ECE 329
Lecture 26

Uniform Plane Waves in Material

Dielectric and Conductor Approximations

Propagation Parameters (General)

- The *general form* of these parameters are always valid.
- However, they are somewhat complicated.
- Based on material properties, simplifications can be made

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$$\alpha = \frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]^{1/2}$$

$$\beta = \frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]^{1/2}$$

$$v_p = \frac{\sqrt{2}}{\sqrt{\mu\varepsilon}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]^{-1/2}$$

$$\lambda = \frac{\sqrt{2}}{f\sqrt{\mu\varepsilon}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]^{-1/2}$$

Propagation Parameters (Perfect Dielectric)

- A perfect dielectric is characterized by a conductivity of zero.
- This implies there will be no conduction current, ohmic loss, or attenuation.
- The E and H fields will be in phase
- Perfect dielectrics act like free space, but with ε_0 replaced with ε and μ_0 replaced with μ.

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu\varepsilon}$$

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = \frac{1}{f\sqrt{\mu\varepsilon}}$$

Propagation Parameters (Imperfect Dielectric)

- An imperfect dielectric is characterized by a loss tangent much less than 1 (σ/ωε << 1)
- This implies there will a small amount of conduction current, and thus attenuation.
- Due to the constraint on the loss tangent, this is the only noticeable difference with the perfect dielectric case.

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\left[\left(1 - \frac{3\sigma^2}{8\omega^2\varepsilon^2}\right) + j\frac{\sigma}{2\omega\varepsilon}\right]$$

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}\left(1 - \frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

$$\beta = \omega\sqrt{\mu\varepsilon}\left(1 + \frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}}\left(1 - \frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

$$\lambda = \frac{1}{f\sqrt{\mu\varepsilon}}\left(1 - \frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

Propagation Parameters (Good Conductor)

- A good conductor is characterized by a loss tangent much greater than 1 ($\sigma/\omega\varepsilon >> 1$)
- This implies there will more conduction current than displacement current, and thus considerable attenuation.
- To a good approximation, the E and H fields in a good conductor are 45° out of phase

$$\eta = \sqrt{\frac{\pi f \mu}{\sigma}}(1 + j)$$

$$\alpha = \sqrt{\pi f \mu \sigma}$$

$$\beta = \sqrt{\pi f \mu \sigma}$$

$$v_p = \sqrt{\frac{4\pi f}{\mu \sigma}}$$

$$\lambda = \sqrt{\frac{4\pi}{f \mu \sigma}}$$

Example

- **For a uniform plane wave of frequency $f = 10^5$ Hz propagating in a good conductor, the fields undergo attenuation by the factor $e^{-\pi}$ in a distance of 2.5 m. Find the following:**
  - The distance in which the fields undergo a change of phase by 2π for $f = 10^5$ Hz
  - The distance a constant phase travels in 1 μs for $f = 10^5$ Hz
  - The distance a constant phase travels in 1 μs for f = 10^4 Hz assuming the material parameters are the same as at 10^5 Hz

Example

- Assuming the properties are not frequency dependent, find the frequencies at which moist ground ($\varepsilon_r = 10$, $\sigma = 10^{-2}$ S/m) behaves as an imperfect dielectric and a good conductor.

Example

- Consider communication with an underwater submarine using an EM wave with an E field given by:

$$\mathbf{E} = 100 \cos(10^7 \pi t)\ \mathbf{a}_x \text{ V/m}$$

at z=0. Given the material parameters for seawater ($\varepsilon_r = 72$, $\mu_r = 1$, $\sigma = 4$ S/m), determine the depth at which the E field is attenuated by a factor of e^{-1} (a skin depth).

Example (cont)

- If the same submarine is at 17.5 m depth and requires an electric field an amplitude of at least 36 V/m to receive the signal, what can be done to make sure it can receive the transmitted signal?

Example

- The propagation constant in a plasma can be approximated by:

$$\gamma = j\omega(\mu_0\varepsilon_0)^{1/2}(1-(f_p/f)^2)^{1/2}$$

where f_p is the plasma frequency and is a function of the electron density of the plasma. The ionosphere is a layer of plasma surrounding the Earth at approximately 300 km. During the daytime, the ionospheric plasma frequency is approximately 20 MHz. What does this imply about the frequencies required for communicating with satellites orbiting Earth?