8 Lighting technology and daylight use

8.1 Introduction to lighting and daylighting technology

Lighting technology deals primarily with the supply of sufficient, glare-free lighting for workplaces and dwellings. Light, however, also assumes the important function of orientation in the interior and time, and enables reference to the exterior. These qualities are supplied predominantly by daylight and contribute crucially to visual comfort.

The human eye is optimally adapted to the visible spectral range of solar radiation, so that with short-wave solar irradiance there is a higher luminous efficiency per Watt of power than with most types of artificial light. Efficient daylight use thus contributes directly to reducing energy consumption, in particular in administrative buildings (Dudda, 2000).

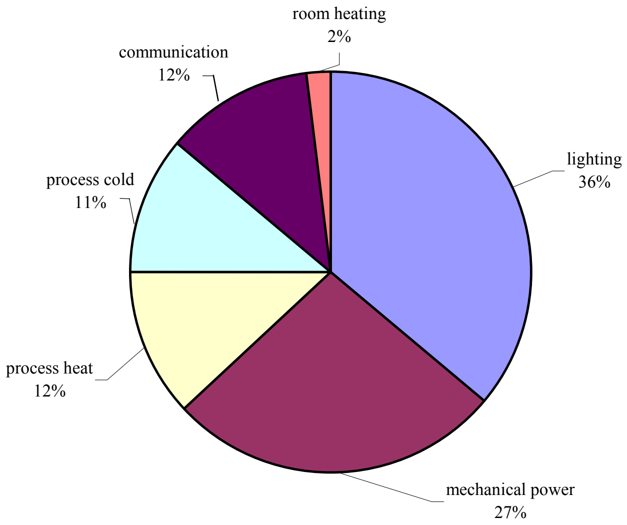


Figure 8.1: Average electricity consumption and lighting contribution for small consumers (trades, service sector and public sector) in Germany, 1998.

The average proportion of lighting in electricity consumption is 36% in an administrative building, compared to only about 5% in the industrial sector. Due to the high luminous efficiency of daylight, the internal thermal loads due to lighting are reduced, and thus also the problem of summer overheating in offices.

Daylight is predominantly used via conventional window openings. Special components are available for glare and sun protection, as well as for light distribution to the depth of the room. In particular in computer workplaces, glare shielding assumes a crucial role in daylight technology. Glare shielding systems should be based on the criteria of brightness reduction, light permeability, visual contact outward and light guidance far into the room. Textile anti-glare blinds reduce the brightness by their partially translucent layers, but cannot redirect the light. Also, with absorption and reflection glass, the decrease in glare luminance is proportional to the light permeability. With glare-reducing blinds, the user has more control. Transparency and visual contact outward can be improved by perforation.

Daylight-guiding elements within the upper window section (mirror-type lamellas, prism systems, etc.) also contribute to the reduction of glare in the window area and to improved illumination-in-depth of side-illuminated rooms. With an overcast sky, adequate illumination cannot be ensured even just 3 m deep into a 3 m high room with a window up to the ceiling. A light-guiding element of 20 cm shifts the area of sufficient illumination to a depth of 4.5 m.

The light guidance of direct radiation to the ceiling as far into the room as possible can be achieved either by fixed mirror-lamellas, prism systems, translucent insulation materials or the like. Cheapest and most widely used are mirror-lamellas divided into two parts, which guide light within the upper section of the window, while the lower area remains closed to reduce glare. Apart from open lamella systems, specially reflecting lamella profiles are also available for the space between panes, consisting of two mirror profiles opposite one another. The light reflected upwards by the lower mirror profile is, with a steep angle of incidence, reflected out from the pane by the mirror in front (sun protection); with a flat angle of incidence it is channelled inwards by the second mirror at the back. While the profiles mainly have a sun-protection function with a high sun position, a light-guiding effect can be obtained when the sun is low. Lamellas between non-ventilated glass panes, however, often cause high glass surface temperatures and thus add to the cooling load of the building.

Prism-profile panels use the total reflection of the light either to block it (sun-protection function) or to channel it. They are usually inserted between the panes. Prism systems are translucent and therefore only used in the skylight area or the overhead area. Sun-protection prisms reduce the sky brightness and thus the glare problem by a factor of 100 even with an overcast sky.

8.1.1 Daylighting of interior spaces

The illuminance level during daylight use in the interior is typically between 2% and 5% of the exterior illuminance, corresponding to a mean illuminance of 200–500 lux (lx) in the interior. The relation of the interior illuminance E_i to the horizontal exterior illuminance E_o is termed the daylight coefficient D.

$$D = \frac{E_i}{E_o} \tag{8.1}$$

A room lit by daylight, for which no perceptible difference in brightness between outside and inside is to exist, must have a daylight coefficient of at least 10%, i.e. illuminances of 1000–3000 lx or more. The visual sensitivity of the eye is almost constant in relation to further brightness increases. The illuminance E is defined by the ratio of the light flux Φ [lumen] and the illuminated surface area A [m²].

$$E = \frac{\Phi}{A} \quad \left[\frac{lm}{m^2} = lx\right] \tag{8.2}$$

In residential buildings, the minimum daylight requirement is essentially characterised by the avoidance of the impression of a dark room, corresponding to daylight coefficients of around 0.9% halfway into the room. For offices in which all workstations are near windows, an illuminance of 300 lx is sufficient; in offices with computer workstations 500 lx is necessary, and for open-plan offices or drawing workstations, 750 lx is required.

Example 8.1

Calculation of the mean illuminance for a 3 m high office of surface area 4 m × 5 m, with a window area of 9 m^2 with 40% total transmission (including the framework proportion). The outside available illuminance on the vertical window area on an overcast day is to be 4500 lux (lx).

Light flux onto the entire window area:	$4500 \text{ lx} \times 9 \text{ m}^2 = 40500 \text{ lm}$
Transmitted light flux into the office:	$40500 \text{ lm} \times 0.4 = 16200 \text{ lm}$
Light flux related to a square metre of office surface:	$\frac{16200 \text{ lm}}{20 \text{ m}^2} = 810 \text{ lx}$

It is not the quantity of light which is the problem, but the unfavourable distribution of the daylight. Purely side-illuminated rooms are characterised by an almost exponential fall in the daylight coefficient, so near to windows glare problems occur, and with increasing room depth illuminances are too low.

For the use of daylight, glazings in the upper window section which enable good deep illumination of the room are particularly favourable. A centrally arranged 1 m high window in a 3 m high room leads to an exponential fall in the daylight coefficient with increasing room depth. An arrangement of the same window height just below the ceiling, on the other hand, increases the daylight coefficient in the room depth. The highest daylight coefficients are obtained with a window front over the entire room height.

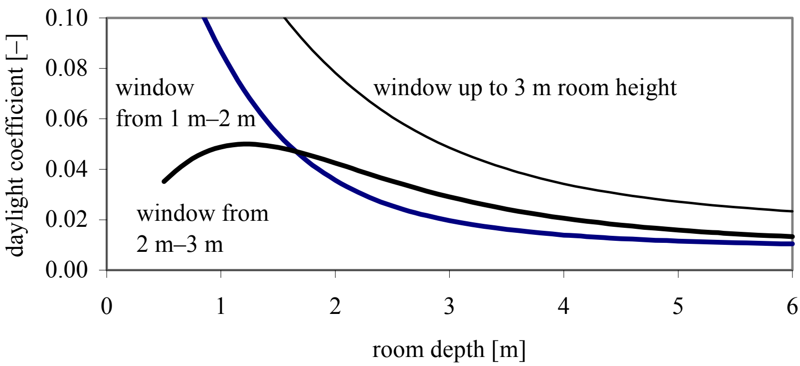


Figure 8.2: Daylight coefficient in the 3 m high side-illuminated room for a window from the floor up to the ceiling or windows in each case 1 m height centrally or within the area below the ceiling.

8.1.2 Luminance contrast and glare

Apart from the supply of sufficient illuminance, the main issues in administrative buildings are problems with workstations which must be illuminated, glare-free and with low contrast. Disturbance effects arise with very uneven distribution of the luminance, which in the worst cases impair the visual function (physiological or disability glare). The contrast between the object and the background luminance is reduced due to scattered luminance entering the eye sideways and the eye functions, such as distinction between nearby objects or contrast sensitivity, are reduced. The luminance is defined by the light flux, which is radiated from a surface into a spatial angle. At luminance levels above 10^4 cd/m^2 absolute glare occurs, where objects can no longer be distinguished within the visual field.

