# Information Aggregating Networks based on Extended Sugeno's Fuzzy Integral

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**Abstract.** Sugeno's fuzzy integral is a functional to aggregate partial evaluations for an object in consideration of importance degrees of evaluation items. This paper presents the issues related to Sugeno's fuzzy integral for information aggregation. For the identification of importance degrees of evaluation items with the properties of fuzzy measures, we suggest to use a genetic algorithm based method. To improve the behavior of the fuzzy integral by avoiding excessive emphasis of pessimistic aspects, we introduce compensatory operators into the fuzzy integral. On the other hand, to tune the parameters for the used compensatory operators and to perform the fuzzy integral in parallel computation, we propose a network model.

## 1 Introduction

In decision making and pattern recognition, we frequently encounter the situations that we should produce an overall evaluation value from several partial evaluation values, where each partial evaluation value is determined in the viewpoint of an evaluation item. As a method to deal with these situations, there is Sugeno's fuzzy integral[6] which has the role to aggregate partial evaluation values with respect to importance degrees of evaluation items.

When we use Sugeno's fuzzy integral, we have the following problems: First, we should determine importance degrees of evaluation items holding the properties of fuzzy measure. It is not easy to provide consistent fuzzy measure values since they have to be subjectively determined. Second, Sugeno's fuzzy integral has the tendency to emphasize the pessimistic aspect by taking the minimum of evaluation values on performing fuzzy integral. Third, there is some computational burden to perform fuzzy integral since its operation is imposed on all elements of the power set of evaluation item set.

This paper proposes some solutions for the above problems. To identify fuzzy measure values for importance degree, we use genetic algorithm-based method that we have already proposed[5]. In the identification, we are interested in $\lambda$-fuzzy measure identification. To avoid excessively pessimistic evaluation, we introduce compensatory operators[11] into fuzzy integral. In addition, we extend the fuzzy integral to reflect certainty factors of evaluation values. On the other hand, to alleviate the computational burden and tune the parameters of compensatory operators, we propose an information aggregating network model based on Sugeno's fuzzy integral. To show the applicability of proposed method, we perform an experiment which evaluates preference degree for secondhand cars.

This paper is organized as follows: Section 2 describes the $\lambda$-fuzzy measure identification based on genetic algorithms. Section 3 extends Sugeno's fuzzy integral to improve its properties. Section 4 presents a network model to carry out the extended Sugeno's fuzzy integral and make it possible to tune some parameters in fuzzy integral and Section 5 shows an experiment and its results. Finally Section 6 draws conclusions.

## 2 Fuzzy Measure Identification

To use fuzzy integral in information aggregation, we should have importance degrees assigned to evaluation items. These importance degrees are required to preserve the properties of fuzzy measures.

### 2.1 Fuzzy Measures

Fuzzy measure $g$ is a set function defined on the power set $\mathcal{B}(X)$ of $X$ satisfying the following properties[10]:

$$g : \mathcal{B}(X) \to [0,1]$$
1) $g(\phi) = 0, \quad g(X) = 1$
2) If $A, B \in \mathcal{B}(X)$ and $A \subset B$, then $g(A) \leq g(B)$.
3) If $F_n \in \mathcal{B}(X)$ for $1 \leq n < \infty$ and a sequence $\{F_n\}$ is monotone (in the sense of inclusion), then $\lim_{n\to\infty} g(F_n) = g(\lim_{n\to\infty} F_n)$.

Among fuzzy measures, $\lambda$-fuzzy measure $g_\lambda$ is a widely used fuzzy measure with the following properties:

$$\forall A, B \in \mathcal{B}(X),\ A \cap B = \phi,$$
$$g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B) + \lambda g_\lambda(A) g_\lambda(B) \text{ for } \lambda > -1.$$

