Unsupervised Image Segmentation Using Markov Random Fields

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Abstract. In this study, we carried out an unsupervised gray level image segmentation based on Markov Random Fields (MRF) model. First, we use the *Expectation Maximization* (EM) algorithm to estimate the distribution of the input image and the number of the components is automatically determined by the *Minimum Message Length* (MML) algorithm. Then the segmentation is done by the Iterated Conditional Modes (ICM) algorithm. For testing the segmentation performance, we use both artificial images and real images. The experimental results are satisfactory.

1 Introduction

Segmentation is an important process in digital image processing which has found extensive applications in several areas. It aims to find the homogeneous regions for labeling objects and background. In other words, image segmentation is the process of grouping pixels of a given image into regions with respect to certain features and with semantic content [1]. Numerous segmentation methods have been proposed in the research literature. Thresholding is the most popular approach [2-3]. Clustering methods [4], region growing and splitting methods [5] and multi resolution [6] techniques are the other proposed approaches.

In this study, we use Markov Random Fields (MRF) for segmenting gray level images without supervision. With a seminal paper by *Geman and Geman* [7], MRF has been introduced to image processing and the computer vision community. MRF is a powerful tool to model the joint probability distribution of the image pixels in the terms of local spatial interactions [8]. *Besag* [9] proposed a method, called the Iterated Conditional Modes (ICM), which uses the local MRF's for segmentation of the true image from the noisy one. Several last decades, MRF is used for color image segmentation [10]. *Kato et al.* used MRF for segmenting the color images [11]. In ref. 11, a MRF based segmentation schema, which can perform better solutions even noisy images, is proposed. The algorithm only needs the number of classes as a priori. Several researchers also use the MRF for refining the coarse segmentation [12].

In this paper, we use MRF for segmentation of the gray level images when there is no prior information about the model parameters. Many methods, assume that the number of classes is known in advance. Here, we use Expectation Maximization (EM) algorithm for parameter estimation [13]. We also use Minimum Message

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Length (MML) for estimating the number of classes [14]. The experimental results are satisfactory. The rest of the paper is organized as follow: in section 2, we review the MRF and ICM algorithms briefly. In section 3, EM and MML algorithms are given. In section 4 we discuss experimental result. In section 5, finally we conclude the paper.

2 MRF for Image Segmentation

This section introduces the general framework to MRF image analysis and gives a brief overview of the MRF theory. MRF is n-dimensional random process defined on a discrete lattice. Usually the lattice is a regular 2-dimensional grid in the plane [7]. A random field can be considered as a MRF, if its probability distribution at any site depends only upon its neighborhood [8]. According to the Cliff-Hammersley theorem, any MRF can be described by a probability distribution of the Gibbs form:

$$p(x) = \frac{1}{Z} e^{(-U(x))} \tag{1}$$

Where x is the random field, Z is the normalization constant and the energy function $U(x)$ is defined as;

$$U(x) = \sum_{c \in C} V_c(x) \tag{2}$$

5	4	3	4	5
4	2	1	2	4
3	1	t	1	3
4	2	1	2	4
5	4	3	4	5

(a)

t:-11	t:-7	t:-6	t:+8	t:+12
t:-9	t:-3	t:-2	t:+4	t:+10
t:-5	t:-1	t	t:+1	t:+5
t:-10	t:-4	t:+2	t:+3	t:+9
t:-12	t:-8	t:+6	t:+7	t:+11

(b)

Fig. 1. Definition of Neighborhoods and Relative Neighborhoods

Where $V_c(x)$ is the potential function. We assume that the image is defined on an *MxN* rectangular lattice $L = \{(i, j), 1 \le i \le M, 1 \le j \le N\}$ and c is a set of pixels, called a clique that consists of either a single pixel or a group of pixels. Figure 1-(a), demonstrates the first-order spatial neighbors of a site t as 1, second order neighbor as 2 so on and figure 1-(b) provides a convenient labeling for neighbors of each pixel. The image observed is denoted by the *MN*-vector random variable Y and is obtained by adding a noise process to the true image. Therefore, the density model for Y given the true image is;

$$f(y \mid X = x) = \prod_{t=1}^{MN} f_t(y_t \mid x_t) \tag{3}$$

Note that $f_t(.\mid x_t)$ is the conditional density function for Y_t, the gray level at pixel *t*. We take $f_t(y_t \mid x_t)$ to be the Gaussian density function with mean μ_{x_t} and standard deviation σ_{x_t}. The standard deviation depends on the both noise and the gray level variation of each image label. The a posteriori probability mass function for the pixel labels *X*, given the observed image *Y=y* also has the form of a Gibbs random fields respect to a neighborhood system cliques.