With psychological or discomfort glare a sensation of disturbance is caused without a real reduction in the eye's capability. The contrast relation between object luminance and surrounding luminance, which is experienced as pleasant, depends on the absolute height of the surrounding luminance and thus on eye adaptation: the higher the surrounding luminance, the lower the subjectively perceived brightness of an object luminance. This relative glare due to too large luminance contrasts in the visual field, which occurs most frequently in lighting technology, can therefore be reduced by raising the average luminance.

Average luminances of a monitor are approximately 100 cd/m^2, while the figure for a window is about 4500 cd/m^2, even with an overcast sky. The luminance contrast of 1:45 is so far over the acceptable contrast of 1:15 that glare reduction measures have to be taken. Ideal luminance contrasts between the visual task and darker immediate surroundings are only 3:1, with more distant surroundings 10:1 and should not exceed values over 20–40:1.

8.2 Solar irradiance and light flux

Optical radiation is part of the electromagnetic radiation with wavelengths above 1 nm (upper boundary of Röntgen radiation) up to 1 mm (lower boundary of radio waves). The radiated power P transported by the electromagnetic waves can be calculated using the Poynting vector $\vec{S}$ as a cross-product of electrical and magnetic field vectors $\vec{E}$ and $\vec{H}$, integrated over a closed surface A surrounding the radiating source.

$$\vec{S} = \vec{E} \times \vec{H}$$
$$P = \oint_A \vec{S}\, d\vec{A} \tag{8.3}$$

The solar irradiance covers a wide spectral range from about 0.3–4 µm due to the high temperature of the sun's surface. The measured radiated power per square metre is the irradiance G [W/m²] and is a so-called radiometric quantity. As the human eye is only sensitive to a small spectral range of 0.38–0.78 µm, the radiometric units have to be converted to eye sensitivity weighted photometric quantities. The light flux Φ [lumen] as a photometric quantity is the sensitivity weighted power. If the light flux is determined per square metre of surface, the illuminance E_v [lumen/m²] is obtained.

The lighting requirements at workstations are between 300 and 1000 lumen per square metre depending on the visual aspect of the work (EN 12464: lighting of workspaces in interior spaces). The illuminance describes the light flux Φ onto the work surface which results from the irradiated power, be it solar radiation or artificial light, weighted with the spectral sensitivity of the eye.

8.2.1 Physiological–optical basics

The human eye is an almost spherical object of 24 mm diameter and about 26 mm length. The pigmented iris functions as a lens with varying opening diameters between 2–8 mm, depending on the object's distance and average luminance. The maximum sensitivity of the retina cells during the day is 555×10^{-9} m (555 nm), i.e. in the green colour area, falling to zero in the short-wave area below 380 nm and in the long-wave area above 780 nm. This spectral area is called visible light. In some international norms the spectral boundaries for the visible part are rounded to 400 nm to 800 nm.

About 6.5 million cone cells, responsible for colour vision, are concentrated in the centre of the retina and contain three different dye stuffs with maximum spectral sensitivities at 419 nm, 531 nm and 558 nm. During illumination the dyes chemically change (isomerisation) and need around six minutes for regeneration. The rod cells, which are responsible for night vision, have a maximum spectral sensitivity at 496 nm, are extremely light-sensitive due to very large numbers (120 million) and coupling of different cells (up to 32 cells on one nerve end). The pigment regeneration time is around 30 minutes.

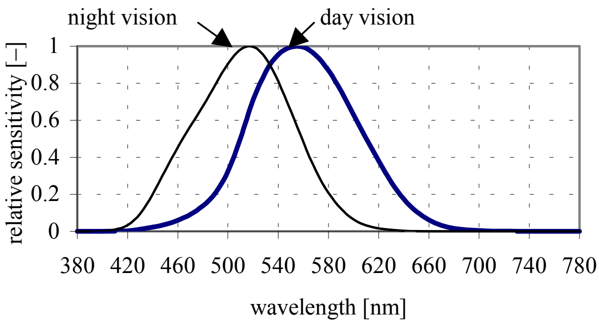


Figure 8.3: Relative spectral sensitivity $V(\lambda)$ of the eye for day vision with retina cells termed cones and for night vision with rods.

8.2.2 Photometric radiation equivalent

To convert an energetic radiant flux Φ_e (index *e* for energetic) in units of Watt into a light flux Φ_v (index *v* for visual) in lumen, the relative eye sensitivity $V(\lambda)$ at a given wavelength λ must be known, as well as an absolute conversion factor between the units, called the photometric radiation equivalent k_{max}. For each wavelength λ, the light flux $\Phi_{v,\lambda}$ is converted from the spectral radiating power $\Phi_{e,\lambda}$:

$$\Phi_{v\lambda} = k_{\max} V(\lambda)\Phi_{e\lambda}$$
$$k_{\max} = \frac{maximum\ light\ flux[lm]}{radiative\ power[W]} \tag{8.4}$$

The conversion factor k_{max} results from the definition of the photometric SI-unit candela [cd], which describes the light flux Φ per spatial angle Ω (in steradiant [sr]), i.e. the luminous intensity I:

$$I = \frac{\Phi}{\Omega} \qquad [cd] = \left[\frac{lm}{sr}\right] \tag{8.5}$$

For the definition of the luminous intensity I in candela, a black cavity emitter (platinum with a melting temperature of 2044.9 K) was historically used, whose spectral radiant emittance can be calculated using Planck's law of radiation. A candela was defined as the light flux per spatial angle, which 1/60 cm^2 of the surface of the black cavity emitter emits. The emitted power of this surface, calculated using Planck's law of radiation, is 1/673 W.

Today a candela is defined as the light flux of a monochromatic radiation source with the somewhat smaller power of 1/683 W, which radiates with a frequency of 540×10^{12} Hz, i.e. 555 nm, into a spatial angle of one steradiant. An emitted power of 1 W per steradiant thus results in a luminous intensity of 683 cd, corresponding to 683 lumen per steradiant.

Since the wavelength 555 nm corresponds to the maximum relative eye sensitivity, the maximum photometric radiation equivalent is today k_{max} = 683 lm/W.

Example 8.2

Calculation of the light flux $\Phi_{v,\lambda}$ for a light source of 10 W power with the wavelength λ = 633 nm (red helium-neon laser) and with λ = 588 nm (yellow resonance line of the sodium vapour low pressure lamp). The relative spectral sensitivity is V(633nm) = 0.25 or V(588nm) = 0.77. Based on Equation (8.4) the light flux is

$$\Phi_{v,633nm} = 683\frac{lm}{W}\times 0.25\times 10W = 1707.5lm \quad \text{or} \quad \Phi_{v,588nm} = 683\frac{lm}{W}\times 0.77\times 10W = 5258lm.$$

To determine the light flux and finally the illuminance on a surface of any radiation source, the entire spectrum must be weighted with the spectral sensitivity of the eye.

A single value of the photometric radiation equivalent for a given spectrum (of the sun or a lamp) is obtained by converting the energetic radiant flux for each wavelength into a light flux, integrating it over the visible area of the spectrum and standardising the value on the integrated total radiation flow of the source of light. A source of light is thus characterised by the following radiation equivalent:

$$k = \frac{\Phi_v}{\Phi_e} = \frac{k_{\max}\int_{380nm}^{780nm}\Phi_{e\lambda}V(\lambda)d\lambda}{\int_0^{\infty}\Phi_{e\lambda}d\lambda} \tag{8.6}$$

For an evenly overcast sky a radiation equivalent of k = 115 lm/W results, based on DIN 5034. The radiation equivalent varies, however, with cloud thickness, the vapour content, the height of the sun etc., and can be between 90 and 120 lm/W. The diffuse radiation of the clear sky can assume values over 140 lm/W; for the direct component values, between 50 and 120 lm/W have been measured. If one relates the light flux to a square metre of recipient surface, the illuminace E is obtained with the unit lumen per square metre (abbreviated to lux [lx]).

$$E = \frac{\Phi_v}{A} \quad \left[\frac{lm}{m^2} = lx\right] \tag{8.7}$$

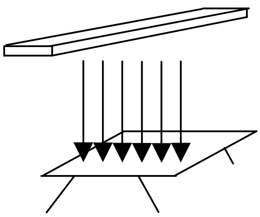


Figure 8.4: Light flux and illuminance on a horizontal surface.