In $\lambda$-fuzzy measure for a finite set $X = \{x_1, x_2, \ldots, x_k\}$, fuzzy density values $g_i = g_\lambda(\{x_i\})$ leads the followings:

$$g_\lambda(\{x_1, \ldots, x_l\}) = \sum_{i=1}^{l} g_i + \lambda \sum_{i_1=1}^{l-1} \sum_{i_2=i_1+1}^{l} g_{i_1} g_{i_2} + \cdots + \lambda^{l-1} g_1 g_2 \cdots g_l = \frac{1}{\lambda}[\Pi_{i=1}^{l}\ (1 + \lambda g_i) - 1]$$

### 2.2 Genetic Algorithms

Genetic algorithms can be viewed as a general-purpose search method, an optimization method, or a learning mechanism, based loosely on Darwinian principles of biological evolution: reproduction and "survival of the fittest" along with genetic recombination[1, 2, 3, 4].

Genetic algorithms maintain a set of candidate solutions called a population. Candidate solutions are usually represented as strings of fixed length, called chromosomes. Given a (random) initial population, genetic algorithms operate in cycles, called generations, as follows:

1. Initialize a population of chromosomes.
2. Evaluate each chromosome in the population.
3. Create new chromosomes by mating current chromosomes; apply genetic operators
4. Delete some chromosomes of the population to make room for the new chromosomes.
5. Evaluate the new chromosomes and insert them into the population.
6. If time is up, stop and return the best chromosome; if not, go to 3.

When we want to use genetic algorithms for solving problems, we should develop the followings:

*Encoding scheme.* Each chromosome corresponds to a candidate solution and should have a format to which genetic operators can be applied. Thus we need to develop some methods to represent candidate solutions in coded strings(chromosomes). In general, chromosomes are represented by binary strings, real value strings, or symbolic strings, etc.

*Initialization of population.* A well-initialized population helps genetic algorithms to find desirable solution(s) more easily and faster than does a poorly-initialized one. Thus we need deliberate methods to create an initial population.

*Evaluation function.* During the operation of genetic algorithms, all chromosomes are evaluated to see how proper they are as solutions to the problem. Thus we have to develop a function to evaluate the fitness of candidate solutions as real solutions.

*Genetic operators.* Although there are typical genetic operators, we can not directly use them in our problems since they are affected by the encoding schemes and properties of problems. Thus, we must develop genetic operators suitable to the problem.

## 2.3 $\lambda$-Fuzzy Measure Identification with Genetic Algorithms

Fuzzy measure identification is to determine fuzzy measure values $g(A)$, $A \subset \mathcal{B}(X)$ for a set $X = \{x_1, x_2, \ldots, x_k\}$. Thus, $\lambda$-fuzzy measure identification is to decide fuzzy density values $g_i$, $i = 1, \ldots, k$ and $\lambda$. Since fuzzy measure values are subjectively determined, it is difficult to acquire consistent values satisfying the properties of fuzzy measures from human experts. Here we review a method to produce such fuzzy measure values from human-provided values with genetic algorithms[1, 2]. In the sequel, $\hat{g}_\lambda(A)$, $A \subset \mathcal{B}(X)$ and $\hat{g}_i$ denote the human-provided values, and $g_\lambda(A)$ and $g_i$ denote the identified values.

The genetic algorithm for $\lambda$-fuzzy measure identification consists of two stages: Genetic Algorithm I and Genetic Algorithm II. Both stages are performed by genetic algorithms. Genetic algorithm I takes charge of determining fuzzy density values $g_i$s, and Genetic Algorithm II finds the $\lambda$ value for the given fuzzy density values $g_i$s obtained by Genetic Algorithm I.

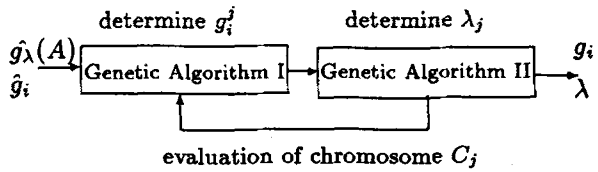


**Fig. 1.** $\lambda$-fuzzy measure identification genetic algorithm

In Genetic Algorithm I, chromosomes encode the fuzzy density values $g_i^j$ and the $\lambda$ value by a vector $C_j = (g_1^j, g_2^j, \ldots, g_k^j; \lambda_j)$. When $\hat{g}_i$ is available, the $i$-th position of each chromosome is initialized with a value near $\hat{g}_i$. When $\hat{g}_i$ is not available, the value for the $i$-th position is randomly selected from the interval $(0, \min\{\hat{g}_\lambda(A) \mid x_i \in A, A \in \mathcal{B}(X)\}]$.

