mathbf{V}_{\text{Th}}^2}{4R_L} = \frac{100}{4 \times 8} = 3.125\ \text{W}$$

PRACTICE PROBLEM 18.9

Find $\mathbf{I}_1$ and $\mathbf{I}_2$ if the transmission parameters for the two-port in Fig. 18.36 are

$$\begin{bmatrix} 5 & 10\ \Omega \\ 0.4\ \text{S} & 1 \end{bmatrix}$$

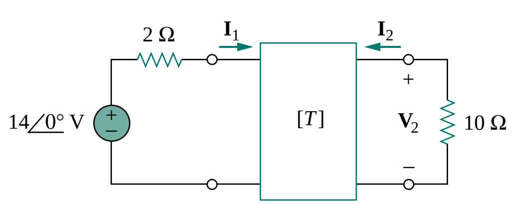

Figure 18.36 For Practice Prob. 18.9.

Answer: 1 A, −0.2 A.

†18.6 RELATIONSHIPS BETWEEN PARAMETERS

Since the six sets of parameters relate the same input and output terminal variables of the same two-port network, they should be interrelated. If two sets of parameters exist, we can relate one set to the other set. Let us demonstrate the process with two examples.

Given the z parameters, let us obtain the y parameters. From Eq. (18.2),

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = \begin{bmatrix}\mathbf{z}_{11} & \mathbf{z}_{12}\\ \mathbf{z}_{21} & \mathbf{z}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} = [\mathbf{z}]\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} \tag{18.31}$$

or

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} = [\mathbf{z}]^{-1}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} \tag{18.32}$$

Also, from Eq. (18.9),

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix}\mathbf{y}_{11} & \mathbf{y}_{12}\\ \mathbf{y}_{21} & \mathbf{y}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = [\mathbf{y}]\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} \tag{18.33}$$

Comparing Eqs. (18.32) and (18.33), we see that

$$[\mathbf{y}] = [\mathbf{z}]^{-1} \tag{18.34}$$

The adjoint of the $[\mathbf{z}]$ matrix is

$$\begin{bmatrix}\mathbf{z}_{22} & -\mathbf{z}_{12}\\ -\mathbf{z}_{21} & \mathbf{z}_{11}\end{bmatrix}$$

and its determinant is

$$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}$$

Substituting these into Eq. (18.34), we get

$$\begin{bmatrix}\mathbf{y}_{11} & \mathbf{y}_{12}\\ \mathbf{y}_{21} & \mathbf{y}_{22}\end{bmatrix} = \frac{\begin{bmatrix}\mathbf{z}_{22} & -\mathbf{z}_{12}\\ -\mathbf{z}_{21} & \mathbf{z}_{11}\end{bmatrix}}{\Delta_z} \tag{18.35}$$

Equating terms yields

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z}, \qquad \mathbf{y}_{12} = -\frac{\mathbf{z}_{12}}{\Delta_z}, \qquad \mathbf{y}_{21} = -\frac{\mathbf{z}_{21}}{\Delta_z}, \qquad \mathbf{y}_{22} = \frac{\mathbf{z}_{11}}{\Delta_z} \tag{18.36}$$

As a second example, let us determine the h parameters from the z parameters. From Eq. (18.1),

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \tag{18.37a}$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \tag{18.37b}$$

Making $\mathbf{I}_2$ the subject of Eq. (18.37b),

$$\mathbf{I}_2 = -\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}}\mathbf{I}_1 + \frac{1}{\mathbf{z}_{22}}\mathbf{V}_2 \tag{18.38}$$

Substituting this into Eq. (18.37a),

$$\mathbf{V}_1 = \frac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}}{\mathbf{z}_{22}}\mathbf{I}_1 + \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}}\mathbf{V}_2 \tag{18.39}$$

Putting Eqs. (18.38) and (18.39) in matrix form,

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix}\dfrac{\Delta_z}{\mathbf{z}_{22}} & \dfrac{\mathbf{z}_{12}}{\mathbf{z}_{22}}\\ -\dfrac{\mathbf{z}_{21}}{\mathbf{z}_{22}} & \dfrac{1}{\mathbf{z}_{22}}\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{V}_2\end{bmatrix} \tag{18.40}$$

From Eq. (18.15),

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix}\mathbf{h}_{11} & \mathbf{h}_{12}\\ \mathbf{h}_{21} & \mathbf{h}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{V}_2\end{bmatrix}$$

Comparing this with Eq. (18.40), we obtain

$$\mathbf{h}_{11} = \frac{\Delta_z}{\mathbf{z}_{22}}, \qquad \mathbf{h}_{12} = \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}}, \qquad \mathbf{h}_{21} = -\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}}, \qquad \mathbf{h}_{22} = \frac{1}{\mathbf{z}_{22}} \tag{18.41}$$

Table 18.1 provides the conversion formulas for the six sets of two-port parameters. Given one set of parameters, Table 18.1 can be used to find other parameters. For example, given the T parameters, we find the corresponding h parameters in the fifth column of the third row. Also, given that $\mathbf{z}_{21} = \mathbf{z}_{12}$ for a reciprocal network, we can use the table to express this condition in terms of other parameters. It can also be shown that

$$[\mathbf{g}] = [\mathbf{h}]^{-1} \tag{18.42}$$

but

$$[\mathbf{t}] \neq [\mathbf{T}]^{-1} \tag{18.43}$$

TABLE 18.1 Conversion of two-port parameters.

	z		**y**		**h**		**g**		**T**		**t**	
z	$\mathbf{z}_{11}$	$\mathbf{z}_{12}$	$\frac{\mathbf{y}_{22}}{\Delta_y}$	$-\frac{\mathbf{y}_{12}}{\Delta_y}$	$\frac{\Delta_h}{\mathbf{h}_{22}}$	$\frac{\mathbf{h}_{12}}{\mathbf{h}_{22}}$	$\frac{1}{\mathbf{g}_{11}}$	$-\frac{\mathbf{g}_{12}}{\mathbf{g}_{11}}$	$\frac{\mathbf{A}}{\mathbf{C}}$	$\frac{\Delta_T}{\mathbf{C}}$	$\frac{\mathbf{d}}{\mathbf{c}}$	$\frac{1}{\mathbf{c}}$
	$\mathbf{z}_{21}$	$\mathbf{z}_{22}$	$-\frac{\mathbf{y}_{21}}{\Delta_y}$	$\frac{\mathbf{y}_{11}}{\Delta_y}$	$-\frac{\mathbf{h}_{21}}{\mathbf{h}_{22}}$	$\frac{1}{\mathbf{h}_{22}}$	$\frac{\mathbf{g}_{21}}{\mathbf{g}_{11}}$	$\frac{\Delta_g}{\mathbf{g}_{11}}$	$\frac{1}{\mathbf{C}}$	$\frac{\mathbf{D}}{\mathbf{C}}$	$\frac{\Delta_t}{\mathbf{c}}$	$\frac{\mathbf{a}}{\mathbf{c}}$
y	$\frac{\mathbf{z}_{22}}{\Delta_z}$	$-\frac{\mathbf{z}_{12}}{\Delta_z}$	$\mathbf{y}_{11}$	$\mathbf{y}_{12}$	$\frac{1}{\mathbf{h}_{11}}$	$-\frac{\mathbf{h}_{12}}{\mathbf{h}_{11}}$	$\frac{\Delta_g}{\mathbf{g}_{22}}$	$\frac{\mathbf{g}_{12}}{\mathbf{g}_{22}}$	$\frac{\mathbf{D}}{\mathbf{B}}$	$-\frac{\Delta_T}{\mathbf{B}}$	$\frac{\mathbf{a}}{\mathbf{b}}$	$-\frac{1}{\mathbf{b}}$
	$-\frac{\mathbf{z}_{21}}{\Delta_z}$	$\frac{\mathbf{z}_{11}}{\Delta_z}$	$\mathbf{y}_{21}$	$\mathbf{y}_{22}$	$\frac{\mathbf{h}_{21}}{\mathbf{h}_{11}}$	$\frac{\Delta_h}{\mathbf{h}_{11}}$	$-\frac{\mathbf{g}_{21}}{\mathbf{g}_{22}}$	$\frac{1}{\mathbf{g}_{22}}$	$-\frac{1}{\mathbf{B}}$	$\frac{\mathbf{A}}{\mathbf{B}}$	$-\frac{\Delta_t}{\mathbf{b}}$	$\frac{\mathbf{d}}{\mathbf{b}}$
h	$\frac{\Delta_z}{\mathbf{z}_{22}}$	$\frac{\mathbf{z}_{12}}{\mathbf{z}_{22}}$	$\frac{1}{\mathbf{y}_{11}}$	$-\frac{\mathbf{y}_{12}}{\mathbf{y}_{11}}$	$\mathbf{h}_{11}$	$\mathbf{h}_{12}$	$\frac{\mathbf{g}_{22}}{\Delta_g}$	$-\frac{\mathbf{g}_{12}}{\Delta_g}$	$\frac{\mathbf{B}}{\mathbf{D}}$	$\frac{\Delta_T}{\mathbf{D}}$	$\frac{\mathbf{b}}{\mathbf{a}}$	$\frac{1}{\mathbf{a}}$
	$-\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}}$	$\frac{1}{\mathbf{z}_{22}}$	$\frac{\mathbf{y}_{21}}{\mathbf{y}_{11}}$	$\frac{\Delta_y}{\mathbf{y}_{11}}$	$\mathbf{h}_{21}$	$\mathbf{h}_{22}$	$-\frac{\mathbf{g}_{21}}{\Delta_g}$	$\frac{\mathbf{g}_{11}}{\Delta_g}$	$-\frac{1}{\mathbf{D}}$	$\frac{\mathbf{C}}{\mathbf{D}}$	$\frac{\Delta_t}{\mathbf{a}}$	$\frac{\mathbf{c}}{\mathbf{a}}$
g	$\frac{1}{\mathbf{z}_{11}}$	$-\frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}$	$\frac{\Delta_y}{\mathbf{y}_{22}}$	$\frac{\mathbf{y}_{12}}{\mathbf{y}_{22}}$	$\frac{\mathbf{h}_{22}}{\Delta_h}$	$-\frac{\mathbf{h}_{12}}{\Delta_h}$	$\mathbf{g}_{11}$	$\mathbf{g}_{12}$	$\frac{\mathbf{C}}{\mathbf{A}}$	$-\frac{\Delta_T}{\mathbf{A}}$	$\frac{\mathbf{c}}{\mathbf{d}}$	$-\frac{1}{\mathbf{d}}$
	$\frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}$	$\frac{\Delta_z}{\mathbf{z}_{11}}$	$-\frac{\mathbf{y}_{21}}{\mathbf{y}_{22}}$	$\frac{1}{\mathbf{y}_{22}}$	$-\frac{\mathbf{h}_{21}}{\Delta_h}$	$\frac{\mathbf{h}_{11}}{\Delta_h}$	$\mathbf{g}_{21}$	$\mathbf{g}_{22}$	$\frac{1}{\mathbf{A}}$	$\frac{\mathbf{B}}{\mathbf{A}}$	$\frac{\Delta_t}{\mathbf{d}}$	$-\frac{\mathbf{b}}{\mathbf{d}}$
T	$\frac{\mathbf{z}_{11}}{\mathbf{z}_{21}}$	$\frac{\Delta_z}{\mathbf{z}_{21}}$	$-\frac{\mathbf{y}_{22}}{\mathbf{y}_{21}}$	$-\frac{1}{\mathbf{y}_{21}}$	$-\frac{\Delta_h}{\mathbf{h}_{21}}$	$-\frac{\mathbf{h}_{11}}{\mathbf{h}_{21}}$	$\frac{1}{\mathbf{g}_{21}}$	$\frac{\mathbf{g}_{22}}{\mathbf{g}_{21}}$	$\mathbf{A}$	$\mathbf{B}$	$\frac{\mathbf{d}}{\Delta_t}$	$\frac{\mathbf{b}}{\Delta_t}$
	$\frac{1}{\mathbf{z}_{21}}$	$\frac{\mathbf{z}_{22}}{\mathbf{z}_{21}}$	$-\frac{\Delta_y}{\mathbf{y}_{21}}$	$-\frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}$	$-\frac{\mathbf{h}_{22}}{\mathbf{h}_{21}}$	$-\frac{1}{\mathbf{h}_{21}}$	$\frac{\mathbf{g}_{11}}{\mathbf{g}_{21}}$	$\frac{\Delta_g}{\mathbf{g}_{21}}$	$\mathbf{C}$	$\mathbf{D}$	$\frac{\mathbf{c}}{\Delta_t}$	$\frac{\mathbf{a}}{\Delta_t}$
t	$\frac{\mathbf{z}_{22}}{\mathbf{z}_{12}}$	$\frac{\Delta_z}{\mathbf{z}_{12}}$	$-\frac{\mathbf{y}_{11}}{\mathbf{y}_{12}}$	$-\frac{1}{\mathbf{y}_{12}}$	$\frac{1}{\mathbf{h}_{12}}$	$\frac{\mathbf{h}_{11}}{\mathbf{h}_{12}}$	$-\frac{\Delta_g}{\mathbf{g}_{12}}$	$-\frac{\mathbf{g}_{22}}{\mathbf{g}_{12}}$	$\frac{\mathbf{D}}{\Delta_T}$	$\frac{\mathbf{B}}{\Delta_T}$	$\mathbf{a}$	$\mathbf{b}$
	$\frac{1}{\mathbf{z}_{12}}$	$\frac{\mathbf{z}_{11}}{\mathbf{z}_{12}}$	$-\frac{\Delta_y}{\mathbf{y}_{12}}$	$-\frac{\mathbf{y}_{22}}{\mathbf{y}_{12}}$	$\frac{\mathbf{h}_{22}}{\mathbf{h}_{12}}$	$\frac{\Delta_h}{\mathbf{h}_{12}}$	$-\frac{\mathbf{g}_{11}}{\mathbf{g}_{12}}$	$-\frac{1}{\mathbf{g}_{12}}$	$\frac{\mathbf{C}}{\Delta_T}$	$\frac{\mathbf{A}}{\Delta_T}$	$\mathbf{c}$	$\mathbf{d}$

$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}, \quad \Delta_h = \mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{12}\mathbf{h}_{21}, \quad \Delta_T = \mathbf{AD} - \mathbf{BC}$