Table 8.1: Photometric radiation equivalent

Light source	*Photometric radiation equivalent k* [lm/W]
evenly overcast sky	115
diffuse radiation of the clear sky	140
direct component of the clear sky	50–120

During full exposure to the sun (1000 W/m^2), with a photometric radiation equivalent of 120 lm/W, an exterior lighting strength of 120 000 lx is achieved on a horizontal surface.

$$\Phi_v = k\Phi_e = 120\frac{\text{lm}}{\text{W}} \times 1000\frac{W}{m^2} = 120\,000\frac{lm}{m^2} = 120\,000 lx$$

The annual average value of the exterior lighting strength is around 10 000 lx during the day.

8.2.3 *Artificial light sources*

50% of all artificial light sources are fluorescent lamps, consisting of a glas tube with two wolfram electrodes with an emitting surface at the ends. The gas filling is a mixture of rare gases such as argon or krypton (at a pressure of 70 Pa) and Hg (at 1 Pa pressure). The Hg atoms emit due to the electric discharge in the ultraviolet region (mainly at 185 and 254 nm) and excite fluorescent material on the tube walls (halogen phosphate or others) to give a continuous visible spectrum. Fluorescent tubes are not well suited for external applications as the light flux decreases to 25% at –10°C of its normal value at 20°C. Compact fluorescent lights are folded fluorescent lights with lifetimes over 10 000 h. Fluorescent lights have photometric radiation equivalents of 50–88 lm/W, clearly lower than daylight, and their power is obviously supplied by electricity. Electrical lightbulbs with hot wolfram wires provide a very low light flux of 6–16 lumen per Watt power. Halogen lights are slightly better, as evaporated wolfram from the hot wire no longer deposes on the light bulb walls. Furthermore, the pressure and operating temperatures are higher within the glass tube. At low power levels, light-emitting diodes start to provide good luminous efficiencies at a wide spectral range.

Table 8.2: Photometric radiation equivalent of artificial light sources

Light source	*Description*	*Power P* [W]	*Photometric radiation equivalent k* [lm/W]
(electric) light bulb	Wolfram glowing wire 2800K	15–200	6–16
halogen light	Wolfram wire at 3000K with higher gas pressure and halogen addition, quartz glass	15–200	8–20
fluorescent lamp	argon/krypton + Hg filled	18–58	50–88
compact fluorescent lamps	as above	5–55	50–88
sodium vapour lamp	widening of double emission line at 589/90 nm at high pressure	180	150
Light emitting diodes		40–80 mW	20–100
LED White	AlInGaN/Phosphor	20 mA, 4V	20
LED Green	506 , 530 , 571 nm, AlInGasN		30, 54, 14
LED Red	611 , 658 nm , AlInGaP	20 mA, 2V	102, 38

8.3 Luminance and illuminance

The light flux onto a surface and thus the illuminance E_v, defined as light flux per square metre, can in the simplest case be calculated, at a given solar irradiance, directly from the radiating power and the photometric radiation equivalent. For the solar irradiance one usually proceeds from an isotropic diffuse energy distribution. Since luminance distributions rising to the zenith are mostly used in lighting engineering, the light flux onto a randomly inclined surface must be calculated from the contributions of the luminance of different spatial angle areas of the sky’s diffuse radiation. In the interior too, the light flux onto a recipient surface A_r consists of the total of the light fluxes of radiation-emitting or radiation-reflecting sender surfaces A_s at different spatial angles.

The task now is to integrate the light fluxes received by a surface over all spatial angle areas and thus to calculate the illuminance E for different luminance distributions. For this, a spatial angle $d\Omega$ is constructed by a two-dimensional surface element dA, which is at distance r from the sending or receiving surface.

$$d\Omega = \frac{dA}{r^2} \tag{8.8}$$

The unit of the three-dimensional spatial angle is steradiant [sr] and corresponds to the definition of a two-dimensional angle in arc measure [rad], which is constructed by a circular arc with radius r. A simple spatial angle is, for example, given by a surface A_{sp} on a sphere, which is calculated as a function of the sphere radius r and of the height h of the cut-out sphere segment.

$$\Omega = \frac{A_{sp}}{r^2} = \frac{2\pi r\, h}{r^2} = \frac{2\pi r^2 (1-\cos\theta)}{r^2} = 2\pi(1-\cos\theta) \tag{8.9}$$

Here the angle θ describes half the opening angle of the spherical cone. The full spatial angle of a hemisphere with half an opening angle $\theta = 90°$ is therefore 2π, and of a sphere 4π.

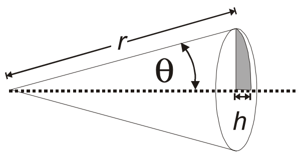


Figure 8.5: Spherical cone spatial angle.

If the light flux of a luminescent or radiation-reflecting surface is to be calculated from a certain spatial angle, first the orientation of the surface relative to the observed radiant emittance direction must be determined. If the radiant emittance is considered not as perpendicular to the sender surface element dA_s, but at an angle θ_s, effectively only the smaller surface $dA_s \cos\theta_s$ is visible.

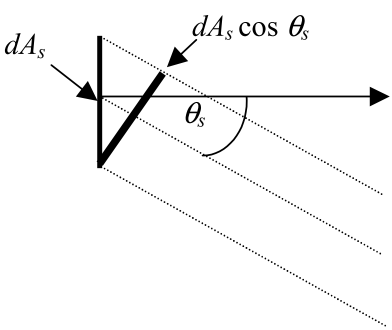


Figure 8.6: Effective size of the radiant two-dimensional element towards θ_s.

The light flux $d\Phi$, which a two-dimensional area dA_s sends at angle θ_s into a spatial angle $d\Omega_1$, is termed luminance L and describes the brightness of the surface.

$$L = \frac{d^2\Phi}{d\Omega_1 dA_s \cos\theta_s} = \frac{dI}{dA_s \cos\theta_s} \quad \left[\frac{lm}{sr\,m^2} = \frac{cd}{m^2}\right] \tag{8.10}$$

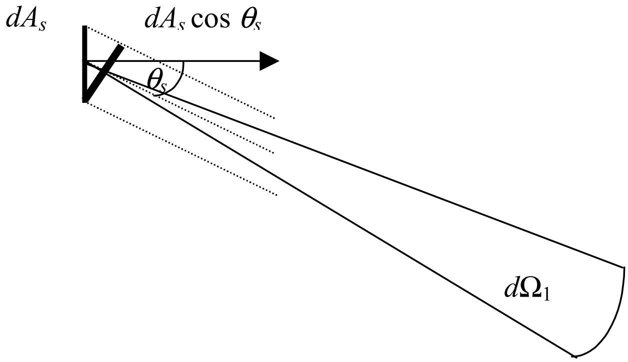


Figure 8.7: Luminance of a two-dimensional area dA_s radiating at an angle θ_s into a spatial angle $d\Omega_1$.

The light flux $d\Phi$ [lm] into a spatial angle $d\Omega$ [sr] can be replaced by the luminous intensity *I*. In many cases the luminance is independent of the angle θ_s, i.e. the surface appears equally bright independent of the viewing angle. Such surfaces are termed Lambert radiators. From Equation (8.10) it follows that the luminous intensity *dI* in Lambert radiators must decrease with the cosine of the angle θ_s: $I(\theta) = I(0)\cos\theta_s$. Rough, diffusely reflecting surfaces such as gypsum walls, paper etc. behave in good approximation like Lambert radiators.

In order to obtain the light flux of a sender surface A_s into a spatial angle Ω_1, integration must take place at a given brightness of this surface over the differential spatial angles $d\Omega_1$, with consideration given to the effective surface size through the cosine of the angle θ_s between the surface and the respective spatial angle element.

$$\Phi = \int_{\Omega_1} \int_{A_s} L dA_s \cos\theta_s d\Omega_1 \tag{8.11}$$

The spatial angle $d\Omega_1$ is constructed by the receiving two-dimensional surface dA_r, for example the work surface, or even the pupil opening surface of the eye. Of course the receiving surface dA_r also need not be perpendicular to the spatial angle regarded in each case, so effectively only $dA_r \cos\theta_r$ is available for radiation reception.