The fitness function $f_1(C_j)$ for chromosome $C_j$ is the sum of the differences between human-provided fuzzy measure value $\hat{g}_\lambda(A)$ and fuzzy measure value obtained by $g_i^j$ and $\lambda_j$.

$$f_1(C_j) = \sum_{A \in \mathcal{B}(X)} |\hat{g}_\lambda(A) - \frac{1}{\lambda_j}[\Pi_{x_i \in A}(1 + \lambda_j g_i^j) - 1]|$$

In Genetic Algorithm I, there are two genetic operators : crossover and mutation. The crossover operator produces a new chromosome $C'$ from two randomly selected chromosomes $C_1$ and $C_2$ as follows:

$$\begin{aligned} C_1 &= (g_1^1, g_2^1, \ldots, g_i^1, g_{i+1}^1, \ldots, g_k^1; \lambda_1) \\ C_2 &= (g_1^2, g_2^2, \ldots, g_i^2, g_{i+1}^2, \ldots, g_k^2; \lambda_2) \\ C' &= (g_1^1, g_2^1, \ldots, g_i^1, g_{i+1}^2, \ldots, g_k^2; \lambda') \end{aligned}$$

The crossover influences on only the fuzzy density values $g_i^j$ but the $\lambda$ value. In the reproduced chromosome $C'$, $\lambda'$ is determined by Genetic Algorithm II. The mutation operator selects a chromosome $C_1$ and a position $i$ of the chromosome, and modifies the $i$-th position value with a randomly selected value $r$ from a specified interval as follows: Here $\lambda'$ is determined by Genetic Algorithm II.

$$\begin{aligned} C_1 &= (g_1^1, g_2^1, \ldots, g_i^1, \ldots, g_k^1; \lambda_1) \\ C' &= (g_1^1, g_2^1, \ldots, g_i^1 + r, \ldots, g_k^1; \lambda') \end{aligned}$$

In Genetic Algorithm II, chromosome is a real value to represent a $\lambda$ value, i.e., $C_j = (\lambda_j)$. To initialize the population, randomly select $\hat{g}_i$, $\hat{g}_j$ and $\hat{g}(\{x_i, x_j\})$, and find $\Lambda$ such that $\hat{g}(\{x_i, x_j\}) = \hat{g}_i + \hat{g}_j + \Lambda \hat{g}_i \hat{g}_j$. Each chromosome is initialized with randomly selected value from the interval $[\Lambda - \Delta, \Lambda + \Delta]$ where $\Delta > 0$.

The fitness function $f_2(C_j)$ of chromosome $C_j$ is the sum of differences between the human-provided measure value $\hat{g}_\lambda(A)$ and measure value obtained by fuzzy density values $g_i$ produced by Genetic Algorithm I and $\lambda_j$.

$$f_2(C_j) = \sum_{A \in \mathcal{B}(X)} |\hat{g}_\lambda(A) - \frac{1}{\lambda_j}[\Pi_{x_i \in A}(1 + \lambda_j \hat{g}_i) - 1]|$$

Genetic Algorithm II uses two genetic operators: crossover and mutation. Crossover operator selects two chromosomes $C_1 = (\lambda_1)$ and $C_2 = (\lambda_2)$, and creates a new chromosome $C' = (r\lambda_1 + (1-r)\lambda_2)$ with a randomly selected value $r$ at the interval $[0,1]$. Mutation operator selects a chromosome $C_j = (\lambda_j)$ and changes it into $C'_j = (\lambda_j + r)$ with a randomly selected value $r$ at a specified interval.