$\Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21}, \quad \Delta_g = \mathbf{g}_{11}\mathbf{g}_{22} - \mathbf{g}_{12}\mathbf{g}_{21}, \quad \Delta_t = \mathbf{ad} - \mathbf{bc}$

EXAMPLE 18.10

Find [**z**] and [**g**] of a two-port network if

$$[\mathbf{T}] = \begin{bmatrix} 10 & 1.5\ \Omega \\ 2\ \text{S} & 4 \end{bmatrix}$$

Solution:

If $\mathbf{A} = 10$, $\mathbf{B} = 1.5$, $\mathbf{C} = 2$, $\mathbf{D} = 4$, the determinant of the matrix is

$$\Delta_T = \mathbf{AD} - \mathbf{BC} = 40 - 3 = 37$$

From Table 18.1,

$$\mathbf{z}_{11} = \frac{\mathbf{A}}{\mathbf{C}} = \frac{10}{2} = 5, \qquad \mathbf{z}_{12} = \frac{\Delta_T}{\mathbf{C}} = \frac{37}{2} = 18.5$$

$$\mathbf{z}_{21} = \frac{1}{\mathbf{C}} = \frac{1}{2} = 0.5, \qquad \mathbf{z}_{22} = \frac{\mathbf{D}}{\mathbf{C}} = \frac{4}{2} = 2$$

$$\mathbf{g}_{11} = \frac{\mathbf{C}}{\mathbf{A}} = \frac{2}{10} = 0.2, \qquad \mathbf{g}_{12} = -\frac{\Delta_T}{\mathbf{A}} = -\frac{37}{10} = -3.7$$

$$\mathbf{g}_{21} = \frac{1}{\mathbf{A}} = \frac{1}{10} = 0.1, \qquad \mathbf{g}_{22} = \frac{\mathbf{B}}{\mathbf{A}} = \frac{1.5}{10} = 0.15$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} 5 & 18.5 \\ 0.5 & 2 \end{bmatrix}\Omega, \qquad [\mathbf{g}] = \begin{bmatrix} 0.2\ \text{S} & -3.7 \\ 0.1 & 0.15\ \Omega \end{bmatrix}$$

PRACTICE PROBLEM 18.10

Determine [**y**] and [**T**] of a two-port network whose z parameters are

$$[\mathbf{z}] = \begin{bmatrix} 6 & 4 \\ 4 & 6 \end{bmatrix}\Omega$$

Answer: $[\mathbf{y}] = \begin{bmatrix} 0.3 & -0.2 \\ -0.2 & 0.3 \end{bmatrix}\text{S}$, $[\mathbf{T}] = \begin{bmatrix} 1.5 & 5\ \Omega \\ 0.25\ \text{S} & 1.5 \end{bmatrix}$.

EXAMPLE 18.11

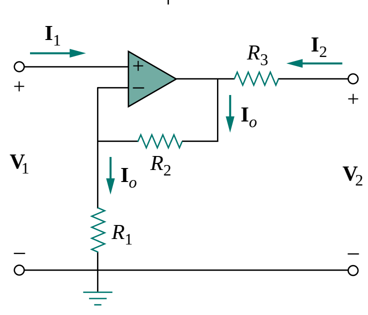

Figure 18.37 For Example 18.11.

Obtain the y parameters of the op amp circuit in Fig. 18.37. Show that the circuit has no z parameters.

Solution:

Since no current can enter the input terminals of the op amp, $\mathbf{I}_1 = 0$, which can be expressed in terms of $\mathbf{V}_1$ and $\mathbf{V}_2$ as

$$\mathbf{I}_1 = 0\mathbf{V}_1 + 0\mathbf{V}_2 \tag{18.11.1}$$

Comparing this with Eq. (18.8) gives

$$\mathbf{y}_{11} = 0 = \mathbf{y}_{12}$$

Also,

$$\mathbf{V}_2 = R_3\mathbf{I}_2 + \mathbf{I}_o(R_1 + R_2)$$

where $\mathbf{I}_o$ is the current through R_1 and R_2. But $\mathbf{I}_o = \mathbf{V}_1/R_1$. Hence,

$$\mathbf{V}_2 = R_3\mathbf{I}_2 + \frac{\mathbf{V}_1(R_1 + R_2)}{R_1}$$

which can be written as

$$\mathbf{I}_2 = -\frac{(R_1 + R_2)}{R_1 R_3}\mathbf{V}_1 + \frac{\mathbf{V}_2}{R_3}$$

Comparing this with Eq. (18.8) shows that

$$\mathbf{y}_{21} = -\frac{(R_1 + R_2)}{R_1 R_3}, \qquad \mathbf{y}_{22} = \frac{1}{R_3}$$

The determinant of the [**y**] matrix is

$$\Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21} = 0$$

Since $\Delta_y = 0$, the [**y**] matrix has no inverse; therefore, the [**z**] matrix does not exist according to Eq. (18.34). Note that the circuit is not reciprocal because of the active element.

PRACTICE PROBLEM 18.11

Find the z parameters of the op amp circuit in Fig. 18.38. Show that the circuit has no y parameters.

Answer: $[\mathbf{z}] = \begin{bmatrix} R_1 & 0 \\ -R_2 & 0 \end{bmatrix}$. Since $[\mathbf{z}]^{-1}$ does not exist, [**y**] does not exist.

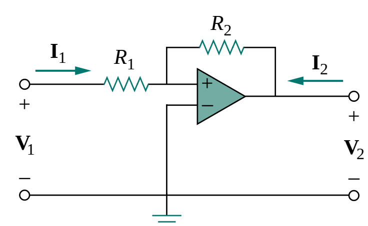

Figure 18.38 For Practice Prob. 18.11.

18.7 INTERCONNECTION OF NETWORKS

A large, complex network may be divided into subnetworks for the purposes of analysis and design. The subnetworks are modeled as two-port networks, interconnected to form the original network. The two-port networks may therefore be regarded as building blocks that can be interconnected to form a complex network. The interconnection can be in series, in parallel, or in cascade. Although the interconnected network can be described by any of the six parameter sets, a certain set of parameters may have a definite advantage. For example, when the networks are in series, their individual z parameters add up to give the z parameters of the larger network. When they are in parallel, their individual y parameters add up to give the y parameters the larger network. When they are cascaded, their individual transmission parameters can be multiplied together to get the transmission parameters of the larger network.

Consider the series connection of two two-port networks shown in Fig. 18.39. The networks are regarded as being in series because their input currents are the same and their voltages add. In addition, each network has a common reference, and when the circuits are placed in series, the common reference points of each circuit are connected together. For network N_a,

$$\mathbf{V}_{1a} = \mathbf{z}_{11a}\mathbf{I}_{1a} + \mathbf{z}_{12a}\mathbf{I}_{2a}$$
$$\mathbf{V}_{2a} = \mathbf{z}_{21a}\mathbf{I}_{1a} + \mathbf{z}_{22a}\mathbf{I}_{2a} \quad (18.44)$$

and for network N_b,

$$\mathbf{V}_{1b} = \mathbf{z}_{11b}\mathbf{I}_{1b} + \mathbf{z}_{12b}\mathbf{I}_{2b}$$
$$\mathbf{V}_{2b} = \mathbf{z}_{21b}\mathbf{I}_{1b} + \mathbf{z}_{22b}\mathbf{I}_{2b} \quad (18.45)$$

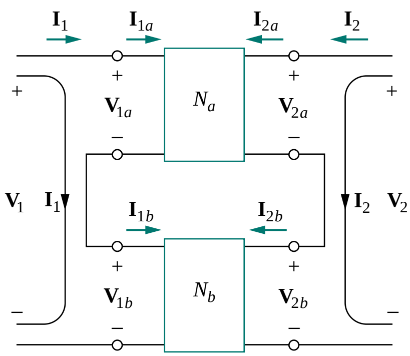

Figure 18.39 Series connection of two two-port networks.

We notice from Fig. 18.39 that

$$\mathbf{I}_1 = \mathbf{I}_{1a} = \mathbf{I}_{1b}, \qquad \mathbf{I}_2 = \mathbf{I}_{2a} = \mathbf{I}_{2b} \quad (18.46)$$

and that

$$\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} = (\mathbf{z}_{11a} + \mathbf{z}_{11b})\mathbf{I}_1 + (\mathbf{z}_{12a} + \mathbf{z}_{12b})\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} = (\mathbf{z}_{21a} + \mathbf{z}_{21b})\mathbf{I}_1 + (\mathbf{z}_{22a} + \mathbf{z}_{22b})\mathbf{I}_2 \quad (18.47)$$

Thus, the z parameters for the overall network are

$$\begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11a} + \mathbf{z}_{11b} & \mathbf{z}_{12a} + \mathbf{z}_{12b} \\ \mathbf{z}_{21a} + \mathbf{z}_{21b} & \mathbf{z}_{22a} + \mathbf{z}_{22b} \end{bmatrix} \quad (18.48)$$

or

$$\boxed{[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b]} \quad (18.49)$$

showing that the z parameters for the overall network are the sum of the z parameters for the individual networks. This can be extended to n networks in series. If two two-port networks in the [**h**] model, for example, are connected in series, we use Table 18.1 to convert the **h** to **z** and then apply Eq. (18.49). We finally convert the result back to **h** using Table 18.1.

Two two-port networks are in parallel when their port voltages are equal and the port currents of the larger network are the sums of the individual port currents. In addition, each circuit must have a common reference and when the networks are connected together, they must all have their common references tied together. The parallel connection of two two-port networks is shown in Fig. 18.40. For the two networks,

$$\mathbf{I}_{1a} = \mathbf{y}_{11a}\mathbf{V}_{1a} + \mathbf{y}_{12a}\mathbf{V}_{2a}$$
$$\mathbf{I}_{2a} = \mathbf{y}_{21a}\mathbf{V}_{1a} + \mathbf{y}_{22a}\mathbf{V}_{2a} \quad (18.50)$$

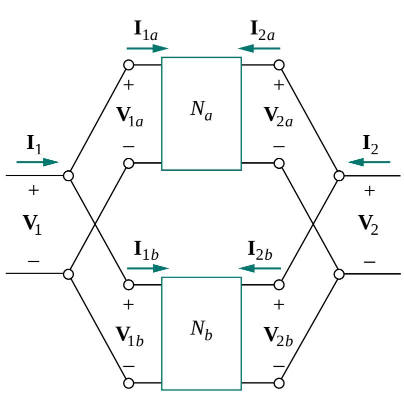

Figure 18.40 Parallel connection of two two-port networks.

and

$$\mathbf{I}_{1b} = \mathbf{y}_{11b}\mathbf{V}_{1b} + \mathbf{y}_{12b}\mathbf{V}_{2b}$$
$$\mathbf{I}_{2a} = \mathbf{y}_{21b}\mathbf{V}_{1b} + \mathbf{y}_{22b}\mathbf{V}_{2b} \quad (18.51)$$

But from Fig. 18.40,

$$\mathbf{V}_1 = \mathbf{V}_{1a} = \mathbf{V}_{1b}, \qquad \mathbf{V}_2 = \mathbf{V}_{2a} = \mathbf{V}_{2b} \quad (18.52a)$$
$$\mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b}, \qquad \mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b} \quad (18.52b)$$

Substituting Eqs. (18.50) and (18.51) into Eq. (18.52b) yields

$$\begin{aligned}\mathbf{I}_1 &= (\mathbf{y}_{11a} + \mathbf{y}_{11b})\mathbf{V}_1 + (\mathbf{y}_{12a} + \mathbf{y}_{12b})\mathbf{V}_2 \\ \mathbf{I}_2 &= (\mathbf{y}_{21a} + \mathbf{y}_{21b})\mathbf{V}_1 + (\mathbf{y}_{22a} + \mathbf{y}_{22b})\mathbf{V}_2\end{aligned} \tag{18.53}$$

Thus, the y parameters for the overall network are

$$\begin{bmatrix}\mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22}\end{bmatrix} = \begin{bmatrix}\mathbf{y}_{11a} + \mathbf{y}_{11b} & \mathbf{y}_{12a} + \mathbf{y}_{12b} \\ \mathbf{y}_{21a} + \mathbf{y}_{21b} & \mathbf{y}_{22a} + \mathbf{y}_{22b}\end{bmatrix} \tag{18.54}$$

or

$$\boxed{[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b]} \tag{18.55}$$

showing that the y parameters of the overall network are the sum of the y parameters of the individual networks. The result can be extended to n two-port networks in parallel.

Two networks are said to be *cascaded* when the output of one is the input of the other. The connection of two two-port networks in cascade is shown in Fig. 18.41. For the two networks,

$$\begin{bmatrix}\mathbf{V}_{1a} \\ \mathbf{I}_{1a}\end{bmatrix} = \begin{bmatrix}\mathbf{A}_a & \mathbf{B}_a \\ \mathbf{C}_a & \mathbf{D}_a\end{bmatrix}\begin{bmatrix}\mathbf{V}_{2a} \\ -\mathbf{I}_{2a}\end{bmatrix} \tag{18.56}$$

$$\begin{bmatrix}\mathbf{V}_{1b} \\ \mathbf{I}_{1b}\end{bmatrix} = \begin{bmatrix}\mathbf{A}_b & \mathbf{B}_b \\ \mathbf{C}_b & \mathbf{D}_b\end{bmatrix}\begin{bmatrix}\mathbf{V}_{2b} \\ -\mathbf{I}_{2b}\end{bmatrix} \tag{18.57}$$

From Fig. 18.41,

$$\begin{bmatrix}\mathbf{V}_1 \\ \mathbf{I}_1\end{bmatrix} = \begin{bmatrix}\mathbf{V}_{1a} \\ \mathbf{I}_{1a}\end{bmatrix}, \qquad \begin{bmatrix}\mathbf{V}_{2a} \\ -\mathbf{I}_{2a}\end{bmatrix} = \begin{bmatrix}\mathbf{V}_{1b} \\ \mathbf{I}_{1b}\end{bmatrix}, \qquad \begin{bmatrix}\mathbf{V}_{2b} \\ -\mathbf{I}_{2b}\end{bmatrix} = \begin{bmatrix}\mathbf{V}_2 \\ -\mathbf{I}_2\end{bmatrix} \tag{18.58}$$

Substituting these into Eqs. (18.56) and (18.57),

$$\begin{bmatrix}\mathbf{V}_1 \\ \mathbf{I}_1\end{bmatrix} = \begin{bmatrix}\mathbf{A}_a & \mathbf{B}_a \\ \mathbf{C}_a & \mathbf{D}_a\end{bmatrix}\begin{bmatrix}\mathbf{A}_b & \mathbf{B}_b \\ \mathbf{C}_b & \mathbf{D}_b\end{bmatrix}\begin{bmatrix}\mathbf{V}_2 \\ -\mathbf{I}_2\end{bmatrix} \tag{18.59}$$

Thus, the transmission parameters for the overall network are the product of the transmission parameters for the individual transmission parameters:

$$\begin{bmatrix}\mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D}\end{bmatrix} = \begin{bmatrix}\mathbf{A}_a & \mathbf{B}_a \\ \mathbf{C}_a & \mathbf{D}_a\end{bmatrix}\begin{bmatrix}\mathbf{A}_b & \mathbf{B}_b \\ \mathbf{C}_b & \mathbf{D}_b\end{bmatrix} \tag{18.60}$$

or

$$\boxed{[\mathbf{T}] = [\mathbf{T}_a][\mathbf{T}_b]} \tag{18.61}$$

It is this property that makes the transmission parameters so useful. Keep in mind that the multiplication of the matrices must be in the order in which the networks N_a and N_b are cascaded.