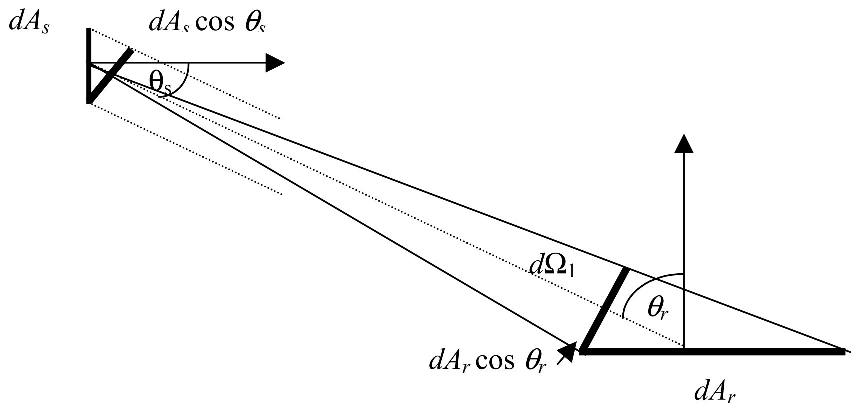


Figure 8.8: Light flux of a radiating two-dimensional surface dA_s onto a recipient surface dA_r.

The light flux falling from the radiating sender surface A_s onto the receiving surface A_r thus results in:

$$\Phi_{s\to r} = \int_{\Omega_1} \int_{A_s} L_s dA_s \cos\theta_s d\Omega_1 = \int_{A_r} \int_{A_s} L_s dA_s \cos\theta_s \frac{dA_r \cos\theta_r}{r^2} \tag{8.12}$$

If, conversely, the view is from the receiving surface A_r from the light flux over a spatial angle Ω_2 which covers the radiating surface A_s, for reasons of energy conservation the received light flux must correspond to the emitted light flux and the so-called Basic Law of Photometry is obtained:

$$\Phi_{r\leftarrow s} = \int_{\Omega_2} \int_{A_r} L_s dA_r \cos\theta_r d\Omega_2 = \int_{A_s} \int_{A_r} L_s dA_r \cos\theta_r \frac{dA_s \cos\theta_s}{r^2} = \Phi_{s\to r} \tag{8.13}$$

The cosine of the angle θ_r refers to the normal of the receiving surface dA_r and the spatial angle $d\Omega_2$, which is constructed by a differential two-dimensional surface area dA_s.

The illuminance E_r on the receiving surface dA_r for any sender surfaces dA_s with luminance L_s results from Equation (8.13).

$$E_r = \frac{d\Phi_r}{dA_r} = \int_{\Omega_2} L_s \cos\theta_r d\Omega_2 = \int_{A_s} L_s \cos\theta_r \frac{dA_s \cos\theta_s}{r^2} \tag{8.14}$$

Example 8.3

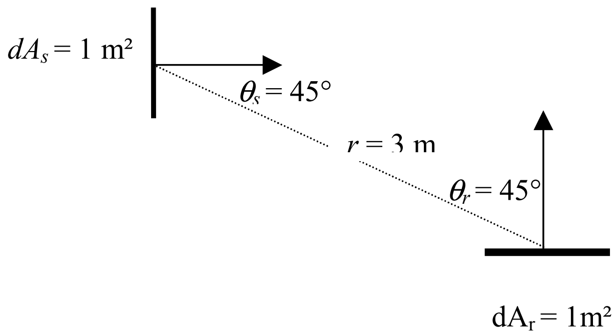
Calculation of the illuminance on a horizontal work surface of 1 m^2 illuminated by a 1 m^2 vertical window with a luminance of 4500 cd/m^2 at an average angle of 45° and 3 m distance (without consideration of the photometric boundary distance).

If the differentials are set as differences, Equation (8.14) is simplified to:

$$E_r \approx \frac{\Delta\Phi_r}{\Delta A_r} = L_s \cos\theta_r \frac{\Delta A_s \cos\theta_s}{r^2}$$

Both the angle of the radiating and of the receiving surface normal are 45° to the spatial angle.

$$E_r = \frac{\Delta\Phi_r}{\Delta A_r} = 4500\frac{\text{cd}}{\text{m}^2}\cos 45° \frac{1\text{m}^2 \cos 45}{(3\text{m})^2} = 250 lx$$



8.3.1 Luminance and adaptation of the eye

The luminance of sources of light or radiation-reflecting surfaces vary over a very wide range of values.

Table 8.3: Luminances of sources of light.

Light source	*Luminance* [cd/m²]
Sun, values depending on sun height	600 000 to 1 600 000 000
Clear sky	2000–12 000
Overcast sky	1000–6000
Lightbulb	20 000–50 000
Compact fluorescent lamp	9000–25 000
Candle flame	7000
Paper in a well lit office	250
Computer screen	20–200
Lower limit of light sensitivity	10^{-5}

The human eye adapts to the mean outside brightness L_o in the visual field. This is termed adaptation, and takes place via modification of the pupil surface A_p, which expands with falling brightness from 2 mm diametre to a maximum of about 8 mm during brightness adaptation (Henschel, 2002). The pupil surface, which varies as a logarithmic function of the luminance, enables adjustment of the penetrating light flux by a factor of 16.

The illuminance reaching the eye is let through with transmittance τ_p and strikes the retina. Brightness-sensitivity is determined by the number of photons which strike a two-dimensional element of the retina, i.e. by the luminance on the retina. This is obtained from the mean luminance of the visual field, which is seen by the spatial angle $d\Omega_1$ of the pupil surface A_p at a distance r between the lens and retina $d\Omega_1 = dA_p / r^2$. Between the spatial angle and the surface normal of the retina recipient surface dA_r, the angle θ_r is zero, so the cosine term for the recipient surface is omitted.

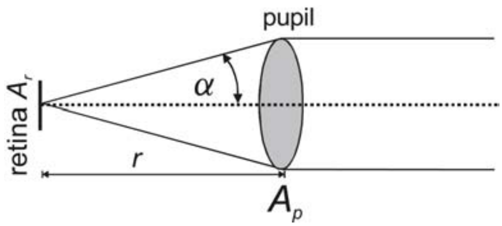


Figure 8.9: Cross-section of the eye with distance *r* between the lens and retina.

$$E_{retina} = \frac{d\Phi}{dA_r} = \int_{\Omega_1} L_o \tau_p \, d\Omega_1 = L_o \tau_p \frac{A_p}{r^2} = \text{const}\, L_o A_p \tag{8.15}$$

Besides the opening area of the pupil (A_p), which depends on the adaptation status of the eye, the brightness sensitivity is therefore determined solely by the luminance striking the eye. The luminance is thus the most important quantity in lighting engineering.

8.3.2 Distribution of the luminous intensity of artificial light sources

The luminance of lamps or lights is rarely constant in all directions in space. Luminous intensity distributions are indicated by the manufacturers in polar diagrams in different sections. Usually the absolute luminous intensity in candelas refers to a fixed lamp light-flux.

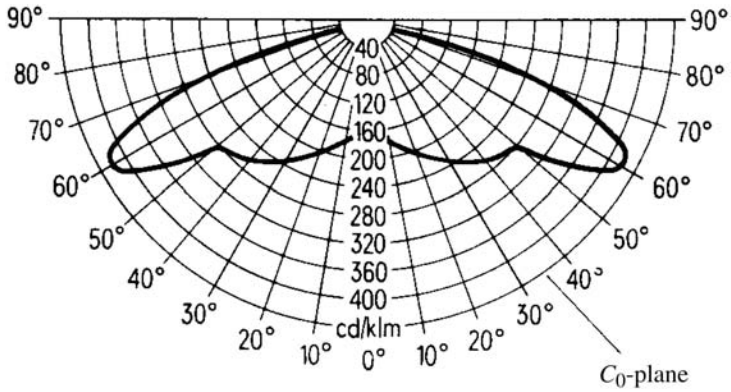


Figure 8.10: Light distribution curve in the C_0-plane, which is perpendicular to the radiating surface (in candelas per Kilolumen).

If in Equation (8.14) the luminance of the sender surface is replaced by the luminous intensity per unit area with

$$L_s = \frac{d^2\Phi_s}{d\Omega_1 dA_s \cos\theta_s} = \frac{dI(\theta)}{dA_s \cos\theta_s} \tag{8.16}$$

the result for the illuminance is

$$E_r = \frac{d\Phi}{dA_r} = \int_{\Omega_2} L_s \cos\theta_e d\Omega_2 = \int_{A_s} \frac{dI(\theta_s)}{dA_s \cos\theta_s} \cos\theta_r \frac{dA_s \cos\theta_s}{r^2} = \int_{A_s} \frac{dI(\theta_s)}{r^2} \cos\theta_r \tag{8.17}$$

with θ_r as the angle between the receiving surface and solid angle $d\Omega_2$.