## 3 Extension of Sugeno's Fuzzy Integral

In the literature, there are several fuzzy integrals which are a kind of Lebesque integral[10]. Among them, Sugeno's fuzzy integral is a typical one. It has the role of aggregating partial evaluations for an object in consideration of importance degrees of evaluation items.

Let $X$ be a set of evaluation items and $g(E)$ the importance degree of evaluation item set $E \subset X$ with the properties of fuzzy measure. $g(x)$ denotes the evaluation value on the standpoint of evaluation item $x$, and $A$ denotes the interest focus of evaluation items. The fuzzy integral $\oint_A h(x) \circ g(\cdot)$ over the set $A \subset X$ of the function $h$ with respect to a fuzzy measure $g$ is defined as follows:

$$\begin{aligned} \oint_A h(x) \circ g(\cdot) &= \sup_{E \subseteq X} \{\min\{\min_{x \in E} h(x), g(A \cap E)\}\} \\ &= \sup_{\alpha \in [0,1]} \{\min\{\alpha, g(A \cap F_\alpha)\}\} \\ & \qquad F_\alpha = \{x \mid h(x) \geq \alpha\} \\ &= \sup_{E \subseteq A} \{\min\{\min_{x \in E} h(x), g(E)\}\} \end{aligned}$$

We can interpret the meaning of the fuzzy integral in the following way: In the above formula, $\min_{x \in E} h(x)$ selects the most pessimistic evaluation among the current evaluation items $E$. $\min\{\min_{x\in E} h(x), g(E)\}$ imposes the restriction that the aggregated evaluation value can not be greater than the importance degree of the current evaluation items $E$. $\sup_{E \subseteq A}\{\min\{\min_{x \in E} h(x), g(E)\}\}$ elicits the most promising evaluation value.

Due to the operation $\min_{x \in E} h(x)$, fuzzy integral has a tendency to produce pessimistic evaluation. Some decision making problems show that although an item has poor evaluation, the item can be compensated by other good items. Thus to provide the same effect to fuzzy integral, we extend it by introducing compensatory operators instead of minimum operator in the operation $\min_{x \in E}$ $h(x)$.

There are two kinds of compensatory operators which can be used in the extended fuzzy integral: mean operators and hybrid operators.

*Mean Operators*

Weighted Arithmetic Mean Operator

$$A \oplus_\gamma B = (1-\gamma)(A \cap B) + \gamma(A \cup B) \qquad 0 \leq \gamma \leq 1$$

Geometric Mean Operator

$$A \otimes_\gamma B = (A \cap B)^{(1-\gamma)}(A \cup B)^\gamma \quad 0 \leq \gamma \leq 1$$

Here $A \cap B$ denotes the result obtained by some t-norm operator[11] such as minimum operation, and $A \cup B$ the result by some t-conorm operator such as maximum operation. $\gamma$ represents the compensation degree. Thus as $\gamma$ comes to have a larger value, the compensatory operators take a more optimistic stance.

*Hybrid Operators*

Multiplicative $\gamma$-model

$$y = (\Pi_{i=1}^{n} x_i^{\delta_i})^{(1-\gamma)}(1 - \Pi_{i=1}^{n}(1 - x_i)^{\delta_i})^\gamma$$
$$\sum_{i=1}^{n} \delta_i = n,\ 0 \leq \gamma \leq 1$$

Additive $\gamma$-model

$$y = (1-\gamma)\Pi_{i=1}^{n} x_i^{\delta_i} + \gamma(\Pi_{i=1}^{n}(1 - x_i)^{\delta_i})$$

Here $\delta_i$ denotes the relative importance degree of evaluation item value $x_i$.