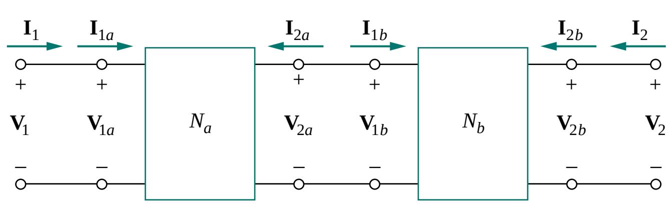

Figure 18.41 Cascade connection of two two-port networks.

EXAMPLE 18.12

Evaluate $\mathbf{V}_2/\mathbf{V}_s$ in the circuit in Fig. 18.42.

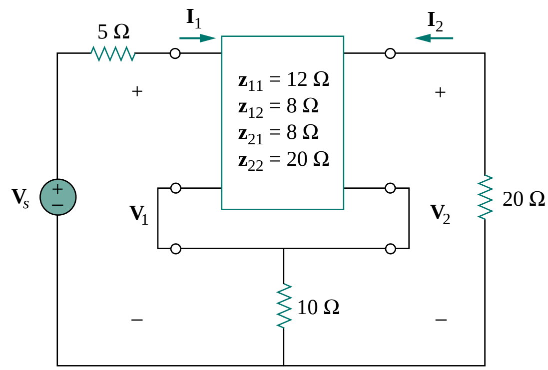

Figure 18.42 For Example 18.12.

Solution:

This may be regarded as two two-ports in series. For N_b,

$$\mathbf{z}_{12b} = \mathbf{z}_{21b} = 10 = \mathbf{z}_{11} = \mathbf{z}_{22}$$

Thus,

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b] = \begin{bmatrix} 12 & 8 \\ 8 & 20 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 22 & 18 \\ 18 & 30 \end{bmatrix}$$

But

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 = 22\mathbf{I}_1 + 18\mathbf{I}_2 \qquad (18.12.1)$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 = 18\mathbf{I}_1 + 30\mathbf{I}_2 \qquad (18.12.2)$$

Also, at the input port

$$\mathbf{V}_1 = \mathbf{V}_s - 5\mathbf{I}_1 \qquad (18.12.3)$$

and at the output port

$$\mathbf{V}_2 = -20\mathbf{I}_2 \quad \Longrightarrow \quad \mathbf{I}_2 = -\frac{\mathbf{V}_2}{20} \qquad (18.12.4)$$

Substituting Eqs. (18.12.3) and (18.12.4) into Eq. (18.12.1) gives

$$\mathbf{V}_s - 5\mathbf{I}_1 = 22\mathbf{I}_1 - \frac{18}{20}\mathbf{V}_2 \quad \Longrightarrow \quad \mathbf{V}_s = 27\mathbf{I}_1 - 0.9\mathbf{V}_2 \qquad (18.12.5)$$

while substituting Eq. (18.12.4) into Eq. (18.12.2) yields

$$\mathbf{V}_2 = 18\mathbf{I}_1 - \frac{30}{20}\mathbf{V}_2 \quad \Longrightarrow \quad \mathbf{I}_1 = \frac{2.5}{18}\mathbf{V}_2 \qquad (18.12.6)$$

Substituting Eq. (18.12.6) into Eq. (18.12.5), we get

$$\mathbf{V}_s = 27 \times \frac{2.5}{18}\mathbf{V}_2 - 0.9\mathbf{V}_2 = 2.85\mathbf{V}_2$$

And so,

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{1}{2.85} = 0.3509$$

PRACTICE PROBLEM 18.12

Find $\mathbf{V}_2/\mathbf{V}_s$ in the circuit in Fig. 18.43.

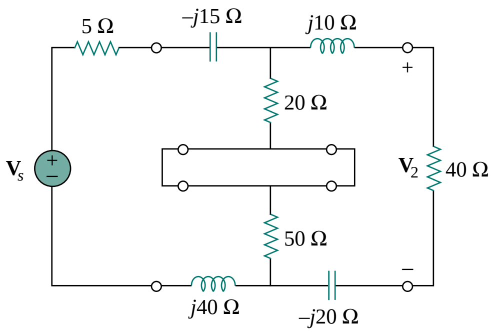

Figure 18.43 For Practice Prob. 18.12.

Answer: $0.58\angle -40^\circ$.

EXAMPLE 18.13

Find the y parameters of the two-port in Fig. 18.44.

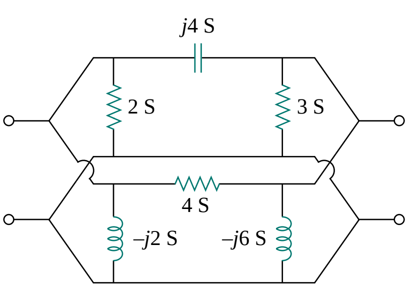

Figure 18.44 For Example 18.13.

Solution:

Let us refer to the upper network as N_a and the lower one as N_b. The two networks are connected in parallel. Comparing N_a and N_b with the circuit in Fig. 18.13(a), we obtain

$$\mathbf{y}_{12a} = -j4 = \mathbf{y}_{21a}, \qquad \mathbf{y}_{11a} = 2 + j4, \qquad \mathbf{y}_{22a} = 3 + j4$$

or

$$[\mathbf{y}_a] = \begin{bmatrix} 2 + j4 & -j4 \\ -j4 & 3 + j4 \end{bmatrix} \text{S}$$

and

$$\mathbf{y}_{12b} = -4 = \mathbf{y}_{21b}, \qquad \mathbf{y}_{11b} = 4 - j2, \qquad \mathbf{y}_{22b} = 4 - j6$$

or

$$[\mathbf{y}_b] = \begin{bmatrix} 4 - j2 & -4 \\ -4 & 4 - j6 \end{bmatrix} \text{S}$$

The overall y parameters are

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} 6 + j2 & -4 - j4 \\ -4 - j4 & 7 - j2 \end{bmatrix} \text{S}$$

PRACTICE PROBLEM 18.13

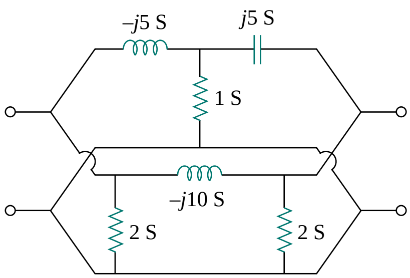

Figure 18.45 For Practice Prob. 18.13.

Obtain the y parameters for the network in Fig. 18.45.

Answer: $\begin{bmatrix} 27 - j15 & -25 + j10 \\ -25 + j10 & 27 - j5 \end{bmatrix}$ S.

EXAMPLE 18.14

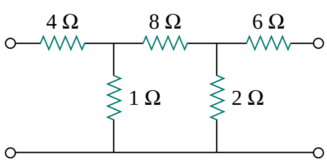

Figure 18.46 For Example 18.14.

Find the transmission parameters for the circuit in Fig. 18.46.

Solution:

We can regard the given circuit in Fig. 18.46 as a cascade connection of two T networks as shown in Fig. 18.47(a). We can show that a T network, shown in Fig. 18.47(b), has the following transmission parameters [see Prob. 18.42(b)]:

$$\mathbf{A} = 1 + \frac{R_1}{R_2}, \qquad \mathbf{B} = R_3 + \frac{R_1(R_2 + R_3)}{R_2}$$

$$\mathbf{C} = \frac{1}{R_2}, \qquad \mathbf{D} = 1 + \frac{R_3}{R_2}$$

Applying this to the cascaded networks N_a and N_b in Fig. 18.47(a), we get

$$\mathbf{A}_a = 1 + 4 = 5, \qquad \mathbf{B}_a = 8 + 4 \times 9 = 44\ \Omega$$
$$\mathbf{C}_a = 1\ \text{S}, \qquad \mathbf{D}_a = 1 + 8 = 9$$

or in matrix form,

$$[\mathbf{T}_a] = \begin{bmatrix} 5 & 44\ \Omega \\ 1\ \text{S} & 9 \end{bmatrix}$$

and

$$\mathbf{A}_b = 1, \qquad \mathbf{B}_b = 6\ \Omega, \qquad \mathbf{C}_b = 0.5\ \text{S}, \qquad \mathbf{D}_b = 1 + \frac{6}{2} = 4$$

i.e.,

$$[\mathbf{T}_b] = \begin{bmatrix} 1 & 6\ \Omega \\ 0.5\ \text{S} & 4 \end{bmatrix}$$

Thus, for the total network in Fig. 18.46,

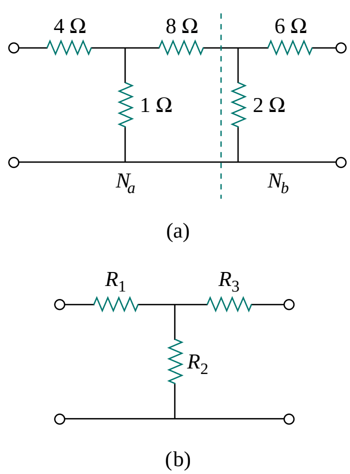

Figure 18.47 For Example 18.14: (a) Breaking the circuit in Fig. 18.46 into two two-ports, (b) a general T two-port.

$$[\mathbf{T}] = [\mathbf{T}_a][\mathbf{T}_b] = \begin{bmatrix} 5 & 44 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0.5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \times 1 + 44 \times 0.5 & 5 \times 6 + 44 \times 4 \\ 1 \times 1 + 9 \times 0.5 & 1 \times 6 + 9 \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 206\ \Omega \\ 5.5\ \text{S} & 42 \end{bmatrix}$$

Notice that

$$\Delta_{T_a} = \Delta_{T_b} = \Delta_T = 1$$

showing that the network is reciprocal.

PRACTICE PROBLEM 18.14

Obtain the **ABCD** parameter representation of the circuit in Fig. 18.48.

Answer: $[\mathbf{T}] = \begin{bmatrix} 29.25 & 2200\ \Omega \\ 0.425\ \text{S} & 32 \end{bmatrix}$.

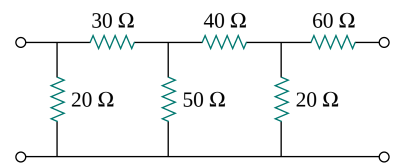

Figure 18.48 For Practice Prob. 18.14.

18.8 COMPUTING TWO-PORT PARAMETERS USING *PSPICE*

Hand calculation of the two-port parameters may become difficult when the two-port is complicated. We resort to *PSpice* in such situations. If the circuit is purely resistive, *PSpice* dc analysis may be used; otherwise, *PSpice* ac analysis is required at a specific frequency. The key to using *PSpice* in computing a particular two-port parameter is to remember how that parameter is defined and to constrain the appropriate port variable with a 1-A or 1-V source while using an open or short circuit to impose the other necessary constraints. The following two examples illustrate the idea.

EXAMPLE 18.15

Find the h parameters of the network in Fig. 18.49.

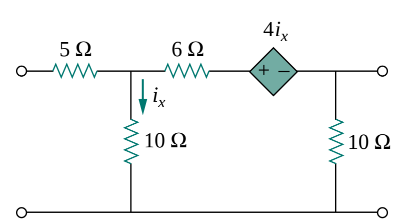

Figure 18.49 For Example 18.15.

Solution:

From Eq. (18.16),

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}$$

showing that $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ can be found by setting $\mathbf{V}_2 = 0$. Also by setting $\mathbf{I}_1 = 1$ A, $\mathbf{h}_{11}$ becomes $\mathbf{V}_1/1$ while $\mathbf{h}_{21}$ becomes $\mathbf{I}_2/1$. With this in mind, we draw the schematic in Fig. 18.50(a). We insert a 1-A dc current source IDC to take care of $\mathbf{I}_1 = 1$ A, the pseudocomponent VIEWPOINT to display $\mathbf{V}_1$ and pseudocomponent IPROBE to display $\mathbf{I}_2$. After saving the schematic, we run *PSpice* by selecting **Analysis/Simulate** and note the values displayed on the pseudocomponents. We obtain

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{1} = 10\ \Omega, \qquad \mathbf{h}_{21} = \frac{\mathbf{I}_2}{1} = -0.5$$

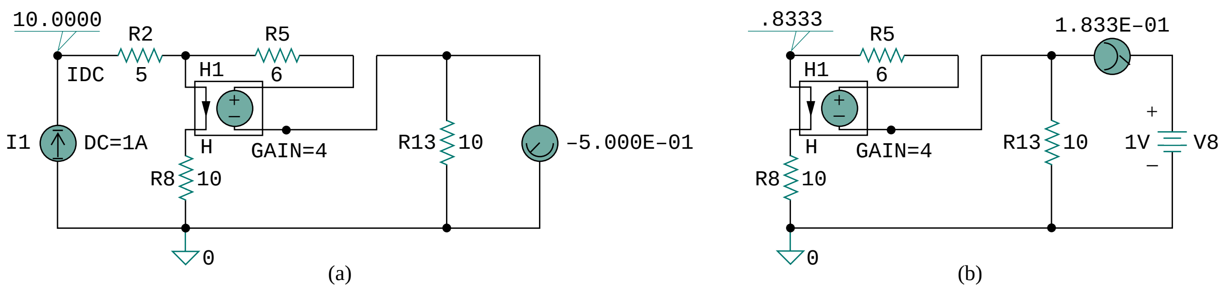

Figure 18.50 For Example 18.15: (a) computing $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, (b) computing $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$.