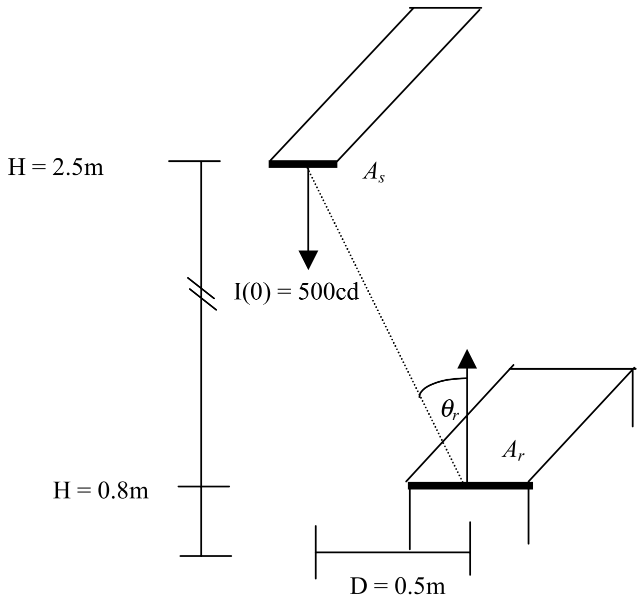
With the usual constancy of the luminous intensity over the radiating surface A_s, the photometric distance law is obtained:

$$E_r = \frac{d\Phi}{dA_r} = \frac{I(\theta_s)}{r^2} \cos\theta_r \tag{8.18}$$

However, the functional characteristic of the illuminance, which decreases in inverse proportion to the square of the radius, only applies starting from the so-called photometric minimum distance, which is about 10 times as large as the largest linear dimension of the lighting surface.

Example 8.4

Calculation of the illuminance on an 80 cm high work surface lit by a Lambert emitting lamp at a height of 2.5 m and a lateral distance of 0.5 m with a luminous intensity *I(0)* = 500 cd.



The angle θ_r between the recipient-surface normal and the light is obtained from the lamp height and lateral distance,

$$\theta_r = \arctan\left(\frac{0.5m}{1.7m}\right) = 16.4°$$

the distance is 1.77 m. The luminous intensity of the light toward the recipient surface is

$$I(0)\cos\theta_s = 500\ cd \times \cos 16.4 = 480\ cd$$

so the result is an illuminance of

$$E = \frac{I(\theta_s)\cos\theta_r}{r^2} = \frac{480\ cd \times \cos(16.4°)}{(1.77\ m)^2} = 147\,lx$$

8.3.3 Units and definitions

An overview of the definitions and units in lighting technology is given in Table 8.4.

Table 8.4: Units in lighting technology.

Photometric unit	*Symbol*	*Definition*	*Unit*
Luminous energy	Q_v	$Q_v = \int \Phi_v dt$	lm s
Luminous flux	Φ_v	–	lm
Luminous exitance	M_v	$M_v = \frac{d\Phi_v}{dA_1}$	lm m^{-2}
Luminous intensity	I_v	$I_v = \frac{d\Phi_v}{d\Omega}$	cd
Luminance	L_v	$L_v = \frac{d\Phi}{d\Omega dA_1 \cos\varepsilon_1}$	cd m^{-2}
Illuminance	E_v	$E_v = \int_{2\pi\ sr} L_v \cos\varepsilon_1 d\Omega$	lx
Luminous exposure	H_v	$H = \frac{dQ}{dA_2}$	lx s

Artificial and daylight sources can be characterised by the following efficiencies.

Table 8.5: Efficiency definitions in lighting technology.

Efficiency	*Symbol*	*definition*	*Unit*
Radiative efficiency	η_e	Φ_e/P	– (W/W)
Luminous efficiency	η_v	Φ_v/P	lm/W
Photometric radiation equivalent	K	$\frac{\Phi}{\Phi_e}$	lm/W
Optical efficiency	O	$\int \frac{d\Phi}{d\lambda} d\lambda \Big/ \Phi_e$	–
Visual efficiency	V	$\frac{\Phi}{k_m \Phi_e}$	–

8.4 Sky luminous intensity models

To calculate the daylight distribution in a room, models of the luminous intensity distribution of the sky are required. From the luminous intensity distribution, the effective surface and the solid angle, the light flux onto any recipient surfaces, for example windows, can then be calculated.

As the simplest model, the illuminance of the sky is set as constant, corresponding to an isotropic sky model. The density of light is calculated using Equation (8.14):

$$E_r = \frac{d\Phi_r}{dA_r} = \int_{\Omega_2} L_s \cos\theta_r d\Omega_2$$

The solid angle $d\Omega_2$ is selected using Equation (8.9) as a spherical cone with half opening angle θ, with $\theta = 0$ corresponding to the surface-normal of a horizontal recipient surface. With a horizontal recipient surface, the opening angle is equal to the angle of incidence θ_r and the following density of light results:

$$E_r = \int_{\Omega_2} L_s \cos\theta_r d\left(2\pi\left(1-\cos\theta_r\right)\right) = 2\pi L_s \int_0^{\pi/2} \cos\theta_r \sin\theta_r d\theta_r = 2\pi L_s \left.\frac{\sin^2\theta_r}{2}\right|_0^{\pi/2} = \pi L_s$$

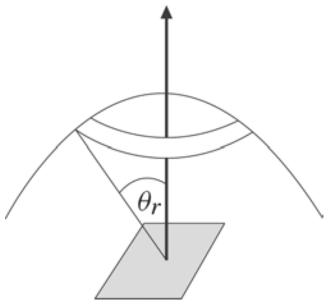


Figure 8.11: Projection of the sky dome with constant luminance onto a horizontal recipient surface.

The sky luminance can thus be calculated from the horizontal irradiance, if the radiating power is converted via the photometric radiation equivalent into a density of light.

Example 8.5

Calculation of the isotropic luminance of the sky for a horizontal irradiance of the overcast sky of 100 W/m^2 and a photometric radiation equivalent of 115 lm/W.

The illuminance on the horizontal surface is 11 500 lm/m^2. The illuminance then produces a sky luminance of

$$L_s = \frac{E_r}{\pi} = 3660 \frac{cd}{m^2}$$

Apart from the isotropic luminance distribution, standardised distributions of the international lighting engineering commission (CIE) for a clear and an overcast sky are used. The overcast, so-called Moon and Spencer sky, is characterised by a rise in the luminance L_α with the elevation angle α, with the zenith luminance L_z assuming three times the value of the horizon luminance,

$$L_\alpha = \frac{1}{3} L_z \left(1 + 2\sin\alpha\right) \tag{8.19}$$

and the zenith luminance with an overcast sky is a function of the sun height angle α_s:

$$L_z = \frac{9}{7}\pi \left(300 + 21000 \sin\alpha_s\right) \tag{8.20}$$

For a horizontal surface, the conical solid angle can be used again and the illuminance can be represented as a function of the angle of incidence θ_r on the horizontal (which corresponds to the zenith angle) instead of the elevation angle α.

$$\begin{aligned}
E_{r,h} &= \int_0^{\pi/2} \left(\frac{1}{3} L_z \left(1 + 2\cos\theta_r\right) \right) \cos\theta_r d\left(2\pi\left(1 - \cos\theta_r\right)\right) \\
&= \frac{2\pi}{3} L_z \int_0^{\pi/2} \left(1 + 2\cos\theta_r\right) \cos\theta_r \sin\theta_r d\theta_r \\
&= \frac{2\pi}{3} L_z \left(\int_0^{\pi/2} \cos\theta_r \sin\theta_r d\theta_r + \int_0^{\pi/2} 2\cos^2\theta_r \sin\theta_r d\theta_r \right) \\
&= \frac{2\pi}{3} L_z \left(\frac{1}{2}\sin^2\theta_r \Big|_0^{\pi/2} + 2\left(-\frac{1}{3}\cos^3\theta_r \Big|_0^{\pi/2} \right) \right) = \frac{L_z \pi}{3}\left(1 + \frac{4}{3}\right) = L_z \frac{7}{9}\pi
\end{aligned} \tag{8.21}$$

For vertical surfaces, the solid angle must be selected in such a way that the angles of incidence on the vertical, which change with the azimuth, can be taken into account. For example, two-dimensional elements on a ring zone of the sky hemisphere can be used, indicated as a function of the zenith and azimuth angles.