When we use a compensatory operator $\Phi$, the fuzzy integral has the following form:

$$\oint_A h(x) \circ g(\cdot) = \sup_{E \subseteq A} \{\min\{\Phi_{x \in E} h(x), g(E)\}\}$$

On the other hand, in the real world, it is possible for evaluation values to have uncertainty. Thus it is helpful to consider certainty factor or reliability of evaluation values in fuzzy integral. For this purpose, we use the interest focus $A$ as a fuzzy set and interpret membership degrees as certainty factors. When we consider certainty factor, fuzzy integral has the following form:

$$\oint_A h(x) \circ g(\cdot) = \sup_{E \subseteq Supp(A)} \{\min\{\min_{x \in E} Q(\mu_A(x), h(x)),\ g(E)\}\}$$

In the formula, $Supp(A)$ denotes the support[11] of fuzzy set $A$, and $Q(\mu_A(x), h(x))$ indicates the application of certainty factor $\mu_A(x)$ to evaluation value $h(x)$. For Q operation, we can use either minimum or product operations. When a compensatory operator is used in fuzzy integral, the above formula comes to be as follows:

$$\oint_A h(x) \circ g(\cdot) = \sup_{E \subseteq Supp(A)} \{\min\{\Phi_{x \in E} Q(\mu_A(x), h(x)),\ g(E)\}\}$$

## 4 Information Aggregating Network

When we extend fuzzy integral by introducing compensatory operators, we should apply compensatory operator to all elements of the power set of evaluation item set $X$. Thus there is computational burden in performing fuzzy integral. In addition, sometimes we should determine compensation degree $\gamma$ and/or relative importance degrees $\delta_i$. It is reasonable to determine such parameters from human-provided data by learning. To tackle these problems, we

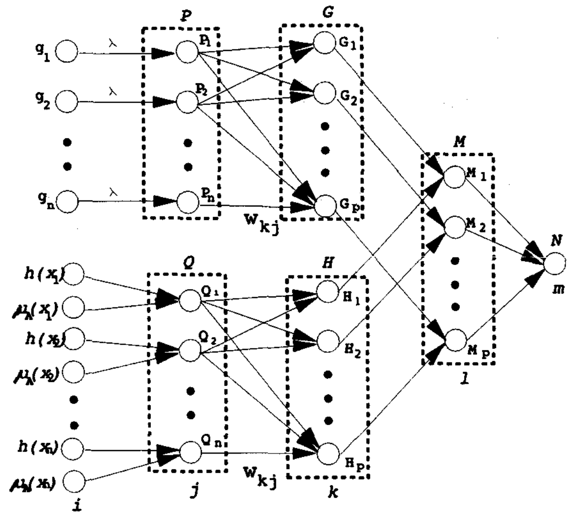


**Fig. 2.** Proposed network model

propose a network model that carries out fuzzy integral by parallel computations and provides a mechanism for tuning parameters such as $\gamma, \delta_i$.

Figure 2 shows the proposed network model which performs the extended fuzzy integral and uses $\lambda$-fuzzy measure values. In the network model, each node has its own role and the connection weights between nodes are fixed to 1 except the connections between layer P and its input layer which have $\lambda$ value.

Let the evaluation item set $X = \{x_1, x_2, \ldots, x_n\}$. The layers $P$ and $Q$ play the role of evaluating $\lambda$-fuzzy measure values. The fuzzy density values $g_i$s and $\lambda$ value obtained by fuzzy measure identification method from human-provided measure value come to be input for the layer $P$. The layer $P$ contains $n$ nodes and the layer $Q$ contains $2^n - 1$ nodes. A node of layer $Q$ corresponds to a subset $E$ of $X (E \subseteq X)$ and produces fuzzy measure value of $E$, i.e., $g(E) = [\Pi_{x_i \in E}(1 + \lambda g_i) - 1]/\lambda$. The node $P_j$ corresponding to evaluation item $x_j$ and the node $G_k$ corresponding to a subset $E$ of $X$, is connected when $x_j \in E$. For this connection, its weight $w_{kj}$ is set to 1. The node operations of layer $P$ and $Q$ are as follows:

$$P_j = \lambda g_j$$
$$G_k = [\Pi_j w_{kj}(1 + P_j) - 1]/\lambda$$

The layer $Q$ plays the role to restrict the domain of fuzzy integral to the interest focus $A$. When $A$ is a crisp set, $\mu_A(x_i) = 1$ if $x_i \in A$ and $\mu_A(x_i) = 0$ otherwise. When $A$ is a fuzzy set, $\mu_A(x_i)$ has a value on $[0, 1]$ and it indicates certainty factor. In the case of fuzzy set, $x_i$s such that $\mu_A(x_i) > 0$ belong to the interest focus. The operation of layer $Q$ is either minimum operation or product operation.