$$P(X = x \mid Y = y) = \frac{e^{-U(x|y)}}{Z} \tag{4}$$

Where Z is the normalizing constant and the energy function is as follows;

$$U(x \mid y) = \sum_{t=1}^{MN} [\frac{1}{2}\ln(\sigma_{x_t}^2) + \frac{(y_t - \mu_{x_t})^2}{2\sigma_{x_t}^2} + \sum_{r=1}^{c} [\theta_r, J(x_t, x_{t:+r})]] \tag{5}$$

Where *J (a, b)= -1* if *a=b, 0* if *a ≠ b* and *c=2* for first-order neighbor model. $[\theta_1,...,\theta_c]$ are the clique parameters. The local properties of an MRF can be derived from Gibbs random fields. Let $X_{\partial t}$ be a random variable presenting the gray level of neighbor of pixel *t* denoted by $[x_{t:+r}\ x_{t:-r}]$ for *r* from *1* to *c*. The conditional probability of X_t can be written as [15];

$$P(X_t = x_t \mid X_{\partial t} = x_{\partial t}, Y = y) = \frac{e^{-U_t(x_t, x_{\partial t}|y)}}{Z} \tag{6}$$

and

$$U(x_t, x_{\partial t} \mid y) = \sum_{t=1}^{MN} [\frac{1}{2}\ln(\sigma_{x_t}^2) + \frac{(y_t - \mu_{x_t})^2}{2\sigma_{x_t}^2} + \sum_{r=1}^{c} \theta_r, [J(x_t, x_{t:+r}), J(x_t, x_{t:-r})] \tag{7}$$

Now the segmentation problem is considered as observing y and estimating the labels in the true image. The Maximum A-Posteriori (MAP) estimate is the vector x' which maximizes *P(X=x | Y=y)* with respect to true image *x*.

2.1 Iterated Conditional Modes (ICM)

ICM is an optimization method. Besag [8] proposed the ICM method as a computationally feasible alternative to MAP. In ICM, all sites are visited iteratively without restriction where the label that yields the maximum a posterior probability accepted as the estimate for the site. It is motivated for reducing the computational time produced by using the stochastic techniques such as Gibbs sampler. The ICM method can be summarized by the following equation where the label of the pixel *t,* given the observed image y and the current estimates $x_{\partial t}$ of the labels of all pixels in the neighborhood of pixel *t*.

$$P(X_t = x_t \mid y, X_{S|r} = x_{S|r}) = f_t(y_t \mid x_t) P(X_t = x_t \mid X_{\partial t} = x_{\partial t}) \tag{8}$$

Maximizing the conditional probability in eq. (8) is equivalent to minimizing the energy function which is given in eq. (7). The ICM algorithm can be represented as follows;

Step 1: Initialize x' by maximizing $f_t(y_t \mid x_t)$ for all pixels.

Step 2: For t=1 to MN, update $x_t^{'}$ to the value of x_t which maximizes energy function in eq. (7).
Step 3: Go to the step 2 for N times.

3 Parameter Estimation with EM Algorithm

Our goal is to segment the observed image using an unsupervised classification algorithm. For estimating the probability distributions of the labels in the observed image, we need to estimate the mean μ_{x_t} and the variance $\sigma_{x_t}^2$ of the each class label. There is no prior information so we can not use maximum likelihood approach for estimating the parameters of the probability distributions of the each class. In statistics, this problem is called as the incomplete data problem [16]. EM algorithm, which has been proposed by Dempster et al., aims to find these parameters [13]. EM algorithm consists of an E-step and an M-step and it starts with initial values $p_m^0, \mu_m^0 \; and \; \sigma_m^0$ for the parameters and iteratively performs these two steps until convergence. Suppose that ^{t} denotes the estimation of obtained after the *t* th iteration of the algorithm. Then at the *(t+1)* th iteration the E-step computes the expected complete log-likelihood function;

$$Q(\theta,\theta^t) = \sum_{k=1}^{K}\sum_{m=1}^{M}\{\log \alpha_m p(x_k \mid \theta_m)\}P(m \mid x_k;\theta^t) \tag{9}$$

where $P(m \mid x_k;\theta^t)$ is a posterior probability and it is computed as follows;

$$P(m \mid x_k;\theta^t) = \frac{\alpha_k^t p(x_k \mid \theta_m^t)}{\sum_{l=1}^{M} \alpha_l^t p(x_k \mid \theta_l^t)} \tag{10}$$