In spherical polar coordinates, a two-dimensional surface in the sky hemisphere with radius r is given by the product of the sides $r\, d\theta$ on the meridian and $r \sin\theta\, d\gamma$ on the parallel circle, with θ corresponding to the zenith angle θ_z. The solid angle $d\Omega$ is thus:

$$d\Omega = \frac{r \sin\theta_z d\theta_z \; r d\gamma}{r^2} = \sin\theta_z d\theta_z \; d\gamma \tag{8.22}$$

and the solid angle of a ring zone

$$\Omega_z = \int_{\gamma=0}^{2\pi} \int_{\theta_1}^{\theta_2} \sin\theta d\theta d\gamma = 2\pi\left(\cos\theta_2 - \cos\theta_1\right) \tag{8.23}$$

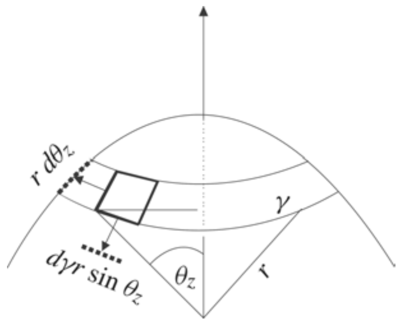


Figure 8.12: Solid angle element of a ring zone on a sphere.

The angle of incidence θ_r between the solid angle element of the sky with the coordinates zenith angle θ_z and azimuth γ and the vertical surface normal (angle of inclination β = 90°) is calculated with the known sun-position equations. Since the luminance does not depend on the azimuth, the vertical surface can be arbitrarily oriented in azimuth direction. For simplification, the surface azimuth $\gamma_s = 0$ is selected.

$$\cos\theta_r = \cos\theta_z \cos\underbrace{\beta}_{90°} + \sin\theta_z \underbrace{\sin\beta}_{1} \cos\left(\gamma - \underbrace{\gamma_s}_{0}\right) = \sin\theta_z \cos\gamma \tag{8.24}$$

The integration over the solid angle is carried out with the integration limits of the zenith angle from 0 to π/2 and of the azimuth of −π/2 to +π/2, since only half the sky is seen from the vertical surface.

$$\begin{aligned} E_v &= \int_{d\Omega} L(\theta_z)\cos\theta_r d\Omega = \int_{-\pi/2}^{\pi/2} \int_0^{\pi/2} \left(\frac{1}{3}L_z\left(1+2\cos\theta_z\right)\right) \underbrace{\sin\theta_z \cos\gamma_{sky}}_{\cos\theta_r} \underbrace{\sin\theta_z d\theta_z d\gamma}_{d\Omega} \\ &= \frac{1}{3}L_z\left(\int_{-\pi/2}^{\pi/2} \int_0^{\pi/2} \sin^2\theta_z d\theta_z \cos\gamma d\gamma + \int_{-\pi/2}^{\pi/2} \int_0^{\pi/2} 2\cos\theta_z \sin^2\theta_z d\theta_z \cos\gamma d\gamma\right) \\ &= \frac{1}{3}L_z\left(\left(\underbrace{\frac{1}{2}\theta_z}_{\pi/4} - \underbrace{\frac{1}{4}\sin\left(2\theta_z\right)}_{0}\right)\Bigg|_0^{\pi/2} \underbrace{\sin\gamma\Big|_{-\pi/2}^{\pi/2}}_{2} + \underbrace{2\left(\frac{1}{3}\sin^3\left(\theta_z\right)\right)\Bigg|_0^{\pi/2}}_{1/3} \underbrace{\sin\gamma\Big|_{-\pi/2}^{\pi/2}}_{2}\right) \\ &= \frac{1}{3}L_z\left(\frac{\pi 2}{4} + 2\frac{1}{3}2\right) = L_z\pi\underbrace{\left(\frac{1}{6} + \frac{4}{9\pi}\right)}_{0.308} \end{aligned} \tag{8.25}$$

In contrast to the isotropic sky model, in which from a vertical surface exactly half of the horizontal illuminance is seen, in the Moon and Spencer sky model only 40% of the horizontal value is obtained.

$$\frac{E_{r,v}}{E_{r,h}} = \frac{L_z \pi \left(\frac{1}{6} + \frac{4}{9\pi} \right)}{L_z \pi 7/9} = 0.4 \tag{8.26}$$

The solid angle element of the spherical ring zone $d\Omega = \sin\theta_z d\theta_z \, d\gamma$ can (as an alternative to the spherical cone) also be used to calculate the horizontal illuminance, with of course the same results being obtained as with the conical solid angle. This sphere ring zone solid angle is used to calculate daylight coefficients, since in this way integration can take place over limited azimuth ranges of windows.

$$\begin{aligned} E_h &= \int_{-\pi/2}^{\pi/2} \int_0^{\pi/2} \underbrace{\left(\frac{1}{3} L_z \left(1 + 2\cos\theta_z\right) \right)}_{L} \underbrace{\cos\theta_z}_{\cos\theta_r} \underbrace{\sin\theta_z d\theta_z d\gamma}_{d\Omega} \\ &= \frac{L_z}{3} \left(\int_{-\pi/2}^{\pi/2} d\gamma \left(\frac{1}{2} \sin^2(\theta_z) \right) \Bigg|_0^{\pi/2} + \int_{-\pi/2}^{\pi/2} d\gamma_{sky} \left(-\frac{2}{3} \cos^3(\theta_z) \right) \Bigg|_0^{\pi/2} \right) \\ &= L_z \frac{7}{9} \pi \end{aligned} \tag{8.27}$$

8.5 Light measurements

The human eye is capable of comparing illuminances of adjacent surfaces with an accuracy of about 2%. This was quantitatively first used in the so-called grease photometer of Bunsen, where luminous intensities were determined by shining two light sources from opposite sides on a paper screen partly covered with grease. If one light source has a known light intensity I_1, the other light intensity I_2 can be determined by varying the distance d from the paper surface, until no difference in illuminance E is detected by the eye.

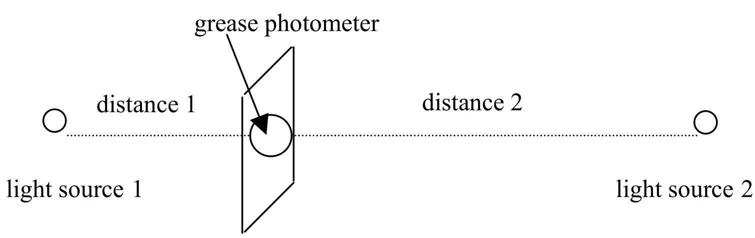


Figure 8.13: Determination of light intensities by illuminance comparison on a greasy screen.

$$E = \frac{I_1}{d_1^{\,2}} = \frac{I_2}{d_2^{\,2}}$$

Today’s photometric receivers measure light or radiation either thermally (thermocouples or thermopiles) or by photo-electric techniques. The most important sensors are silicium photodiodes, which at short-circuit current operation linearily measure irradiance over about 6 orders of magnitude. Very low light fluxes can be measured with a photomultiplier tube.

Charged coupled devices within CCD cameras have up to 2048 × 2048 individual sensors organised in a matrix, where electrons are generated within the sensor pixels proportionally to the illuminance level and time: $n_{electrons} \propto \int E dt$. At a frequency of 20 MHz the whole matrix can be analysed within fractions of a second. The dynamic range of CCD receivers is very large. CCD cameras can be used to measure luminance or spectral distributions of large areas.

To provide the correct visible spectrum to the detector, either integral filtering, combining a range of different filters completely covering the sensor surface (f_1-error is 1.5%), or partial filtering, combining different filters only partially covering the sensor (1% f_1-error) can be used. The f_1-error is an energy-weighted error using the spectral sensitivity of the eye and a standardised light source (at 2856 K) divided by the energy-weighted measured curve of the filters. If the incident light flux is not perpendicular to the detector surface, additional reflection losses occur, which can be compensated by round domes or additional transmitting surfaces on the detector sides. Further measurement errors are due to linearity, temperature coefficients etc.. The best photometric receivers have a total error of 3% (highest accuracy class) up to 20% (lowest accuracy class).

8.6 Daylight distribution in interior spaces

With the asymmetrical luminance distribution of the Moon and Spencer sky, the daylight distribution in the interior can now be calculated. First, as an overview, an illustration will be made of which zenith angle areas the main part of the light-flux falls from, onto a vertical or horizontal glazing surface, in order to make simple estimations of the depth illumination of rooms.