$$Q_j = \begin{cases} \min\{h(x_j), \mu_A(x_j)\} & \text{if min is used} \\ h(x_j) \cdot \mu_A(x_j) & \text{if product is used} \end{cases}$$

In the layer $H$, each node corresponds to a subset $E$ of $X$ and it has the role to carry out $\min_{x \in E} h(x)$ or $\Phi_{x \in E} h(x)$ where $\Phi$ denotes a compensatory operator. The connections between layer $Q$ and $H$ are established in the same way as the connections between layer $P$ and $G$. The nodes of layer $H$ perform the following operation.

$$H_k = \begin{cases} \min_j\{w_{kj} Q_j\} & \text{if min is used} \\ \Phi\{w_{kj} Q_j\} & \text{if } \Phi \text{ is used} \end{cases}$$

Each node of layer $M$ performs the minimum operation to impose the restriction that the aggregated evaluation value can not be greater than the importance degree of the corresponding evaluation items. In the layers $H$ and $G$, the nodes $H_l$ and $G_l$ correspond to the same evaluation items.

$$M_l = \min\{G_l, H_l\}$$

The node of layer $N$ produces the overall evaluation value by sup operation.

$$N = \sup_l M_l$$

By these node operations, the proposed network model carries out both Sugeno's fuzzy integral and its extended one.

When we use a compensatory operator in fuzzy integral, the layer $H$ itself has a parameter for the compensation degree $\gamma$, and each node of $Q$ has a parameter for its relative importance degree $\delta_i$. With training data, we can tune these parameters by the gradient descent method. Let the error $E$ be the square sum of the differences between the real output $N_i$ and the desired output $d_i$. The tuning of parameter $\gamma$ is performed as follows:

$$E = \tfrac{1}{2} \sum_i (N_i - d_i)^2$$

$$\gamma(t+1) = \gamma(t) + \eta \tfrac{\partial E}{\partial \gamma}$$

$$\tfrac{\partial E}{\partial \gamma} = \sum_k \tfrac{\partial E}{\partial H_k} \tfrac{\partial H_k}{\partial \gamma}$$

$$\tfrac{\partial E}{\partial H_k} = \tfrac{\partial E}{\partial M_k} \tfrac{\partial M_k}{\partial H_k}$$

$$\tfrac{\partial M_k}{\partial H_k} = \begin{cases} 1 & \text{if } M_k = H_k \\ 0 & \text{otherwise} \end{cases}$$

When $\gamma$-model is used as a compensatory operator, the relative importance degrees $\delta_j$ of evaluation items are tuned as follows:

$$\delta_j(t+1) = \delta_j(t) + \eta \tfrac{\partial E}{\partial \delta_j}$$

$$\tfrac{\partial E}{\partial \delta_j} = \tfrac{\partial E}{\partial Q_j} \tfrac{\partial Q_j}{\partial \delta_j}$$

$$\tfrac{\partial E}{\partial Q_j} = \sum_k \tfrac{\partial E}{\partial H_k} \tfrac{\partial H_k}{\partial Q_j}$$

## 5 Experiment

To show the applicability of proposed method, we performed an experiment to determine preference degree of secondhand car with fuzzy integral, where evaluation items were mileage, appearance, expected maintenance cost, age of operation, and history of accidents.

To apply fuzzy integral, we first should determine the importance degrees for the evaluation items with the properties of fuzzy measures. Thus we intuitively assigned the human-provided measure values for the evaluation items as shown in Table 1. In Table 1, a row consisting of $m, ap, c, ag, h$ represents a subset for the evaluation item set $X$, and the value of $g(\cdot)$ represents the importance degree for the evaluation items.