Similarly, from Eq. (18.16),

$$\mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}, \qquad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

indicating that we obtain $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$ by open-circuiting the input port ($\mathbf{I}_1 = 0$). By making $\mathbf{V}_2 = 1$ V, $\mathbf{h}_{12}$ becomes $\mathbf{V}_1/1$ while $\mathbf{h}_{22}$ becomes $\mathbf{I}_2/1$. Thus, we use the schematic in Fig. 18.50(b) with a 1-V dc voltage source VDC inserted at the output terminal to take care of $\mathbf{V}_2 = 1$ V. The pseudocomponents VIEWPOINT and IPROBE are inserted to display the values of $\mathbf{V}_1$ and $\mathbf{I}_2$, respectively. (Notice that in Fig. 18.50(b), the 5-Ω resistor is ignored because the input port is open-circuited and *PSpice* will not allow such. We may include the 5-Ω resistor if we replace the open circuit with a very large resistor, say, 10 MΩ.) After simulating the schematic, we obtain the values displayed on the pseudocomponents as shown in Fig. 18.50(b). Thus,

$$\mathbf{h}_{12} = \frac{\mathbf{V}_1}{1} = 0.8333, \qquad \mathbf{h}_{22} = \frac{\mathbf{I}_2}{1} = 0.1833\ \mathrm{S}$$

PRACTICE PROBLEM 18.15

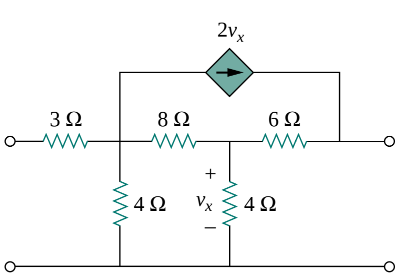

Figure 18.51 For Practice Prob. 18.15.

Obtain the h parameters for the network in Fig. 18.51 using *PSpice*.

Answer: $h_{11} = 4.238\ \Omega$, $h_{21} = -0.6190$, $h_{12} = -0.7143$, $h_{22} = -0.1429$ S.

EXAMPLE 18.16

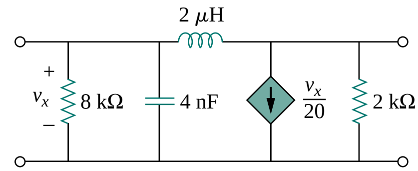

Figure 18.52 For Example 18.16.

Find the z parameters for the circuit in Fig. 18.52 at $\omega = 10^6$ rad/s.

Solution:

Notice that we used dc analysis in Example 18.15 because the circuit in Fig. 18.49 is purely resistive. Here, we use ac analysis at $f = \omega/2\pi = 0.15915$ MHz, because L and C are frequency dependent.

In Eq. (18.3), we defined the z parameters as

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}$$

This suggests that if we let $\mathbf{I}_1 = 1$ A and open-circuit the output port so that $\mathbf{I}_2 = 0$, then we obtain

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{1} \qquad \text{and} \qquad \mathbf{z}_{21} = \frac{\mathbf{V}_2}{1}$$

We realize this with the schematic in Fig. 18.53(a). We insert a 1-A ac current source IAC at the input terminal of the circuit and two VPRINT1 pseudocomponents to obtain $\mathbf{V}_1$ and $\mathbf{V}_2$. The attributes of each VPRINT1 are set as *AC* = *yes*, *MAG* = *yes*, and *PHASE* = *yes* to print the magnitude and phase values of the voltages. We select **Analysis/Setup/AC Sweep** and enter 1 as *Total Pts*, 0.1519MEG as *Start Freq*, and 0.1519MEG as *Final Freq* in the **AC Sweep and Noise Analysis** dialog box. After saving the schematic, we select **Analysis/Simulate** to simulate it. We obtain $\mathbf{V}_1$

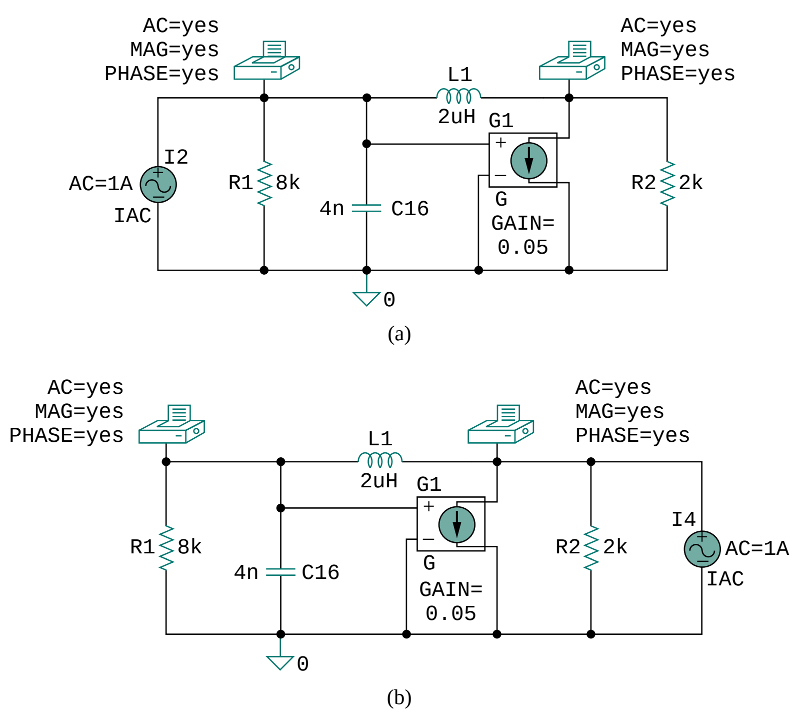

Figure 18.53 For Example 18.16: (a) circuit for determining $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, (b) circuit for determining $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$.

and $\mathbf{V}_2$ from the output file. Thus,

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{1} = 19.70\underline{/175.7^\circ}\ \Omega, \qquad \mathbf{z}_{21} = \frac{\mathbf{V}_2}{1} = 19.79\underline{/170.2^\circ}\ \Omega$$

In a similar manner, from Eq. (18.3),

$$\mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}, \qquad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

suggesting that if we let $\mathbf{I}_2 = 1$ A and open-circuit the input port,

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{1} \quad \text{and} \quad \mathbf{z}_{22} = \frac{\mathbf{V}_2}{1}$$

This leads to the schematic in Fig. 18.53(b). The only difference between this schematic and the one in Fig. 18.53(a) is that the 1-A ac current source IAC is now at the output terminal. We run the schematic in Fig. 18.53(b) and obtain $\mathbf{V}_1$ and $\mathbf{V}_2$ from the output file. Thus,

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{1} = 19.70\underline{/175.7^\circ}\ \Omega, \qquad \mathbf{z}_{22} = \frac{\mathbf{V}_2}{1} = 19.56\underline{/175.7^\circ}\ \Omega$$

PRACTICE PROBLEM 18.16

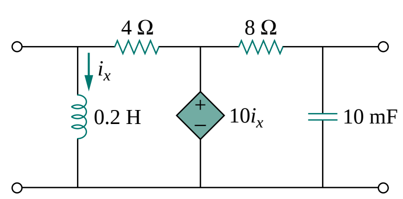

Figure 18.54 For Practice Prob. 18.16.

Obtain the z parameters of the circuit in Fig. 18.54 at $f = 60$ Hz.

Answer: $z_{11} = 3.987\underline{/175.5^\circ}$, $z_{21} = 0.0175\underline{/-2.65^\circ}$, $z_{12} = 0$, $z_{22} = 0.2651\underline{/91.9^\circ}\ \Omega$.

†18.9 APPLICATIONS

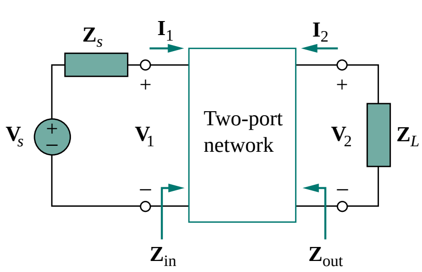

Figure 18.55 Two-port network isolating source and load.

We have seen how the six sets of network parameters can be used to characterize a wide range of two-port networks. Depending on the way two-ports are interconnected to form a larger network, a particular set of parameters may have advantages over others, as we noticed in Section 18.7. In this section, we will consider two important application areas of two-port parameters: transistor circuits and synthesis of ladder networks.

18.9.1 Transistor Circuits

The two-port network is often used to isolate a load from the excitation of a circuit. For example, the two-port in Fig. 18.55 may represent an amplifier, a filter, or some other network. When the two-port represents an amplifier, expressions for the voltage gain A_v, the current gain A_i, the input impedance Z_{in}, and the output impedance Z_{out} can be derived with ease. They are defined as follows:

$$A_v = \frac{V_2(s)}{V_1(s)} \tag{18.62}$$

$$A_i = \frac{I_2(s)}{I_1(s)} \tag{18.63}$$

$$Z_{\text{in}} = \frac{V_1(s)}{I_1(s)} \tag{18.64}$$

$$Z_{\text{out}} = \left.\frac{V_2(s)}{I_2(s)}\right|_{V_s=0} \tag{18.65}$$

Any of the six sets of two-port parameters can be used to derive the expressions in Eqs. (18.62) to (18.65). Here, we will specifically use the hybrid parameters to obtain them for transistor amplifiers.

The hybrid (h) parameters are the most useful for transistors; they are easily measured and are often provided in the manufacturer's data or spec sheets for transistors. The h parameters provide a quick estimate of the performance of transistor circuits. They are used for finding the exact voltage gain, input impedance, and output impedance of a transistor.

The h parameters for transistors have specific meanings expressed by their subscripts. They are listed by the first subscript and related to the general h parameters as follows:

$$h_i = h_{11}, \qquad h_r = h_{12}, \qquad h_f = h_{21}, \qquad h_o = h_{22} \tag{18.66}$$

The subscripts i, r, f, and o stand for input, reverse, forward, and output. The second subscript specifies the type of connection used: e for common emitter (CE), c for common collector (CC), and b for common base (CB). Here we are mainly concerned with the common-emitter connection. Thus, the four h parameters for the common-emitter amplifier are:

$$\begin{aligned} h_{ie} &= \text{Base input impedance} \\ h_{re} &= \text{Reverse voltage feedback ratio} \\ h_{fe} &= \text{Base-collector current gain} \\ h_{oe} &= \text{Output admittance} \end{aligned} \tag{18.67}$$

These are calculated or measured in the same way as the general h parameters. Typical values are $h_{ie} = 6$ kΩ, $h_{re} = 1.5 \times 10^{-4}$, $h_{fe} = 200$, $h_{oe} = 8\ \mu$S. We must keep in mind that these values represent ac characteristics of the transistor, measured under specific circumstances.

Figure 18.56 shows the circuit schematic for the common-emitter amplifier and the equivalent hybrid model. From the figure, we see that

$$\mathbf{V}_b = h_{ie}\mathbf{I}_b + h_{re}\mathbf{V}_c \tag{18.68a}$$

$$\mathbf{I}_c = h_{fe}\mathbf{I}_b + h_{oe}\mathbf{V}_c \tag{18.68b}$$

Consider the transistor amplifier connected to an ac source and a load as in Fig. 18.57. This is an example of a two-port network embedded within a larger network. We can analyze the hybrid equivalent circuit as usual with Eq. (18.68) in mind. (See Example 18.6.) Recognizing from Fig. 18.57 that $\mathbf{V}_c = -R_L\mathbf{I}_c$ and substituting this into Eq. (18.68b) gives

$$\mathbf{I}_c = h_{fe}\mathbf{I}_b - h_{oe}R_L\mathbf{I}_c$$

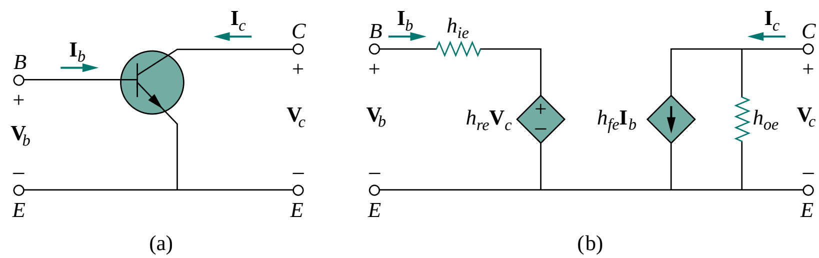

Figure 18.56 Common emitter amplifier: (a) circuit schematic, (b) hybrid model.