Using Equation (8.25) the illuminance on a vertical glazing for a zenith angle range of $\theta_{z,1}$ to $\theta_{z,2}$ can be calculated as follows (angles in arc measure):

$$E_v\Big|_{\theta_{z,1}}^{\theta_{z,2}} = \frac{1}{3} L_z \left(\left(\theta_{z,2} - \theta_{z,1}\right) - \frac{1}{2}\left(\sin\left(2\theta_{z,2}\right) - \sin\left(2\theta_{z,2}\right)\right) + \frac{4}{3}\left(\sin^3\left(\theta_{z,2}\right) - \sin^3\left(\theta_{z,1}\right)\right) \right) \quad (8.28)$$

The luminance falls with rising zenith angle, but the average angles of incidence onto the vertical surface become smaller and the solid angles of the spherical ring zones with constant zenith angle steps (for example 15°) likewise become larger with rising zenith angle. All three effects taken together lead to the fact that despite the highest luminance in the zenith, the light-flux from the high zenith angle areas is most relevant.

Zenith angle [°]	*Luminance ratio* L/L_z [–]	*Normalised vertical illuminance* E_v/L_z [sr]
0–15	0.99	0.01
15–30	0.95	0.07
30–45	0.86	0.17
45–60	0.74	0.24
60–75	0.59	0.26
75–90	0.42	0.21

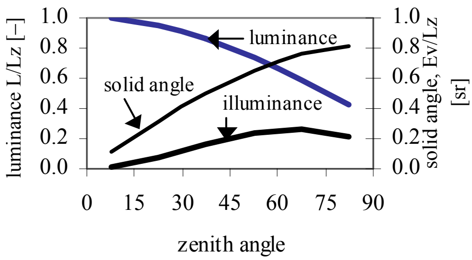


Table 8.6 and Figure 8.14: Reduction in luminance, and increase in the solid angle as well as in the vertical illuminance based on Equation (8.28), with the zenith angle.

The maximum of the vertical illuminance, here standardised on the zenith luminance L_z, comes from the zenith angle interval of 60–75°. The total of the vertical illuminance over all zenith angle areas results in

$$\sum \frac{E_v\big|_{\theta_{z,1}}^{\theta_{z,2}}}{L_z} = 0.986 = \pi\left(\frac{1}{6}+\frac{4}{9\pi}\right)$$

so again the result from Equation (8.26) is obtained. The angle of incidence is, based on Equation (8.25), given by $\cos\theta_r = \sin\theta_z \cos\gamma$, it is thus not constant for a given zenith angle interval, as the azimuth γ varies from $-\pi/2$ to $+\pi/2$ respectively. The average angle of incidence onto the vertical surface falls from 83.7° in the zenith angle interval of 0–15° to 51.3° for the interval of 75–90°.

When light passes through the window, the curve shifts to even higher zenith angles, since the reflection losses for steeply incident light are large. As a rule of thumb for the design of window openings it follows that for sufficient illumination in depth, at least the lower 30° of the sky should be seen. After all, from these zenith angle intervals (60–90°) originate (0.21 + 0.26)/0.986 = 0.48, i.e. 48% of the entire illuminance of the sky hemisphere.

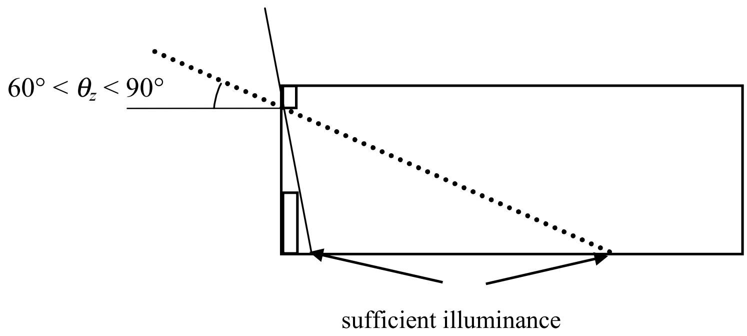


Figure 8.15: Sufficient illuminance of a room.

High-positioned windows without obstructions therefore result in good depth illumination. Narrow, high windows up to the ceiling are, photometrically, clearly better than broad strip windows. With horizontal skylights the illuminance is calculated from different zenith angle areas using Equation (8.21):

$$E_{r,h}\Big|_{\theta_{z,1}}^{\theta_{z,2}} = \frac{2\pi}{3} L_Z \left(\frac{1}{2}\left(\sin^2\theta_{z,2} - \sin^2\theta_{z,1}\right) - \frac{2}{3}\left(\cos^3\theta_{z,2} - \cos^3\theta_{z,1}\right) \right)$$

The maximum of the horizontal illuminance standardised on the zenith luminance is now in the zenith angle area 30–45°. From the zenith angle area 0–30° comes a total of 31% of the entire illuminance.

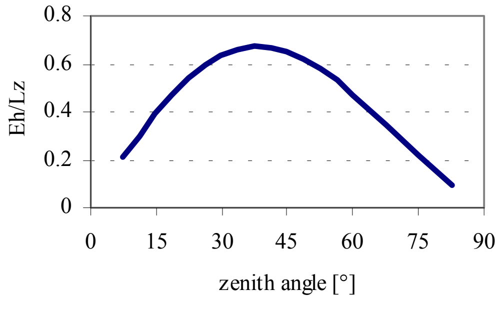


Figure 8.16: The quantity of light striking the horizontal glazing with an overcast sky, depending on the zenith angle.

Illuminating a room with skylights is sufficient if no more than a zenith angle area of 0–30° is cut off.

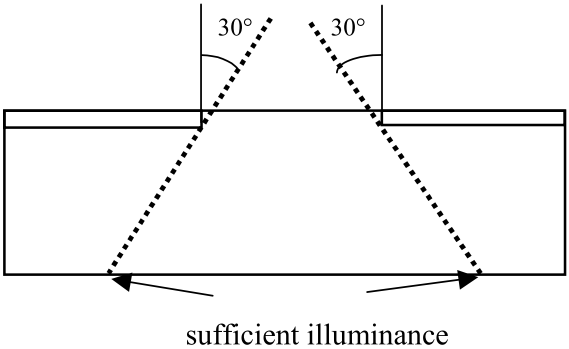


Figure 8.17: Sufficient lighting of a room using skylights.

8.6.1 Calculation of daylight coefficients

The daylight coefficient D as the relation of interior illuminance to exterior illuminance based on Equation (8.1) is defined as standard for two measuring points at a height of 0.85 m, and 1 m away from the side walls. The daylight coefficient consists of a skylight proportion D_{sky}, a proportion of diffuse reflection of shading obstructions D_{sh} and an interior reflection proportion D_r.

$$D = D_{sky} + D_{sh} + D_r \tag{8.29}$$

Since about 1920, graphic methods have been used to determine the proportion of the sky D_{sky} seen from the work surface and of the shaded proportion D_{sh}, which only reflects light. The so-called Waldram diagram contains the projection of the sky dome onto a horizontal surface and takes into account the luminance increase to the zenith. Shading buildings obstructing the horizon are included with their solid angle and reflection coefficient. The more complex inter-reflections in the interior (interior reflection proportion D_r) were only later included in the calculation of the daylight coefficient and are today usually calculated in a simplified way by the so-called "split flux method", in which only sky light reflections from the interior floor and the lower wall sections plus reflection of ground reflected light from the ceiling and the upper wall sections are regarded separately.

The daylight coefficient is determined first in dependence on the room geometry from the raw dimensions of the window openings (index r) and afterwards multiplied by light-reducing factors (window transmittance τ, a framework proportion factor k_1, a dirt factor k_2 and a correction factor for non-vertical incident k_3.).

$$D = \left(D_{sky,r} + D_{sh,r} + D_{r,r}\right)\tau k_1 k_2 k_3 \tag{8.30}$$

For the calculation of the sky light proportion seen from the point of reference P, first the effective elevation angle α_w of the window's upper edge and the lateral delimitations of the window by a left and right azimuth angle γ_{wl} and γ_{wr} have to be determined.

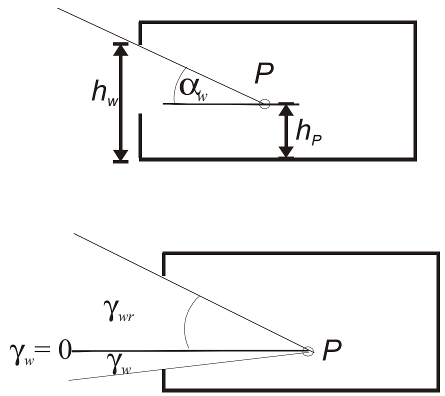


Figure 8.18: Geometrical characteristics of the window in cross-section and plane view.