For these data, we applied the genetic algorithms for $\lambda$-fuzzy measure identification. Thus we obtained the following values: mileage 0.177170, appearance 0.302570, maintenance cost 0.309290, age of operation 0.233480, history of accident 0.350640, and $\lambda: -0.464951$.

With these identified values, we should apply fuzzy integral to the evaluation values for producing preference degrees.

**Table 1.** Data for importance degrees

| $m$ | $ap$ | $c$ | $ag$ | $h$ | $g(\cdot)$ | $m$ | $ap$ | $c$ | $ag$ | $h$ | $g(\cdot)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0.2 | 0 | 1 | 0 | 0 | 0 | 0.3 |
| 0 | 0 | 1 | 0 | 0 | 0.3 | 0 | 0 | 0 | 1 | 0 | 0.25 |
| 0 | 0 | 0 | 0 | 1 | 0.35 | 1 | 1 | 0 | 0 | 0 | 0.45 |
| 1 | 0 | 1 | 0 | 0 | 0.45 | 1 | 0 | 0 | 1 | 0 | 0.4 |
| 1 | 0 | 0 | 0 | 1 | 0.5 | 0 | 1 | 1 | 0 | 0 | 0.6 |
| 0 | 1 | 0 | 1 | 0 | 0.55 | 0 | 1 | 0 | 0 | 1 | 0.6 |
| 0 | 0 | 1 | 1 | 0 | 0.5 | 0 | 0 | 1 | 0 | 1 | 0.65 |
| 0 | 0 | 0 | 1 | 1 | 0.5 | 1 | 1 | 1 | 0 | 0 | 0.7 |
| 1 | 1 | 0 | 1 | 0 | 0.55 | 1 | 1 | 0 | 0 | 1 | 0.75 |
| 1 | 0 | 1 | 1 | 0 | 0.56 | 1 | 0 | 1 | 0 | 1 | 0.74 |
| 1 | 0 | 0 | 1 | 1 | 0.65 | 0 | 1 | 1 | 1 | 0 | 0.78 |
| 0 | 1 | 1 | 0 | 1 | 0.83 | 0 | 1 | 0 | 1 | 1 | 0.73 |
| 0 | 0 | 1 | 1 | 1 | 0.81 | 1 | 1 | 1 | 1 | 0 | 0.87 |
| 1 | 1 | 1 | 0 | 1 | 0.93 | 1 | 1 | 0 | 1 | 1 | 0.89 |
| 1 | 0 | 1 | 1 | 1 | 0.89 | 0 | 1 | 1 | 1 | 1 | 0.95 |
| 1 | 1 | 1 | 1 | 1 | 1.0 | | | | | | |

$m$: mileage, $ap$: appearance, $c$: maintenance cost,
$ag$: age of operation, $h$: history of accident

In the experiment we used the extended fuzzy integral, where the weighted arithmetic mean is used as a compensatory operator. Now we should determine the compensation parameter $\gamma$ for the compensatory operator. Thus we constructed an information aggregating network proposed in the previous section. To tune the parameter, we gathered data consisting of evaluation values and preference degree as shown in Table 2. In Table 2, the values in the columns $m, ap, c, ag, h$

denote the evaluation values with respect to the corresponding evaluation items, and the values in the column $v$ denotes the preference degree for the given evaluation values. Using the data in Table 2, we tuned the parameter by the method mentioned in the previous section. The tuning produced the $\gamma$ value 0.15.

Through this procedure, we determine the fuzzy density values $g_i$s and $\lambda$ value, and the compensation parameter $\gamma$. Now we could produce the preference degree for the given evaluation values of evaluation items with the proposed network model. The produced results by the network showed similar to human-provided preference degrees.