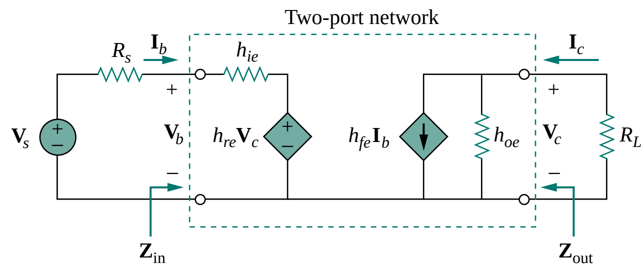

Figure 18.57 Transistor amplifier with source and load resistance.

or

$$(1 + h_{oe}R_L)\mathbf{I}_c = h_{fe}\mathbf{I}_b \tag{18.69}$$

From this, we obtain the current gain as

$$A_i = \frac{\mathbf{I}_c}{\mathbf{I}_b} = \frac{h_{fe}}{1 + h_{oe}R_L} \tag{18.70}$$

From Eqs. (18.68b) and (18.70), we can express $\mathbf{I}_b$ in terms of $\mathbf{V}_c$:

$$\mathbf{I}_c = \frac{h_{fe}}{1 + h_{oe}R_L}\mathbf{I}_b = h_{fe}\mathbf{I}_b + h_{oe}\mathbf{V}_c$$

or

$$\mathbf{I}_b = \frac{h_{oe}\mathbf{V}_c}{\dfrac{h_{fe}}{1 + h_{oe}R_L} - h_{fe}} \tag{18.71}$$

Substituting Eq. (18.71) into Eq. (18.68a) and dividing by $\mathbf{V}_c$ gives

$$\begin{aligned}\frac{\mathbf{V}_b}{\mathbf{V}_c} &= \frac{h_{oe}h_{ie}}{\dfrac{h_{fe}}{1 + h_{oe}R_L} - h_{fe}} + h_{re} \\ &= \frac{h_{ie} + h_{ie}h_{oe}R_L - h_{re}h_{fe}R_L}{-h_{fe}R_L}\end{aligned} \tag{18.72}$$

Thus, the voltage gain is

$$A_v = \frac{\mathbf{V}_c}{\mathbf{V}_b} = \frac{-h_{fe}R_L}{h_{ie} + (h_{ie}h_{oe} - h_{re}h_{fe})R_L} \tag{18.73}$$

Substituting $\mathbf{V}_c = -R_L\mathbf{I}_c$ into Eq. (18.68a) gives

$$\mathbf{V}_b = h_{ie}\mathbf{I}_b - h_{re}R_L\mathbf{I}_c$$

or

$$\frac{\mathbf{V}_b}{\mathbf{I}_b} = h_{ie} - h_{re}R_L\frac{\mathbf{I}_c}{\mathbf{I}_b} \tag{18.74}$$

Replacing $\mathbf{I}_c/\mathbf{I}_b$ by the current gain in Eq. (18.70) yields the input impedance as

$$Z_{\text{in}} = \frac{\mathbf{V}_b}{\mathbf{I}_b} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L} \tag{18.75}$$

The output impedance Z_{out} is the same as the Thevenin equivalent at the output terminals. As usual, by removing the voltage source and placing a 1-V source at the output terminals, we obtain the circuit in Fig. 18.58, from which Z_{out} is determined as $1/\mathbf{I}_c$. Since $\mathbf{V}_c = 1$ V, the input loop gives

$$h_{re}(1) = -\mathbf{I}_b(R_s + h_{ie}) \quad \Longrightarrow \quad \mathbf{I}_b = -\frac{h_{re}}{R_s + h_{ie}} \tag{18.76}$$

For the output loop,

$$\mathbf{I}_c = h_{oe}(1) + h_{fe}\mathbf{I}_b \tag{18.77}$$

Substituting Eq. (18.76) into Eq. (18.77) gives

$$\mathbf{I}_c = \frac{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}}{R_s + h_{ie}} \tag{18.78}$$

From this, we obtain the output impedance Z_{out} as $1/\mathbf{I}_c$; that is,

$$Z_{\text{out}} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}} \tag{18.79}$$

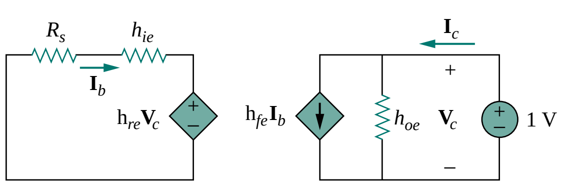

Figure 18.58 Finding the output impedance of the amplifier circuit in Fig. 18.57.

EXAMPLE 18.17

Consider the common-emitter amplifier circuit of Fig. 18.59. (a) Determine the voltage gain, current gain, input impedance, and output impedance using these h parameters:

$$h_{ie} = 1\text{ k}\Omega, \qquad h_{re} = 2.5\times 10^{-4}, \qquad h_{fe} = 50, \qquad h_{oe} = 20\ \mu\text{S}$$

(b) Find the output voltage $\mathbf{V}_o$.

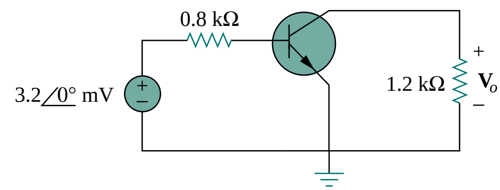

Figure 18.59 For Example 18.17.

Solution:

(a) We note that $R_s = 0.8$ kΩ and $R_L = 1.2$ kΩ. We treat the transistor of Fig. 18.59 as a two-port network and apply Eqs. (18.70) to (18.79).

$$h_{ie}h_{oe} - h_{re}h_{fe} = 10^3 \times 20 \times 10^{-6} - 2.5 \times 10^{-4} \times 50 = 7.5 \times 10^{-3}$$

$$A_v = \frac{-h_{fe}R_L}{h_{ie} + (h_{ie}h_{oe} - h_{re}h_{fe})R_L} = \frac{-50 \times 1200}{1000 + 7.5 \times 10^{-3} \times 1200} = -59.46$$

$$A_i = \frac{h_{fe}}{1 + h_{oe}R_L} = \frac{50}{1 + 20 \times 10^{-6} \times 1200} = 48.83$$

$$Z_{\text{in}} = h_{ie} - h_{re}A_iR_L = 1000 - 2.5 \times 10^{-4} \times 48.83 \times 1200 = 985.4\ \Omega$$

$$(R_s + h_{ie})h_{oe} - h_{re}h_{fe} = (800 + 1000) \times 20 \times 10^{-6} - 2.5 \times 10^{-4} \times 50 = 23.5 \times 10^{-3}$$

$$Z_{\text{out}} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}} = \frac{800 + 1000}{23.5 \times 10^{-3}} = 76.6\ \text{k}\Omega$$

(b) The output voltage is

$$\mathbf{V}_o = A_v\mathbf{V}_s = -59.46(3.2\angle 0^\circ)\ \text{mV} = 0.19\angle 180^\circ\ \text{V}$$

PRACTICE PROBLEM 18.17

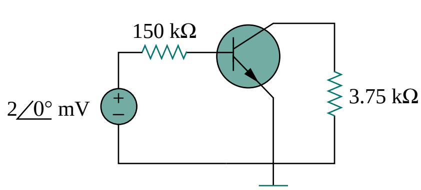

Figure 18.60 For Practice Prob. 18.17.

For the transistor amplifier of Fig. 18.60, find the voltage gain, current gain, input impedance, and output impedance. Assume that

$$h_{ie} = 6\ \text{k}\Omega, \qquad h_{re} = 1.5 \times 10^{-4}, \qquad h_{fe} = 200, \qquad h_{oe} = 8\ \mu\text{S}$$

Answer: −123.61, 194.17, 6 kΩ, 128.08 kΩ.

18.9.2 Ladder Network Synthesis

Another application of two-port parameters is the synthesis (or building) of ladder networks which are found frequently in practice and have particular use in designing passive lowpass filters. Based on our discussion of second-order circuits in Chapter 8, the order of the filter is the order of the characteristic equation describing the filter and is determined by the number of reactive elements that cannot be combined into single elements (e.g., through series or parallel combination). Figure 18.61(a) shows an LC ladder network with an odd number of elements (to realize an odd-order filter), while Fig. 18.61(b) shows one with an even number of elements (for realizing an even-order filter). When either network is terminated by the load impedance Z_L and the source impedance Z_s, we obtain the structure in Fig. 18.62. To make the design less complicated, we will assume that $Z_s = 0$. Our goal is to synthesize the transfer function of the LC ladder network. We begin by characterizing the ladder network by its admittance parameters, namely,

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \tag{18.80a}$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \tag{18.80b}$$

(Of course, the impedance parameters could be used instead of the admittance parameters.) At the input port, $\mathbf{V}_1 = \mathbf{V}_s$ since $\mathbf{Z}_s = 0$. At the output port, $\mathbf{V}_2 = \mathbf{V}_o$ and $\mathbf{I}_2 = -\mathbf{V}_2/\mathbf{Z}_L = -\mathbf{V}_o\mathbf{Y}_L$. Thus Eq. (18.80b) becomes

$$-\mathbf{V}_o\mathbf{Y}_L = \mathbf{y}_{21}\mathbf{V}_s + \mathbf{y}_{22}\mathbf{V}_o$$

or

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{y}_{21}}{\mathbf{Y}_L + \mathbf{y}_{22}} \tag{18.81}$$

We can write this as

$$\boxed{\mathbf{H}(s) = -\frac{\mathbf{y}_{21}/\mathbf{Y}_L}{1 + \mathbf{y}_{22}/\mathbf{Y}_L}} \tag{18.82}$$

We may ignore the negative sign in Eq. (18.82) because filter requirements are often stated in terms of the magnitude of the transfer function. The main objective in filter design is to select capacitors and inductors so that the parameters $\mathbf{y}_{21}$ and $\mathbf{y}_{22}$ are synthesized, thereby realizing the desired transfer function. To achieve this, we take advantage of an important property of the LC ladder network: all z and y parameters are ratios of polynomials that contain only even powers of s or odd powers of s—that is, they are ratios of either Od(s)/Ev(s) or Ev(s)/Od(s), where Od and Ev are odd and even functions, respectively. Let

$$\mathbf{H}(s) = \frac{\mathbf{N}(s)}{\mathbf{D}(s)} = \frac{\mathbf{N}_o + \mathbf{N}_e}{\mathbf{D}_o + \mathbf{D}_e} \tag{18.83}$$

where $\mathbf{N}(s)$ and $\mathbf{D}(s)$ are the numerator and denominator of the transfer

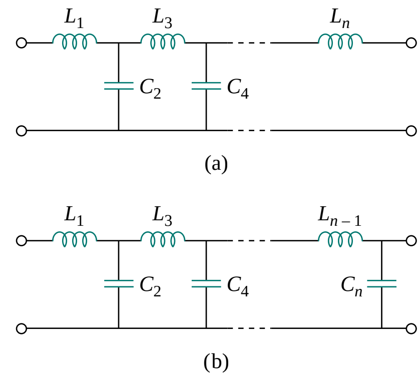

Figure 18.61 LC ladder networks for lowpass filters of: (a) odd order, (b) even order.

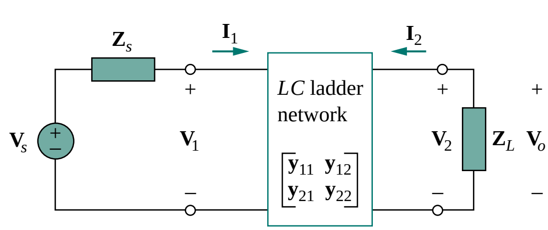

Figure 18.62 LC ladder network with terminating impedances.

function $\mathbf{H}(s)$; $\mathbf{N}_o$ and $\mathbf{N}_e$ are the odd and even parts of $\mathbf{N}$; $\mathbf{D}_o$ and $\mathbf{D}_e$ are the odd and even parts of $\mathbf{D}$. Since $\mathbf{N}(s)$ must be either odd or even, we can write Eq. (18.83) as

$$\mathbf{H}(s) = \begin{cases} \dfrac{\mathbf{N}_o}{\mathbf{D}_o + \mathbf{D}_e}, & (\mathbf{N}_e = 0) \\ \dfrac{\mathbf{N}_e}{\mathbf{D}_o + \mathbf{D}_e}, & (\mathbf{N}_o = 0) \end{cases} \tag{18.84}$$

and can rewrite this as

$$\mathbf{H}(s) = \begin{cases} \dfrac{\mathbf{N}_o/\mathbf{D}_e}{1 + \mathbf{D}_o/\mathbf{D}_e}, & (\mathbf{N}_e = 0) \\ \dfrac{\mathbf{N}_e/\mathbf{D}_o}{1 + \mathbf{D}_e/\mathbf{D}_o}, & (\mathbf{N}_o = 0) \end{cases} \tag{18.85}$$

Comparing this with Eq. (18.82), we obtain the y parameters of the network as

$$\frac{\mathbf{y}_{21}}{\mathbf{Y}_L} = \begin{cases} \dfrac{\mathbf{N}_o}{\mathbf{D}_e}, & (\mathbf{N}_e = 0) \\ \dfrac{\mathbf{N}_e}{\mathbf{D}_o}, & (\mathbf{N}_o = 0) \end{cases} \tag{18.86}$$

and

$$\frac{\mathbf{y}_{22}}{\mathbf{Y}_L} = \begin{cases} \dfrac{\mathbf{D}_o}{\mathbf{D}_e}, & (\mathbf{N}_e = 0) \\ \dfrac{\mathbf{D}_e}{\mathbf{D}_o}, & (\mathbf{N}_o = 0) \end{cases} \tag{18.87}$$

The following example illustrates the procedure.

EXAMPLE 18.18

Design the LC ladder network terminated with a 1-Ω resistor that has the normalized transfer function

$$\mathbf{H}(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

(This transfer function is for a Butterworth lowpass filter.)

Solution:

The denominator shows that this is a third-order network, so that the LC ladder network is shown in Fig. 18.63(a), with two inductors and one capacitor. Our goal is to determine the values of the inductors and capacitor. To achieve this, we group the terms in the denominator into odd or even parts:

$$\mathbf{D}(s) = (s^3 + 2s) + (2s^2 + 1)$$

so that

$$\mathbf{H}(s) = \frac{1}{(s^3 + 2s) + (2s^2 + 1)}$$

Divide the numerator and denominator by the odd part of the denominator to get

$$\mathbf{H}(s) = \frac{\dfrac{1}{s^3 + 2s}}{1 + \dfrac{2s^2 + 1}{s^3 + 2s}} \tag{18.18.1}$$

From Eq. (18.82), when $\mathbf{Y}_L = 1$,

$$\mathbf{H}(s) = \frac{-y_{21}}{1 + y_{22}} \tag{18.18.2}$$

Comparing Eqs. (18.18.1) and (18.18.2), we obtain

$$\mathbf{y}_{21} = -\frac{1}{s^3 + 2s}, \qquad \mathbf{y}_{22} = \frac{2s^2 + 1}{s^3 + 2s}$$

Any realization of y_{22} will automatically realize y_{21}, since y_{22} is the output driving-point admittance, that is, the output admittance of the network with the input port short-circuited. We determine the values of L and C in Fig. 18.63(a) that will give us $\mathbf{y}_{22}$. Recall that $\mathbf{y}_{22}$ is the short-circuit output admittance. So we short-circuit the input port as shown in Fig. 18.63(b). First we get L_3 by letting

$$\mathbf{Z}_A = \frac{1}{\mathbf{y}_{22}} = \frac{s^3 + 2s}{2s^2 + 1} = sL_3 + \mathbf{Z}_B \tag{18.18.3}$$

By long division,

$$\mathbf{Z}_A = 0.5s + \frac{1.5s}{2s^2 + 1} \tag{18.18.4}$$

Comparing Eqs. (18.18.3) and (18.18.4) shows that

$$L_3 = 0.5 \text{ H}, \qquad \mathbf{Z}_B = \frac{1.5s}{2s^2 + 1}$$

Next, we seek to get C_2 as in Fig. 18.63(c) and let

$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = \frac{2s^2 + 1}{1.5s} = 1.333s + \frac{1}{1.5s} = sC_2 + Y_C$$

from which $C_2 = 1.33$ F and

$$\mathbf{Y}_C = \frac{1}{1.5s} = \frac{1}{sL_1} \qquad \Longrightarrow \qquad L_1 = 1.5 \text{ H}$$

Thus, the LC ladder network in Fig. 18.63(a) with $L_1 = 1.5$ H, $C_2 = 1.333$ F, and $L_3 = 0.5$ H has been synthesized to provide the given transfer function $\mathbf{H}(s)$. This result can be confirmed by finding $\mathbf{H}(s) = \mathbf{V}_2/\mathbf{V}_1$ in Fig. 18.63(a) or by confirming the required y_{21}.