The M-step finds the *(t+1)* estimation $^{t+1}$ of by maximizing Q (, ^{t})

$$\alpha_m^{t+1} = \frac{1}{K}\sum_{k=1}^{K} P(m \mid x_k;\theta^t) \tag{11}$$

$$\mu_m^{t+1} = \frac{\sum_{k=1}^{K} x_k P(m \mid x_k;\theta^t)}{\sum_{k=1}^{K} P(m \mid x_k;\theta^t)} \tag{12}$$

$$\sigma_m^{t+1} = \sqrt{\frac{1}{D}\frac{\sum_{k=1}^{K} P(m \mid x_k;\theta^t) \| x_k - \mu_m^{t+1} \|^2}{\sum_{k=1}^{K} P(m \mid x_k;\theta^t)}} \tag{13}$$

3.1 MML Algorithm

Main issue of using MRF model in image segmentation is the difficulty to estimate the number of components. Many methods, such as the method which Kato et al. proposed, assume the number of components is determined by the users. In this study we use the MML [14] criterion for overcoming this situation. For an M-components mixture model, the number of M can be detected where the $\arg\min L(\theta, y)$. The function of L is defined as follows;

$$L(\theta, y) = \frac{N}{2}\sum_{m=1}^{M} \log(\frac{n\alpha_m}{12}) + \frac{M}{2}\log(\frac{n}{12}) + \frac{M(N+1)}{2} - \sum_{k=1}^{K} \log \sum_{m=1}^{M} \alpha_m p(x_k \mid \theta_m) \tag{14}$$

where N is the number of parameters specifying each component, n is the number of pixels and M is the number of components. and are the estimated parameters from the EM algorithm. In this study N is chosen 2.

4 Experimental Study and Results

In this study, an unsupervised gray level image segmentation schema is proposed based on ICM algorithm. The parameters of the components are estimated by using the EM algorithm and the number of the components is determined with MML criterion so the overall process is unsupervised. For testing the performance of the unsupervised segmentation algorithm, we generate several artificial gray level images of size 128x128 on MATLAB environment. These images can be seen at figure 2. The images are generated as they constitute of three, four, five and six classes respectively. Before running the ICM algorithm for segmentation, the unknown parameters such as number of classes and mean and variance of the components are obtained. Firstly, we assume that the number of components M is a fixed number such as 2. According to this assumption we employ EM algorithm thus the component parameters can be determined. The initial values for EM algorithm are selected randomly. These values are then used for calculation the L value in MML algorithm. Then we increase the M and we run the aforesaid process again. In this study the maximum value of the M is restricted 7. The component parameters of the artificial images, which are estimated, are represented in the following tables. Table 1 shows the estimated parameters of artificial image which is labeled as (a) in figure 2. The actual parameter values for the each component is μ_{c1}= 10 $_{c1}$=5, μ_{c2}= 100 $_{c2}$=25 and μ_{c3}= 180 $_{c3}$=45.

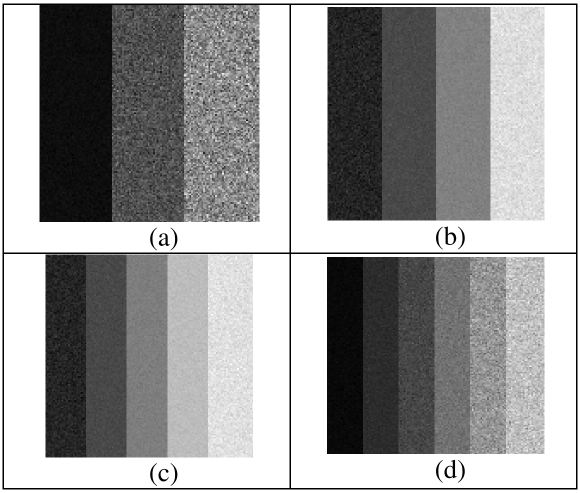

Fig. 2. Artificial images (a) 3 classes (b) 4 classes (c) 5 classes (d) 6 classes

Table 1. L values for image (a)

Number of component	$L(\theta, y)$
M=2	8.6140e+004
M=3	8.5493e+004
M=4	8.5505e+004
M=5	8.5531e+004
M=6	8.5533e+004
M=7	8.5540e+004

Table 2 shows the estimated parameters of artificial image which is labeled as (b) in figure 2. The actual parameter values for the each component is μ_{c1}= 20 $_{c1}$=10, μ_{c2}= 60 $_{c2}$=5, μ_{c3}= 120 $_{c3}$=10 and μ_{c4}= 220 $_{c4}$=5.