**Table 2.** Data for parameter tuning

| $m$ | $ap$ | $c$ | $ag$ | $h$ | $v$ | $m$ | $ap$ | $c$ | $ag$ | $h$ | $v$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.2 | 0.6 | 0.4 | 0.5 | 0.52 | 0.5 | 1.0 | 0.3 | 0.5 | 1.0 | 0.60 |
| 0.3 | 0.2 | 0.3 | 0.1 | 0.7 | 0.36 | 0.5 | 0.6 | 0.3 | 0.2 | 0.8 | 0.60 |
| 0.4 | 0.3 | 0.8 | 0.2 | 0.4 | 0.46 | 0.5 | 0.3 | 0.8 | 0.1 | 0.4 | 0.47 |
| 0.6 | 0.3 | 0.9 | 0.2 | 0.5 | 0.56 | 0.8 | 0.8 | 0.2 | 0.7 | 0.6 | 0.64 |
| 0.2 | 0.7 | 0.5 | 0.4 | 0.2 | 0.53 | 0.5 | 0.3 | 0.4 | 0.1 | 0.4 | 0.42 |
| 0.6 | 0.2 | 0.3 | 0.2 | 0.5 | 0.51 | 1.0 | 0.2 | 0.2 | 0.3 | 0.4 | 0.49 |
| 0.7 | 0.3 | 0.2 | 0.3 | 0.3 | 0.36 | 0.3 | 0.8 | 0.6 | 0.2 | 0.7 | 0.63 |

$m$: mileage, $ap$: appearance, $c$: maintenance cost,
$ag$: age of operation, $h$: history of accident

## 6 Conclusion

In this paper, we investigated the issues to be considered when we use Sugeno's fuzzy integral to aggregate information, and proposed solutions to deal with them. First, to identify importance degrees of evaluation items with the properties of fuzzy measures, we suggested the genetic algorithm-based $\lambda$-fuzzy measure identification method. Second, to improve the properties of fuzzy integral, we introduce compensatory operators into fuzzy integral. Thus we can avoid excessively emphasizing the pessimistic aspect in the information aggregation. Third, to lessen the computational burden in performing extended Sugeno's fuzzy integral, we proposed a network model to perform fuzzy integral. The proposed network model makes it possible to tune the parameters when a compensatory operator is used. In the experiment to determine preference degrees for second-hand car, we could see that the proposed method is useful.

The tuning method of the proposed network model is based on the gradient-descent method. Hence there is some possibility of falling into a local minima. As an alternative method for determining parameters of extended Sugeno's fuzzy integral, the genetic algorithm-based approach seems to be promising.

## References

1. L. Davis, *Handbook of Genetic Algorithms*(eds.), Van Nostrand Reinhold:New York, 1991.

2. K.De Jong, Learning with Genetic Algorithms: An Overview, *Machine Learning*, Vol.3, pp.121-138, 1988.
3. J.M. Fitzpatrick, J.J. Grefenstette, Genetic Algorithms in Noisy Environments, *Machine Learning*, Vol.3, pp.101-120, 1988.
4. D.E. Goldberg, *Genetic Algorithms in Search, Optimization & Machine Learning*, Addison-Wesley, 1989.
5. K.-M. Lee, H. Leekwang, Genetic Algorithms for Fuzzy Measure Identification, *the 3rd International Conference on Fuzzy Logic, Neural Networks, and Soft Computing*(Iizuka, Japan), pp.461-463, 1994.
6. D. Ralescu, G. Adams, The Fuzzy Integral, *Journal of Mathematical Analysis and Applications*, Vol.75, pp.562-570, 1980.
7. M. Sugeno, Fuzzy Decision-Making Problems, *Trans. S.I.C.E.* Vol.11, No.6, pp.709-714, 1975.
8. H. Tahani, J. M. Keller, Information Fusion in Computer Vision using the Fuzzy Integral, *IEEE Systems, Man, and Cybernetics*, Vol.SMC-20, No.3, 1990.
9. S.T. Wierzchoń, An Algorithm for Identification of Fuzzy Measure, *Fuzzy Sets and Systems*, Vol.9, pp.69-78, 1983.
10. S.T. Wierzchoń, On Fuzzy Measure and Fuzzy Integral, *Fuzzy Information and Decision Processes*:M.M. Gupta and E. Sanchez(eds.), pp.79-86, 1982.
11. H.-J. Zimmermann, *Fuzzy Set Theory - and Its Applications*, Kluwer-Nijhoff Publishing: Boston, 364p, 1985.