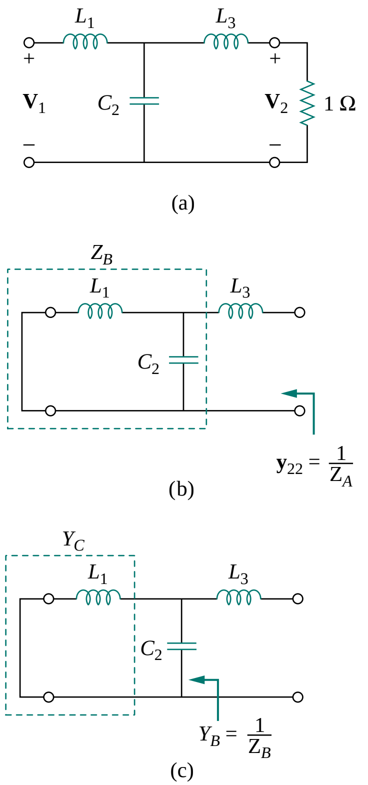

Figure 18.63 For Example 18.18.

PRACTICE PROBLEM 18.18

Realize the following transfer function using an LC ladder network terminated in a 1-Ω resistor:

$$H(s) = \frac{2}{s^3 + s^2 + 4s + 2}$$

Answer: Ladder network in Fig. 18.63(a) with $L_1 = L_3 = 1.0$ H and $C_2 = 0.5$ F.

18.10 SUMMARY

1. A two-port network is one with two ports (or two pairs of access terminals), known as input and output ports.

2. The six parameters used to model a two-port network are the impedance [**z**], admittance [**y**], hybrid [**h**], inverse hybrid [**g**], transmission [**T**], and inverse transmission [**t**] parameters.
3. The parameters relate the input and output port variables as

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = [\mathbf{z}]\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}, \qquad \begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} = [\mathbf{y}]\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}, \qquad \begin{bmatrix}\mathbf{V}_1\\ \mathbf{I}_2\end{bmatrix} = [\mathbf{h}]\begin{bmatrix}\mathbf{I}_1\\ \mathbf{V}_2\end{bmatrix}$$

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{V}_2\end{bmatrix} = [\mathbf{g}]\begin{bmatrix}\mathbf{V}_1\\ \mathbf{I}_2\end{bmatrix}, \qquad \begin{bmatrix}\mathbf{V}_1\\ \mathbf{I}_1\end{bmatrix} = [\mathbf{T}]\begin{bmatrix}\mathbf{V}_2\\ -\mathbf{I}_2\end{bmatrix}, \qquad \begin{bmatrix}\mathbf{V}_2\\ \mathbf{I}_2\end{bmatrix} = [\mathbf{t}]\begin{bmatrix}\mathbf{V}_1\\ -\mathbf{I}_1\end{bmatrix}$$

4. The parameters can be calculated or measured by short-circuiting or open-circuiting the appropriate input or output port.
5. A two-port network is reciprocal if $\mathbf{z}_{12} = \mathbf{z}_{21}$, $\mathbf{y}_{12} = \mathbf{y}_{21}$, $\mathbf{h}_{12} = -\mathbf{h}_{21}$, $\mathbf{g}_{12} = -\mathbf{g}_{21}$, $\Delta_T = 1$ or $\Delta_t = 1$. Networks that have dependent sources are not reciprocal.
6. Table 18.1 provides the relationships between the six sets of parameters. Three important relationships are

$$[\mathbf{y}] = [\mathbf{z}]^{-1}, \qquad [\mathbf{g}] = [\mathbf{h}]^{-1}, \qquad [\mathbf{t}] \neq [\mathbf{T}]^{-1}$$

7. Two-port networks may be connected in series, in parallel, or in cascade. In the series connection the z parameters are added, in the parallel connection the y parameters are added, and in the cascade connection the transmission parameters are multiplied in the correct order.
8. One can use *PSpice* to compute the two-port parameters by constraining the appropriate port variables with a 1-A or 1-V source while using an open or short circuit to impose the other necessary constraints.
9. The network parameters are specifically applied in the analysis of transistor circuits and the synthesis of ladder LC networks. Network parameters are especially useful in the analysis of transistor circuits because these circuits are easily modeled as two-port networks. LC ladder networks, important in the design of passive lowpass filters, resemble cascaded T networks and are therefore best analyzed as two-ports.

REVIEW QUESTIONS

18.1 For the single-element two-port network in Fig. 18.64(a), $\mathbf{z}_{11}$ is:

(a) 0 (b) 5 (c) 10
(d) 20 (e) nonexistent

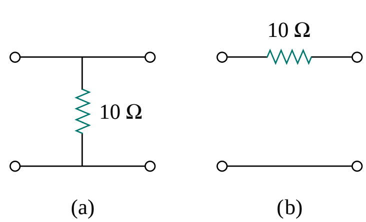

Figure 18.64 For Review Questions.

18.2 For the single-element two-port network in Fig. 18.64(b), $\mathbf{z}_{11}$ is:
(a) 0 (b) 5 (c) 10
(d) 20 (e) nonexistent

18.3 For the single-element two-port network in Fig. 18.64(a), $\mathbf{y}_{11}$ is:
(a) 0 (b) 5 (c) 10
(d) 20 (e) nonexistent

18.4 For the single-element two-port network in Fig. 18.64(b), $\mathbf{h}_{21}$ is:
(a) -0.1 (b) -1 (c) 0
(d) 10 (e) nonexistent

18.5 For the single-element two-port network in Fig. 18.64(a), **B** is:
(a) 0 (b) 5 (c) 10
(d) 20 (e) nonexistent

18.6 For the single-element two-port network in Fig. 18.64(b), **B** is:
(a) 0 (b) 5 (c) 10
(d) 20 (e) nonexistent

18.7 When port 1 of a two-port circuit is short-circuited, $\mathbf{I}_1 = 4\mathbf{I}_2$ and $\mathbf{V}_2 = 0.25\mathbf{I}_2$. Which of the following is true?
(a) $y_{11} = 4$ (b) $y_{12} = 16$
(c) $y_{21} = 16$ (d) $y_{22} = 0.25$

18.8 A two-port is described by the following equations:

$$\mathbf{V}_1 = 50\mathbf{I}_1 + 10\mathbf{I}_2$$
$$\mathbf{V}_2 = 30\mathbf{I}_1 + 20\mathbf{I}_2$$

Which of the following is *not* true?
(a) $\mathbf{z}_{12} = 10$ (b) $\mathbf{y}_{12} = -0.0143$
(c) $\mathbf{h}_{12} = 0.5$ (d) $\mathbf{B} = 50$

18.9 If a two-port is reciprocal, which of the following is *not* true?
(a) $\mathbf{z}_{21} = \mathbf{z}_{12}$ (b) $\mathbf{y}_{21} = \mathbf{y}_{12}$
(c) $\mathbf{h}_{21} = \mathbf{h}_{12}$ (d) $AD = BC + 1$

18.10 If the two single-element two-port networks in Fig. 18.64 are cascaded, then **D** is:
(a) 0 (b) 0.1 (c) 2
(d) 10 (e) nonexistent

Answers: 18.1c, 18.2e, 18.3e, 18.4b, 18.5a, 18.6c, 18.7b, 18.8d, 18.9c, 18.10c.

PROBLEMS

Section 18.2 Impedance Parameters

18.1 Obtain the z parameters for the network in Fig. 18.65.

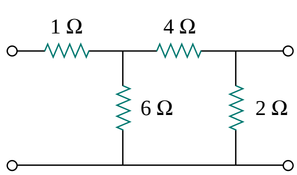

Figure 18.65 For Probs. 18.1 and 18.22.

***18.2** Find the impedance parameter equivalent of the network in Fig. 18.66.

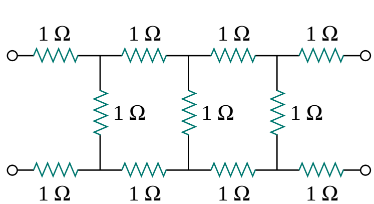

Figure 18.66 For Prob. 18.2.

18.3 Determine the z parameters of the two-ports shown in Fig. 18.67.

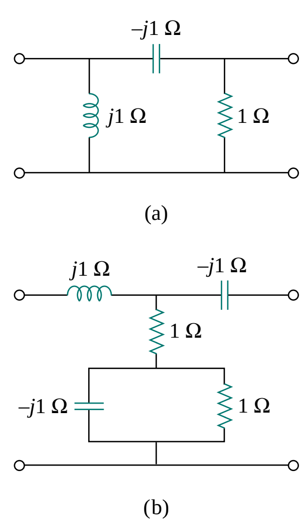

Figure 18.67 For Prob. 18.3.

*An asterisk indicates a challenging problem.

18.4 Calculate the z parameters for the circuit in Fig. 18.68.

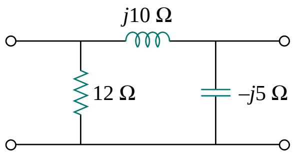

Figure 18.68 For Prob. 18.4.

18.5 Obtain the z parameters for the network in Fig. 18.69 as functions of s.

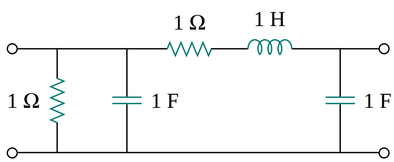

Figure 18.69 For Prob. 18.5.

18.6 Obtain the z parameters for the circuit in Fig. 18.70.

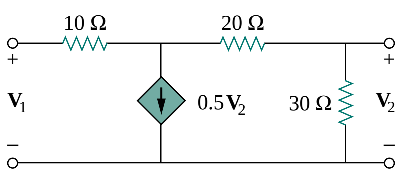

Figure 18.70 For Prob. 18.6.

18.7 Find the impedance-parameter equivalent of the circuit in Fig. 18.71.

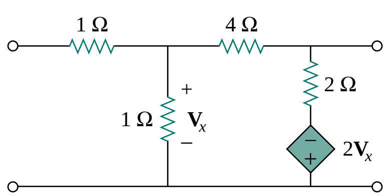

Figure 18.71 For Prob. 18.7.

18.8 Construct a circuit that realizes the following z parameters

$$[\mathbf{z}] = \begin{bmatrix} 10 & 4 \\ 4 & 6 \end{bmatrix}$$

18.9 Construct a two-port that realizes each of the following z parameters.

(a) $[\mathbf{z}] = \begin{bmatrix} 25 & 20 \\ 5 & 10 \end{bmatrix} \Omega$

(b) $[\mathbf{z}] = \begin{bmatrix} 1 + \frac{3}{s} & \frac{1}{s} \\ \frac{1}{s} & 2s + \frac{1}{s} \end{bmatrix} \Omega$

18.10 For a two-port network,

$$[\mathbf{z}] = \begin{bmatrix} 12 & 4 \\ 4 & 6 \end{bmatrix} \Omega$$

find V_2/V_1 if the network is terminated with a 2-Ω resistor.

18.11 If $[\mathbf{z}] = \begin{bmatrix} 50 & 10 \\ 30 & 20 \end{bmatrix} \Omega$ in the two-port of Fig. 18.72, calculate the average power delivered to the 100-Ω resistor.

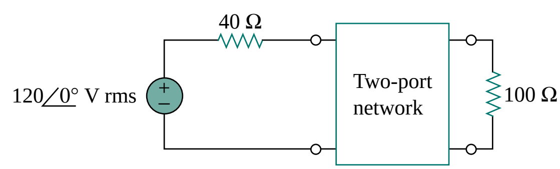

Figure 18.72 For Prob. 18.11.

18.12 For the two-port network shown in Fig. 18.73, show that

$$\mathbf{Z}_{Th} = \mathbf{z}_{22} - \frac{\mathbf{z}_{12}\mathbf{z}_{21}}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

and

$$\mathbf{V}_{Th} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11} + \mathbf{Z}_s}\mathbf{V}_s$$

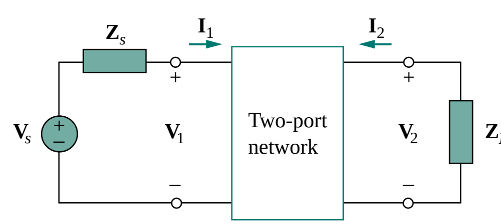

Figure 18.73 For Probs. 18.12 and 18.33.

18.13 For the circuit in Fig. 18.74, at $\omega = 2$ rad/s, $\mathbf{z}_{11} = 10\ \Omega$, $\mathbf{z}_{12} = \mathbf{z}_{21} = j6\ \Omega$, $\mathbf{z}_{22} = 4\ \Omega$. Obtain the Thevenin equivalent circuit at terminals a-b and calculate v_o.

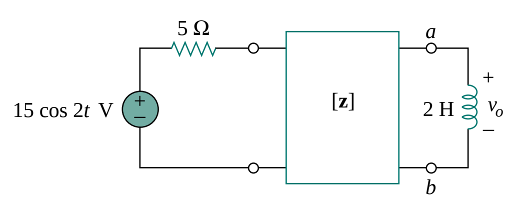

Figure 18.74 For Prob. 18.13.

Section 18.3 Admittance Parameters

*18.14 Determine the z and y parameters for the circuit in Fig. 18.75.

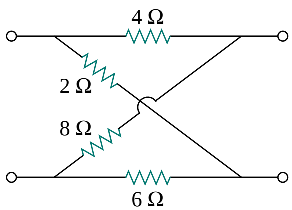

Figure 18.75 For Prob. 18.14.