To determine the effective elevation angle α_w, first determine the maximum elevation angle $\alpha_{w,\max}$ from window height h_w (upper edge) and the shortest distance between the window and point of observation, and then calculate the lateral reduction of the elevation angle with the azimuth γ.

$$\alpha_w = \arctan\left(\tan\alpha_{w,\max}\cos\gamma\right) \tag{8.31}$$

To take obstructions into account, the obstruction elevation angles $\alpha_{sh}(\gamma)$ must be known as a function of the azimuth. The illuminance on a horizontal surface in the unshaded exterior is $E_{h,o} = L_Z\pi 7/9$, based on Equation (8.27). The illuminance on a horizontal surface in the interior can likewise be calculated, taking into account the reduced height and azimuth angles of the window:

$$E_{h,i} = \frac{L_z}{3}\left(\int_{-\pi/2}^{\pi/2} d\gamma \left(\frac{1}{2}\sin^2(\theta_z)\right)\Bigg|_{\theta_{z,1}}^{\theta_{z,2}} + \int_{-\pi/2}^{\pi/2} d\gamma \left(-\frac{2}{3}\cos^3(\theta_z)\right)\Bigg|_{\theta_{z,1}}^{\theta_{z,2}}\right) \tag{8.32}$$

The zenith angle is replaced by the window height angle $\alpha_w = \pi/2 - \theta_z$, with

$$\cos(\theta_z) = \sin(\alpha_w),\quad \sin(\theta_z) = \cos(\alpha_w) \quad \text{and} \quad \sin^2(\theta_z) = \cos^2(\alpha_w) = 1 - \sin^2(\alpha_w).$$

The delimitation angles for the luminance integral are the left and right azimuth angle of the window opening γ_{wl} and γ_{wr}, the lower elevation angle is the obstruction elevation angle α_{sh} (corresponding to the larger zenith angle $\theta_{z,2}$) and the upper elevation angle is the window height angle α_w (corresponding to $\theta_{z,1}$).

$$E_{h,i} = \frac{L_z}{3}\left(\int_{\gamma_{wl}}^{\gamma_{wr}} \left(\left(\frac{1}{2}\left(\sin^2\left(\theta_{z,2}\right)-\sin^2\left(\theta_{z,1}\right)\right)\right)-\frac{2}{3}\left(\cos^3\left(\theta_{z,2}\right)-\cos^3\left(\theta_{z,1}\right)\right)\right)d\gamma\right)$$
$$= \frac{L_z}{3}\left(\int_{\gamma_{wl}}^{\gamma_{wr}} \left(\left(\frac{1}{2}\left(-\sin^2\left(\alpha_{sh}\right)+\sin^2\left(\alpha_{w}\right)\right)\right)-\frac{2}{3}\left(\sin^3\left(\alpha_{sh}\right)-\sin^3\left(\alpha_{w}\right)\right)\right)d\gamma\right) \quad (8.33)$$

Thus the daylight coefficient results in:

$$D_{sky,r} = \frac{E_{h,i}}{E_{h,o}}$$
$$= \frac{\frac{L_z}{3}\left(\int_{\gamma_{wl}}^{\gamma_{wr}} \left(\left(\frac{1}{2}\left(-\sin^2\left(\alpha_{sh}\right)+\sin^2\left(\alpha_{w}\right)\right)\right)-\frac{2}{3}\left(\sin^3\left(\alpha_{sh}\right)-\sin^3\left(\alpha_{w}\right)\right)\right)d\gamma\right)}{L_z\frac{7}{9}\pi} \quad (8.34)$$
$$= \frac{3}{7\pi}\int_{\gamma_{wl}}^{\gamma_{wr}}\left(\frac{2}{3}\left(\sin^3\alpha_w(\gamma)-\sin^3\alpha_{sh}(\gamma)\right)+\frac{1}{2}\left(\sin^2\alpha_w(\gamma)-\sin^2\alpha_{sh}(\gamma)\right)\right)d\gamma$$

The externally reflected proportion $D_{sh,r}$ results as a function of the obstruction angles and the reflection coefficient ρ_{sh} of the obstruction (typically 20%), by integration from the elevation angle $\alpha = 0$ up to the obstruction elevation angle α_{sh} as well as over the azimuth angles of the obstruction $\gamma_{sh,l}$ and $\gamma_{sh,r}$. In the German standard DIN 5034 the external reflection proportion is reduced at a flat rate by a factor of 0.75.

$$D_{sh,r} = 0.75\rho_{sh}\frac{3}{7\pi}\int_{\gamma_{sh,l}}^{\gamma_{sh,r}}\left(\frac{2}{3}\sin^3\alpha_{sh}+\frac{1}{2}\sin^2\alpha_{sh}\right)d\gamma \quad (8.35)$$

The interior reflection proportion calculated using the split flux method is calculated depending on the surface-weighted reflection coefficient of the floor and of the wall lower part ρ_{fw} (without window walls, wall lower part to height of window centre) as well as on the corresponding surface-weighted reflection coefficient of the ceiling and wall upper sections ρ_{cw} (likewise without window walls). In contrast, the average reflection coefficient of the room $\bar{\rho}$ includes all walls.

$$D_{r,r} = \frac{\sum b_w h_w}{A_{room}}\frac{\bar{\rho}}{1-\bar{\rho}^2}\left(f_{up}\rho_{fw}+f_{low}\rho_{cw}\right) \quad (8.36)$$

A_{room}: total room confinement surface [m^2]
b_w, h_w: Window width and height [m]

The upper window factor f_{up} describes the integrated luminance of the Moon and Spencer sky model on the vertical surface, depending on an average obstruction angle α

(obstruction elevation angle in arc measure, measured from the window centre). The lower window factor f_{low} takes into account the diffuse radiation reflected by the floor.

$$\begin{aligned} f_{up}(\alpha) &= 0.3188 - 0.1822\sin\alpha + 0.0773\cos(2\alpha) \\ f_{low}(\alpha) &= 0.03286\cos\alpha' - 0.03638\alpha' + 0.01819\sin(2\alpha') + 0.06714 \end{aligned} \tag{8.37}$$

with $\alpha' = \arctan(2\tan\alpha)$.

Example 8.6

Calculation of the daylight coefficient of a side-illuminated room without obstructions, with window transmittance $\tau = 0.65$, a glazing proportion of 80%, a dirt factor $k_2 = 0.9$ (low contamination) and a factor $k_3 = 0.85$ to take account of the non-vertical incidence angle of the irradiance.

Room geometry:

Width B:	4 m
Depth T:	6 m
Height H:	3 m
Height of window upper edge h_w:	2.5 m
Height of window bottom edge h_{wb}:	0.85 m
Width of window b_w:	4 m

Reflection coefficients of the surfaces:

ρ_{floor}:	0.3
$\rho_{ceiling}$:	0.7
ρ_{wall}:	0.5
$\rho_{windows}$:	0.15

From this results a surface-weighted reflection degree of

$$\begin{aligned} \overline{\rho} &= \frac{\rho_{floor}A_{floor} + \rho_{ceiling}A_{ceiling} + \rho_{wall}A_{wall} + \rho_{window}A_{window} + \rho_{wall}A_{wall,window}}{\sum A} \\ &= \frac{0.3\times 24m^2 + 0.7\times 24m^2 + 0.5\times 48m^2 + 0.15\times 6.6m^2 + 0.5\times 5.4m^2}{108m^2} = 0.48 \end{aligned}$$

Without obstructions the window factors are $f_{up} = 0.3961$ and $f_{low} = 0.1$. The interior reflection proportion thus becomes $D_{r,r} = 0.008$, less than 1%!

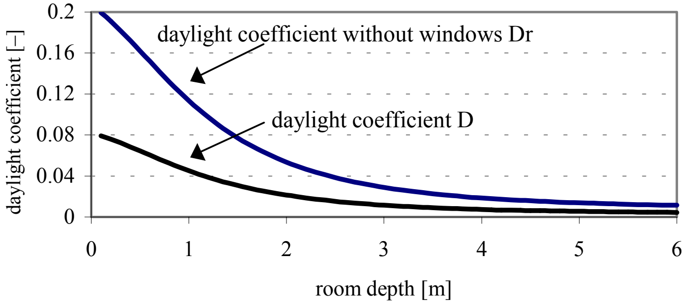


Figure 8.19: Daylight coefficient of the side-illuminated room.

If the reflection coefficient of the walls is increased to 0.7, the interior reflection proportion rises to 1.28%. Raising the window's upper edge to room height (3 m) increases the interior reflection proportion further to 1.7%.

If the reduction in the daylight coefficient by transmittance, framework proportion etc. is taken into account (factor 0.4!), at a depth of 4 m a daylight coefficient of about 1% is obtained.