Table 2. L values for image (b)

Number of component	$L(\theta, y)$
M=2	9.1350e+004
M=3	8.1330e+004
M=4	7.7953e+004
M=5	7.7966e+004
M=6	7.7981e+004
M=7	7.7995e+004

Table 3 shows the estimated parameters of artificial image which is labeled as (c) in figure 2. The actual parameter values for the each component is $\mu_{c1}=20$ ${}_{c1}=5$, $\mu_{c2}=60$ ${}_{c2}=10$, $\mu_{c3}=120$ ${}_{c3}=5$, $\mu_{c4}=190$ ${}_{c4}=5$ and $\mu_{c5}=230$ ${}_{c5}=10$.

Table 3. L values for image (c)

Number of Component	$L(\theta, y)$
M=2	9.0908e+004
M=3	8.6061e+004
M=4	8.3381e+004
M=5	8.0629e+004
M=6	8.0642e+004
M=7	8.0651e+004

Table 4. L values for image (d)

Number of Component	$L(\theta, y)$
M=2	9.2028e+004
M=3	9.1309e+004
M=4	9.1191e+004
M=5	9.1005e+004
M=6	8.7400e+004
M=7	8.7414e+004

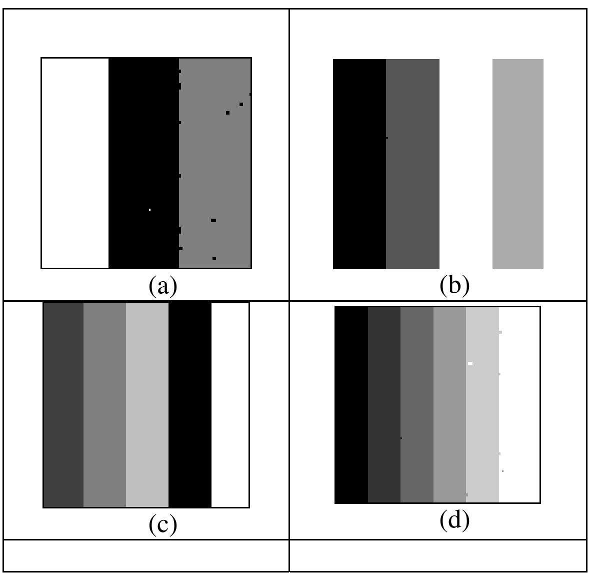

Fig. 3. ICM segmentation results

And finally, Table 4 shows the estimated parameters of artificial image which is labeled as (d) in figure 2. The actual parameter values for the each component is μ_{c1}= 20 $_{c1}$=3, μ_{c2}= 60 $_{c2}$=5, μ_{c3}= 90 $_{c3}$=10, μ_{c4}= 140 $_{c4}$=10, μ_{c5}= 180 $_{c5}$=20 and μ_{c6}= 220 $_{c6}$=20.

While using the ICM algorithm for segmentation of the gray level images, we use first-order spatial neighbors and we choose Θ_r values as 2. The ICM segmentation results can be seen in figure 3.

Table 5. L values for the real image

Number of Component	$L(\theta, y)$
M=2	3.1833e+004
M=3	3.1711e+004
M=4	3.1725e+004
M=5	3.1739e+004
M=6	3.1754e+004
M=7	3.1791e+004

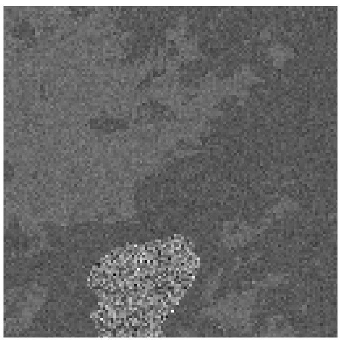

Fig. 4. A real world image

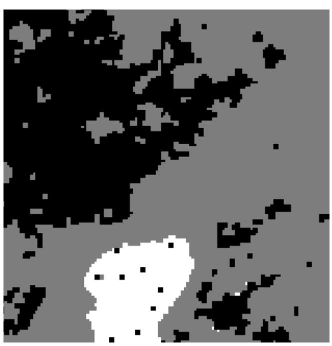

Fig. 5. Segmentation using first-order spatial neighbor

In Fig. 4, we use a real image for testing the algorithm. The image is obtained from the web site of Lund University [17]. The segmentation result of the image is seen at figure 5. In figure 6, the segmentation result of the real image is shown by using the first and the second order spatial neighbors.

Fig. 6. Segmentation using first and second order spatial neighbor

5 Conclusion

In this paper, we have examined an unsupervised gray image segmentation algorithm. The segmentation model is defined in a MRF framework. The examined algorithm is fully unsupervised because the number of the component is determined by the algorithm. No user information is needed. To estimate the component parameters, we use an iterative algorithm. EM and MML algorithms are employed for obtaining the crucial values. Then we use ICM algorithm for completing the segmentation procedure. The algorithm has been tested on a variety of artificial and real images and results are very satisfactory.

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