18.15 Calculate the y parameters for the two-port in Fig. 18.76.

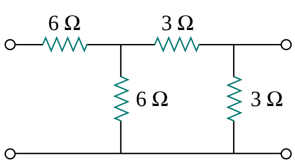

Figure 18.76 For Probs. 18.15 and 18.30.

18.16 Find the y parameters of the two-port in Fig. 18.77 in terms of s.

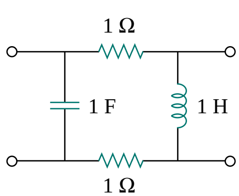

Figure 18.77 For Prob. 18.16.

18.17 Obtain the admittance parameter equivalent circuit of the two-port in Fig. 18.78.

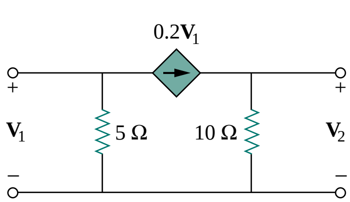

Figure 18.78 For Prob. 18.17.

18.18 Determine the y parameters for the two-ports in Fig. 18.79.

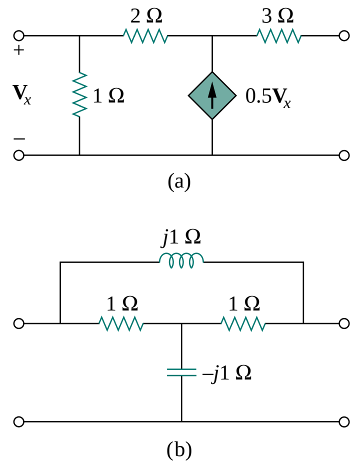

Figure 18.79 For Prob. 18.18.

18.19 Find the resistive circuit that represents these y parameters:

$$[\mathbf{y}] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}$$

18.20 Calculate [**y**] for the two-port in Fig. 18.80.

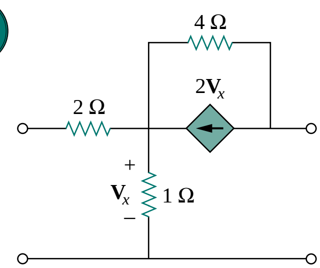

Figure 18.80 For Prob. 18.20.

18.21 Find the y parameters for the circuit in Fig. 18.81.

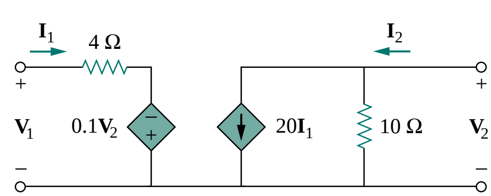

Figure 18.81 For Prob. 18.21.

18.22 In the circuit of Fig. 18.65, the input port is connected to a 1-A dc current source. Calculate the power dissipated by the 2-Ω resistor by using the y parameters. Confirm your result by direct circuit analysis.

18.23 In the bridge circuit of Fig. 18.82, $I_1 = 10$ A and $I_2 = -4$ A.
(a) Find V_1 and V_2 using y parameters.
(b) Confirm the results in part (a) by direct circuit analysis.

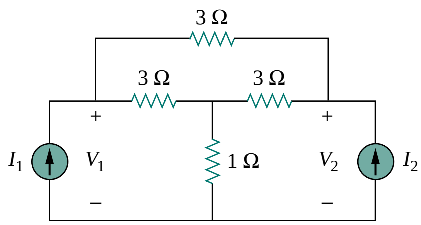

Figure 18.82 For Prob. 18.23.

Section 18.4 Hybrid Parameters

18.24 Find the h parameters for the networks in Fig. 18.83.

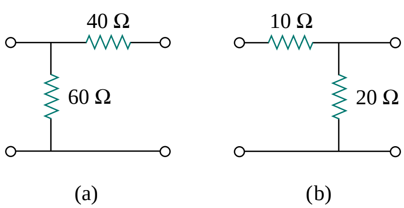

Figure 18.83 For Prob. 18.24.

18.25 Determine the hybrid parameters for the network in Fig. 18.84.

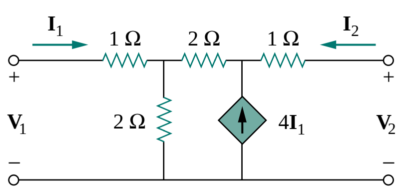

Figure 18.84 For Prob. 18.25.

18.26 Find the h and g parameters of the two-port network in Fig. 18.85 as functions of s.

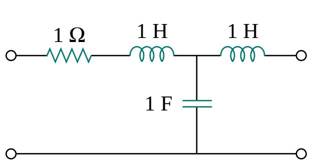

Figure 18.85 For Prob. 18.26.

18.27 Obtain the h and g parameters of the two-port in Fig. 18.86.

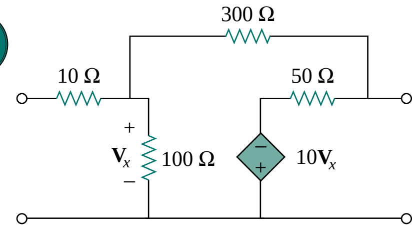

Figure 18.86 For Prob. 18.27.

18.28 Determine the h parameters for the network in Fig. 18.87.

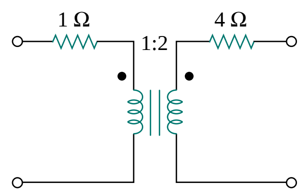

Figure 18.87 For Prob. 18.28.

18.29 For the two-port in Fig. 18.88,

$$[\mathbf{h}] = \begin{bmatrix} 16\ \Omega & 3 \\ -2 & 0.01\ \text{S} \end{bmatrix}$$

Find:
(a) V_2/V_1 (b) I_2/I_1
(c) I_1/V_1 (d) V_2/I_1

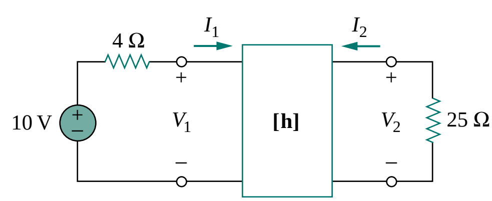

Figure 18.88 For Prob. 18.29.

18.30 The input port of the circuit in Fig. 18.76 is connected to a 10-V dc voltage source while the output port is terminated by a 5-Ω resistor. Find the voltage across the 5-Ω resistor by using h parameters of the circuit. Confirm your result by using direct circuit analysis.

18.31 For the circuit in Fig. 18.89, $\mathbf{h}_{11} = 800\ \Omega$, $\mathbf{h}_{12} = 10^{-4}$, $\mathbf{h}_{21} = 50$, $\mathbf{h}_{22} = 0.5 \times 10^{-5}$ S. Find the input impedance Z_{in}.

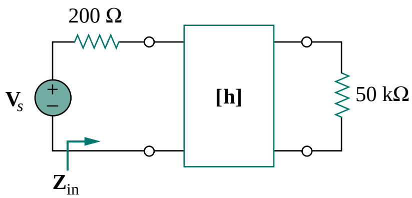

Figure 18.89 For Prob. 18.31.

18.32 Find the g parameters for the circuit in Fig. 18.90.

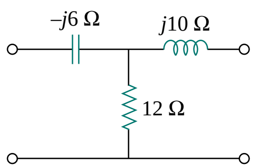

Figure 18.90 For Prob. 18.32.

18.33 For the two-port in Fig. 18.73, show that

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-\mathbf{g}_{21}}{\mathbf{g}_{11}\mathbf{Z}_L + \Delta_g}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\mathbf{Z}_L}{(1 + \mathbf{g}_{11}\mathbf{Z}_s)(\mathbf{g}_{22} + \mathbf{Z}_L) - \mathbf{g}_{21}\mathbf{g}_{12}\mathbf{Z}_s}$$

where Δ_g is the determinant of [**g**] matrix.

18.34 Find the network which realizes each of the following g parameters:

(a) $\begin{bmatrix} 0.01 & -0.5 \\ 0.5 & 20 \end{bmatrix}$ (b) $\begin{bmatrix} 0.1 & 0 \\ 12 & s+2 \end{bmatrix}$

Section 18.5 Transmission Parameters

18.35 Find the transmission parameters for the single-element two-port networks in Fig. 18.91.

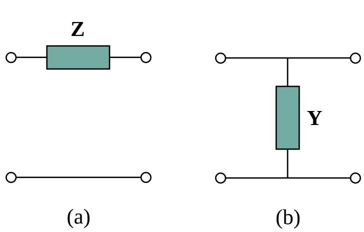

Figure 18.91 For Prob. 18.35.

18.36 Determine the transmission parameters of the circuit in Fig. 18.92.

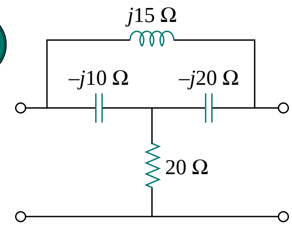

Figure 18.92 For Prob. 18.36.

18.37 Find the transmission parameters for the circuit in Fig. 18.93.

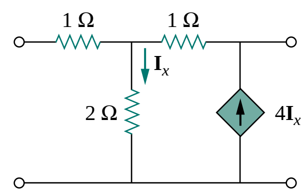

Figure 18.93 For Prob. 18.37.

18.38 For a two-port, let $\mathbf{A} = 4$, $\mathbf{B} = 30\ \Omega$, $\mathbf{C} = 0.1$ S, and $\mathbf{D} = 1.5$. Calculate the input impedance $\mathbf{Z}_{\text{in}} = \mathbf{V}_1/\mathbf{I}_1$, when:

(a) the output terminals are short-circuited,
(b) the output port is open-circuited,
(c) the output port is terminated by a 10-Ω load.

18.39 Using impedances in the s domain, obtain the transmission parameters for the circuit in Fig. 18.94.

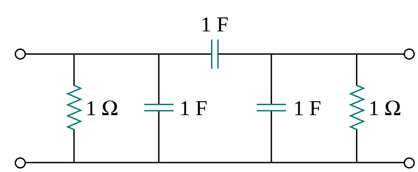

Figure 18.94 For Prob. 18.39.

18.40 Find the t parameters of the network in Fig. 18.95 as functions of s.

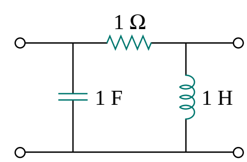

Figure 18.95 For Prob. 18.40.

18.41 Obtain the t parameters for the network in Fig. 18.96.

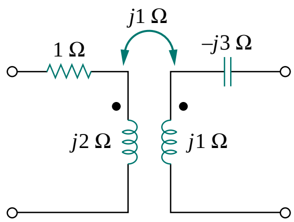

Figure 18.96 For Prob. 18.41.

Section 18.6 Relationships between Parameters

18.42 (a) For the T network in Fig. 18.97, show that the h parameters are:

$$\mathbf{h}_{11} = R_1 + \frac{R_2 R_3}{R_1 + R_3}, \qquad \mathbf{h}_{12} = \frac{R_2}{R_2 + R_3}$$

$$\mathbf{h}_{21} = -\frac{R_2}{R_2 + R_3}, \qquad \mathbf{h}_{22} = \frac{1}{R_2 + R_3}$$

(b) For the same network, show that the transmission parameters are:

$$\mathbf{A} = 1 + \frac{R_1}{R_2}, \qquad \mathbf{B} = R_3 + \frac{R_1}{R_2}(R_2 + R_3)$$

$$\mathbf{C} = \frac{1}{R_2}, \qquad \mathbf{D} = 1 + \frac{R_3}{R_2}$$

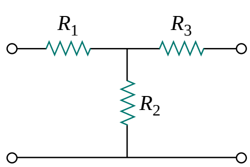

Figure 18.97 For Prob. 18.42.

18.43 Through derivation, express the z parameters in terms of the **ABCD** parameters.

18.44 Show that the transmission parameters of a two-port may be obtained from the y parameters as:

$$\mathbf{A} = -\frac{\mathbf{y}_{22}}{\mathbf{y}_{21}}, \qquad \mathbf{B} = -\frac{1}{\mathbf{y}_{21}}$$

$$\mathbf{C} = -\frac{\Delta_y}{\mathbf{y}_{21}}, \qquad \mathbf{D} = -\frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}$$

18.45 Prove that the g parameters can be obtained from the z parameters as

$$\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}}, \qquad \mathbf{g}_{12} = -\frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}$$

$$\mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}, \qquad \mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}$$

18.46 Given the transmission parameters

$$[\mathbf{T}] = \begin{bmatrix} 3 & 20 \\ 1 & 7 \end{bmatrix}$$

obtain the other five two-port parameters.

18.47 A two-port is described by

$$\mathbf{V}_1 = \mathbf{I}_1 + 2\mathbf{V}_2, \qquad \mathbf{I}_2 = -2\mathbf{I}_1 + 0.4\mathbf{V}_2$$

Find: (a) the y parameters, (b) the transmission parameters.

18.48 Given that

$$[\mathbf{g}] = \begin{bmatrix} 0.06\text{ S} & -0.4 \\ 0.2 & 2\ \Omega \end{bmatrix}$$

determine:

(a) [**z**] (b) [**y**] (c) [**h**] (d) [**T**]

18.49 Let $[\mathbf{y}] = \begin{bmatrix} 0.6 & -0.2 \\ -0.1 & 0.5 \end{bmatrix}$ (S). Find:

(a) [**z**] (b) [**h**] (c) [**t**]

18.50 For the bridge circuit in Fig. 18.98, obtain:

(a) the z parameters
(b) the h parameters
(c) the transmission parameters

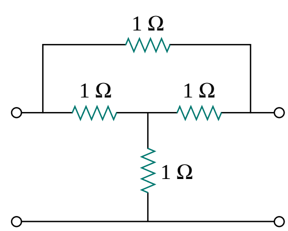

Figure 18.98 For Prob. 18.50.

18.51 Find the z parameters of the op amp circuit in Fig. 18.99. Obtain the transmission parameters.

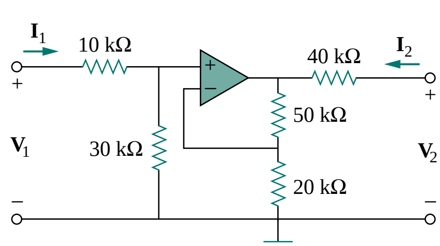

Figure 18.99 For Prob. 18.51.

18.52 Determine the y parameters at $\omega = 1{,}000$ rad/s for the op amp circuit in Fig. 18.100. Find the corresponding h parameters.

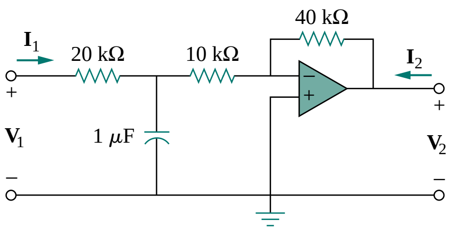

Figure 18.100 For Prob. 18.52.

Section 18.7 Interconnection of Networks

18.53 What is the y parameter presentation of the circuit in Fig. 18.101?

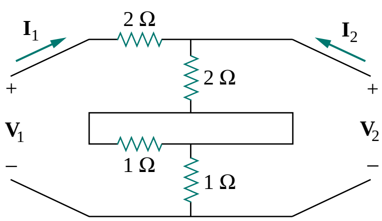

Figure 18.101 For Prob. 18.53.

18.54 In the two-port of Fig. 18.102, let $\mathbf{y}_{12} = \mathbf{y}_{21} = 0$, $\mathbf{y}_{11} = 2$ mS, and $\mathbf{y}_{22} = 10$ mS. Find $\mathbf{V}_o/\mathbf{V}_s$.

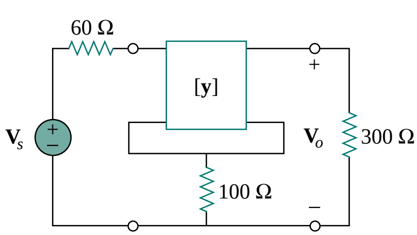

Figure 18.102 For Prob. 18.54.

18.55 Figure 18.103 shows two two-ports in series. Find the transmission parameters.

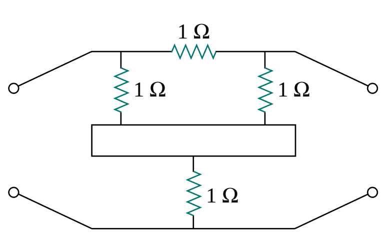

Figure 18.103 For Prob. 18.55.

18.56 Obtain the h parameters for the network in Fig. 18.104.

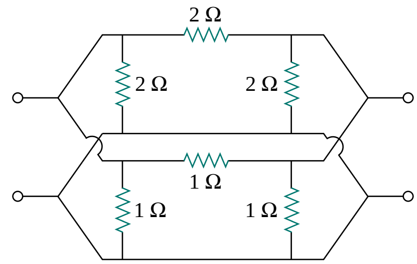

Figure 18.104 For Prob. 18.56.

18.57 Determine the y parameters of the two two-ports in parallel shown in Fig. 18.105.

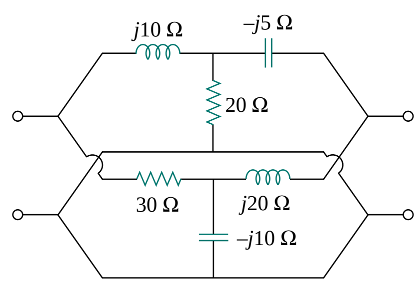

Figure 18.105 For Prob. 18.57.

***18.58** The circuit in Fig. 18.106 may be regarded as two two-ports connected in parallel. Obtain the y parameters as functions of s.

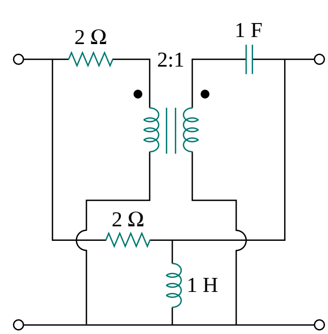

Figure 18.106 For Prob. 18.58.

*18.59 For the parallel-series connection of the two two-ports in Fig. 18.107, find the g parameters.

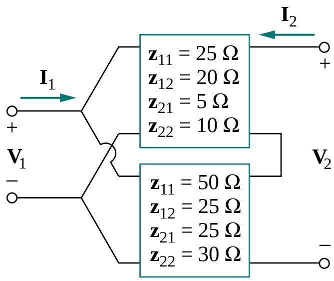

Figure 18.107 For Prob. 18.59.

*18.60 A series-parallel connection of two two-ports is shown in Fig. 18.108. Determine the z parameter representation of the network.

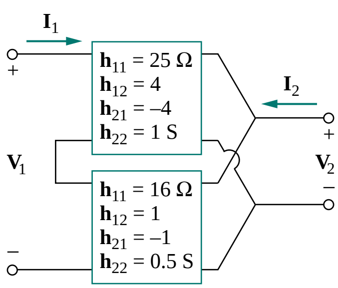

Figure 18.108 For Prob. 18.60.

18.61 Find the transmission parameters for the cascaded two-ports shown in Fig. 18.109. Obtain $\mathbf{Z}_{\text{in}} = \mathbf{V}_1/\mathbf{I}_1$ when the output is short-circuited.

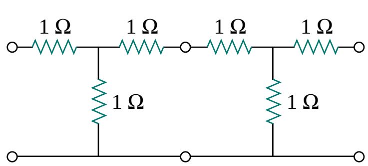

Figure 18.109 For Prob. 18.61.

*18.62 Determine the **ABCD** parameters of the circuit in Fig. 18.110 as functions of s.
(*Hint:* Partition the circuit into subcircuits and cascade them using the results of Prob. 18.35.)

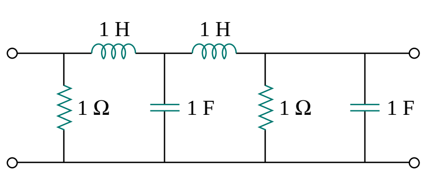

Figure 18.110 For Prob. 18.62.

Section 18.8 Computing Two-Port Parameters Using *PSpice*

18.63 Use *PSpice* to compute the y parameters for the circuit in Fig. 18.111.

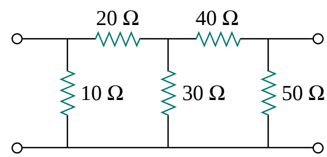

Figure 18.111 For Prob. 18.63.

18.64 Using *PSpice*, find the h parameters of the network in Fig. 18.112. Take $\omega = 1$ rad/s.

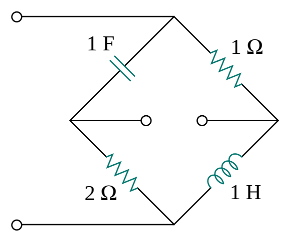

Figure 18.112 For Prob. 18.64.

18.65 Use *PSpice* to determine the z parameters of the circuit in Fig. 18.113. Take $\omega = 2$ rad/s.

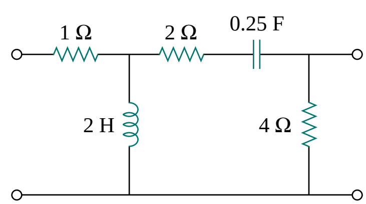

Figure 18.113 For Prob. 18.65.

18.66 Rework Prob. 18.7 using *PSpice*.

18.67 Repeat Prob. 18.20 using *PSpice*.

18.68 Use *PSpice* to rework Prob. 18.25.

18.69 Using *PSpice*, find the transmission parameters for the network in Fig. 18.114.

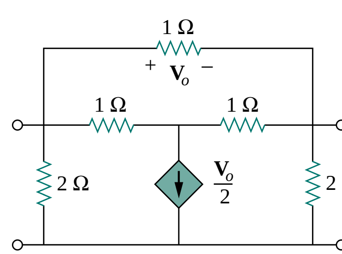

Figure 18.114 For Prob. 18.69.

18.70 At $\omega = 1$ rad/s, find the transmission parameters of the network in Fig. 18.115 using *PSpice*.

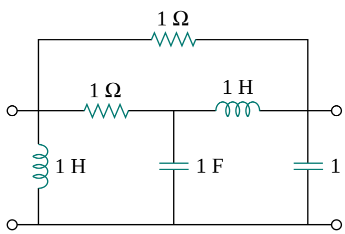

Figure 18.115 For Prob. 18.70.

18.71 Obtain the g parameters for the network in Fig. 18.116 using *PSpice*.

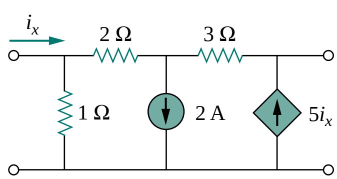

Figure 18.116 For Prob. 18.71.

18.72 For the circuit shown in Fig. 18.117, use *PSpice* to obtain the t parameters. Assume $\omega = 1$ rad/s.

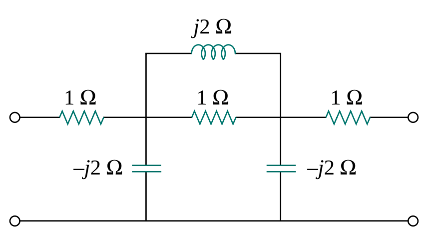

Figure 18.117 For Prob. 18.72.

Section 18.9 Applications

18.73 Using the y parameters, derive formulas for Z_{in}, Z_{out}, A_i, and A_v for the common-emitter transistor circuit.

18.74 A transistor has the following parameters in a common-emitter circuit:

$$h_{ie} = 2640\ \Omega, \qquad h_{re} = 2.6 \times 10^{-4}$$
$$h_{fe} = 72, \qquad h_{oe} = 16\ \mu\text{S}, \qquad R_L = 100\ \text{k}\Omega$$

What is the voltage amplification of the transistor? How many decibels gain is this?

18.75 A transistor with

$$h_{fe} = 120, \qquad h_{ie} = 2\ \text{k}\Omega$$
$$h_{re} = 10^{-4}, \qquad h_{oe} = 20\ \mu\text{S}$$

is used for a CE amplifier to provide an input resistance of 1.5 kΩ.

(a) Determine the necessary load resistance R_L.

(b) Calculate A_v, A_i, and Z_{out} if the amplifier is driven by a 4 mV source having an internal resistance of 600 Ω.

(c) Find the voltage across the load.

18.76 For the transistor network of Fig. 18.118,

$$h_{fe} = 80, \qquad h_{ie} = 1.2\ \text{k}\Omega$$
$$h_{re} = 1.5 \times 10^{-4}, \qquad h_{oe} = 20\ \mu\text{S}$$

Determine the following:

(a) voltage gain $A_v = V_o/V_s$,

(b) current gain $A_i = I_o/I_i$,

(c) input impedance Z_{in},

(d) output impedance Z_{out}.

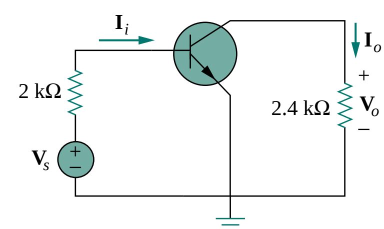

Figure 18.118 For Prob. 18.76.

***18.77** Determine A_v, A_i, Z_{in}, and Z_{out} for the amplifier shown in Fig. 18.119. Assume that

$$h_{ie} = 4\ \text{k}\Omega, \qquad h_{re} = 10^{-4}$$
$$h_{fe} = 100, \qquad h_{oe} = 30\ \mu\text{S}$$

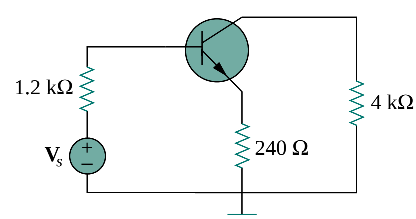

Figure 18.119 For Prob. 18.77.

***18.78** Calculate A_v, A_i, Z_{in}, and Z_{out} for the transistor network in Fig. 18.120. Assume that

$$h_{ie} = 2\ \text{k}\Omega, \qquad h_{re} = 2.5 \times 10^{-4}$$
$$h_{fe} = 150, \qquad h_{oe} = 10\ \mu\text{S}$$

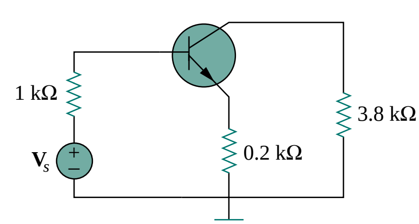

Figure 18.120 For Prob. 18.78.

18.79 A transistor in its common-emitter mode is specified by

$$[\mathbf{h}] = \begin{bmatrix} 200\ \Omega & 0 \\ 100 & 10^{-6}\ \text{S} \end{bmatrix}$$

Two such identical transistors are connected in cascade to form a two-stage amplifier used at audio frequencies. If the amplifier is terminated by a 4-kΩ resistor, calculate the overall A_v and Z_{in}.

18.80 Realize an LC ladder network such that

$$y_{22} = \frac{s^3 + 5s}{s^4 + 10s^2 + 8}$$

18.81 Design an LC ladder network to realize a lowpass filter with transfer function

$$H(s) = \frac{1}{s^4 + 2.613s^2 + 3.414s^2 + 2.613s + 1}$$

18.82 Synthesize the transfer function

$$H(s) = \frac{V_o}{V_s} = \frac{s^3}{s^3 + 6s + 12s + 24}$$

using the LC ladder network in Fig. 18.121.

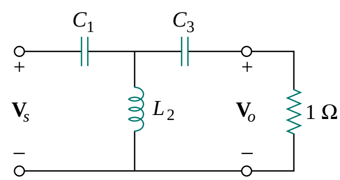

Figure 18.121 For Prob. 18.82.

COMPREHENSIVE PROBLEMS

18.83 Assume that the two circuits in Fig. 18.122 are equivalent. The parameters of the two circuits must be equal. Using this factor and the z parameters, derive Eqs. (9.67) and (9.68).

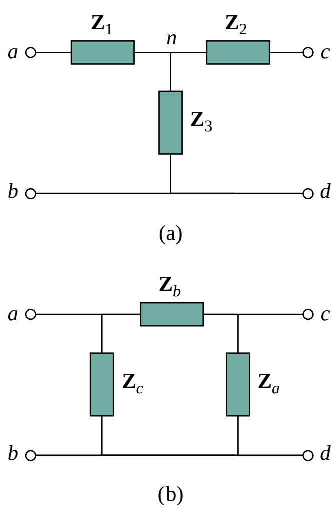

Figure 18.122 For Prob. 18.83.